بسم ا الر ن الرحيم

Mechanics of Materials
Fall 2003
First Exam

MENG 270
Time: 2 hrs
13/8/1424 H

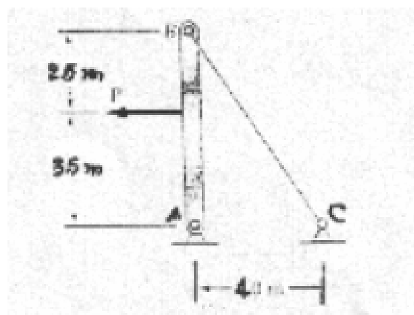
Name:			Computer No.:	
Q.1	Q.2	Q.3	Q.4	Total

Q.1 [10 points]

The 4-mm-diameter cable BC is made of steel with E = 200 GPa. Knowing that the maximum stress in the cable must not exceed 190 MPa and that the elongation of the cable must not exceed 6 mm, find the maximum load P that can be applied as shown.

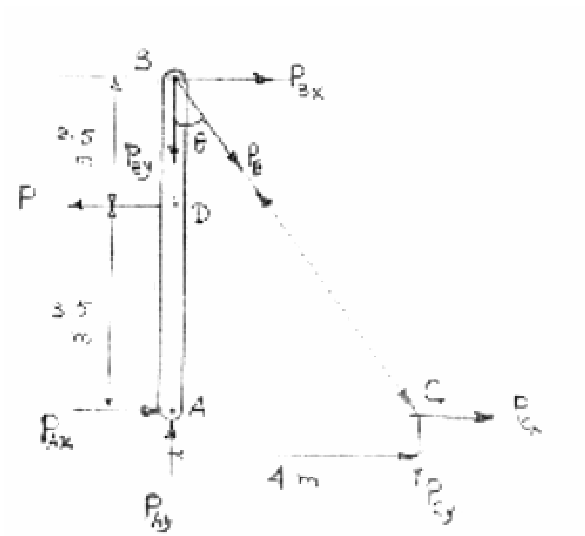
Given:

$E = 200$ GPa
$\sigma_{all} = 190$ MPa
$\delta = 6$ mm
$d_{BC} = 4$ mm
$P = ?$



Solution:

Figure shows F.B.D



Consider link AB

$\rightarrow \Sigma F_x = 0$

$$P_{BX} - P + P_{AX} = 0$$

$$P = P_{BX} + P_{AX} \text{ ------------ } (1)$$

$\uparrow \Sigma F_Y = 0$

$$P_{AY} = P_{BY} \text{ --------------- } (2)$$

$\Sigma M_D = 0$

$$2.5 P_{BX} = 3.5 P_{AX}$$

$$P_{AX} = \frac{2.5}{3.5} P_{BX} \text{ -------------- } (3)$$

Initial length of cable BC

$$L_{BC} = \sqrt{6^2 + 4^2} = 7.21 \ m$$

$$\delta l = \frac{P_B L_{BC}}{A_{BC} E_{BC}} \Rightarrow P_B = \frac{\delta l \ A_{BC} \ E_{BC}}{L_{BC}}$$

$$A_{BC} = \frac{\pi}{4} d_{BC}^2 = \frac{\pi}{4} \times \left(4 \times 10^{-3}\right)^2 = 12.6 \times 10^{-6} \ m^2$$

$$\sigma_{all} = \frac{P_B}{A_{BC}} \Rightarrow P_B = \sigma_{all} \times A_{BC}$$

$$P_B = 190 \times 10^6 \times 12.6 \times 10^{-6} = 2.3 \ kN$$

considering allowable deflection we get

$$P_B = \frac{6 \times 10^{-3} \times 12.6 \times 10^{-6} \times 200 \times 10^9}{7.21} = 2.091 \ kN$$

Take smaller value of P_B

$$\theta = \tan^{-1} \frac{4}{6}$$

$$\theta = 33.7$$

$$P_{BX} = P_B \sin\theta$$

$$P_{Bx} = 1.16 \ kN$$

substituting value of P_{Bx} *in equation* 3

$$P_{AX} = 0.82 \, kN$$

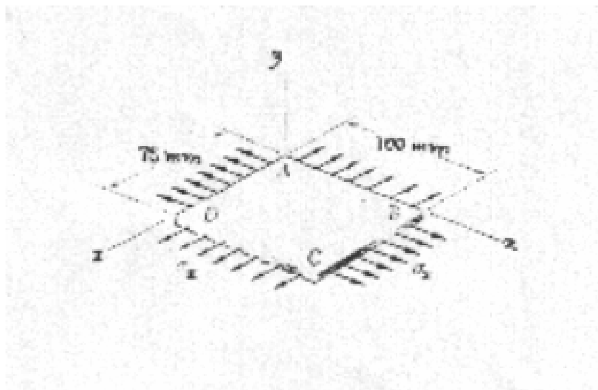
$\therefore$ *from equation* 1

$$P = 1.16 + 0.82$$

$$P = 1.988 \ kN \qquad \textit{Answer}$$

Q.2 [10 points]

A fabric used in an air-inflated structure is subjected to biaxial loading that results in normal stresses σ_x = 120 MPa and σ_z = 160 MPa. Knowing that the properties of fabric can be approximated as E = 87 GPa and υ =0.34, determine change in length of (a) side AB, (b) side BC, (c) diagonal AC.



Given:

$$\sigma_x = 120\ MPa$$
$$\sigma_z = 160\, MPa$$
$$E\ = 87\ GPa$$
$$\nu\ =\ 0.34$$
$$Change\ in\ AB, BC, AC\ = ??$$

Solution:

$$\varepsilon_x = \frac{\sigma_x}{E} - \frac{\nu\sigma_y}{E} - \frac{\nu\sigma_z}{E}$$
$$\varepsilon_y = \frac{\sigma_y}{E} - \frac{\nu\sigma_x}{E} - \frac{\nu\sigma_z}{E}$$
$$\varepsilon_z = \frac{\sigma_z}{E} - \frac{\nu\sigma_x}{E} - \frac{\nu\sigma_y}{E}$$

Subsitute the given values in the above equations

$$\varepsilon_x = \frac{120\times10^6}{87\times10^9} - 0 - \frac{0.34\times160\times10^6}{87\times10^9}$$
$$\varepsilon_x = 0.000754$$
$$\varepsilon_y = 0 - \frac{0.34\times120\times10^6}{87\times10^9} - \frac{0.34\times160\times10^6}{87\times10^9}$$
$$\varepsilon_y = 0.001$$
$$\varepsilon_z = \frac{160\times10^6}{87\times10^9} - \frac{0.34\times120\times10^6}{87\times10^9} - 0$$
$$\varepsilon_z = 0.00137$$

$$\varepsilon_x = \frac{\delta_{AB}}{L_{AB}} \Rightarrow \delta_{AB} = \varepsilon_x \times L_{AB}$$

$$\therefore \delta_{AB} = .000754 \times 100$$

$\delta_{AB} = .0754\ mm$ *Answer*

$$\delta_{BC} = \varepsilon_z \times L_{BC}$$

$\delta_{BC} = 0.10275\ mm$ *Answer*

$$L'_{AB} = L_{AB} + \delta_{AB}$$

$$L_{AB} = 100.0754$$

$$L'_{Bc} = L_{Bc} + \delta_{Bc}$$

$$L'_{BC} = 75.10275$$

$$AC = \sqrt{100^2 + 75^2}$$

$$AC' = \sqrt{100.075^2 + 75.10275^2}$$

$$\delta_{AC} = AC' - AC$$

$\delta_{AC} = 0.1216\ mm$ *Answer*

Q.3 [10 points]

Knowing that the ultimate strength of the bar shown is 300 MPa, determine the maximum allowable centric axial force P which may applied to the bar with a factor of safety 2.

Given:

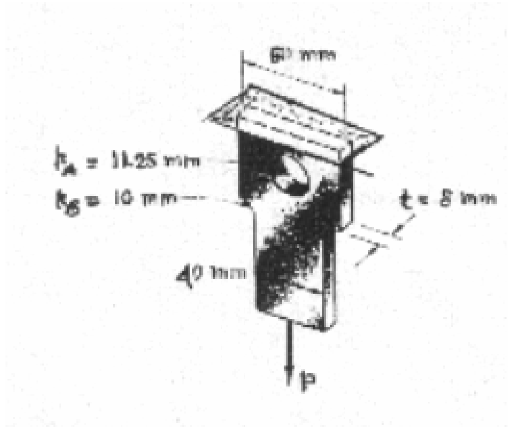
$D = 60 \;\; mm$

$d = 40 \;\; mm$

$r = 10 \;\; mm$

$\sigma_{\max} = 150 \; MPa$

$P_{all} = \; ??$



Solution:

$$\frac{D}{d} = \frac{60}{40} = 1.5$$

$$\frac{r}{d} = \frac{10}{40} = 0.25$$

$$K = 1.61 \quad (\mathit{from} \quad \mathit{figure})$$

$$K = \frac{\sigma_{\max}}{\sigma_{av}}$$

$$\sigma_{av} = \frac{\sigma_{\max}}{K} = \frac{150}{1.61} = 93.12 \quad MPa$$

$$P_{all} = \sigma_{av} \times A$$

$$P_{all} = 93.12 \times 40 \times 8 = 29 \quad kN$$

For Hole

$$\frac{r}{d} = \frac{11.25}{(60 - 22.5)} = 0.3$$

$$K = 2.35 \; (\mathit{fromfigure})$$

$$\sigma_{av} = \frac{\sigma_{\max}}{K} = \frac{150}{2.35} = 63.829 \; MPa$$

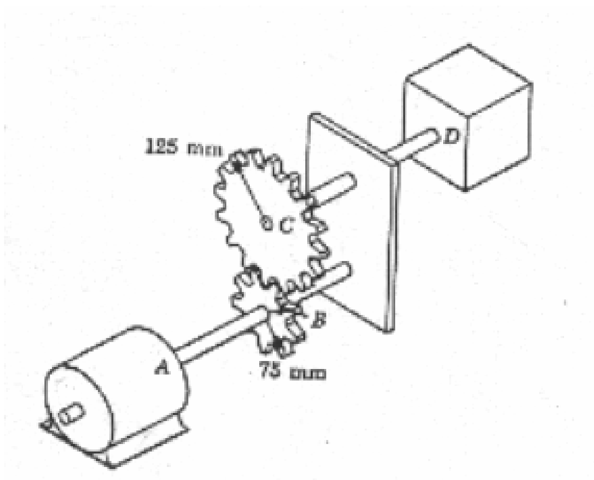
$$P_{all} = \sigma_{av} \times A = 63.829 \times 8 \times (60 - 22.5)$$

$$P_{all} = 19.148 \;\; kN$$

Answer

Q.4 [10 points]

The two solid shafts and gears shown are used to transmit 12 kW from the motor at A, which rotates at frequency of 20 Hz, to a machine tool at D. Each shaft has diameter of 25 mm, determine the maximum shear stress in each shaft. If the maximum allowable shearing stress for the shaft material is 60 MPa, what is factor of safety of the system?



Given :

$P = 12\ kW$

$f = 20\ Hz$

$d = 25\ mm$

$\tau_{AB} = ??$

$\tau_{CD} = ??$

$F.S = ??$

Solution:

$P = 2\pi fT$

$$T = \frac{P}{2\pi f} = \frac{12 \times 1000}{2\pi \times 20}$$

$T = 95.492\ N-m$

$\therefore\ T_{AB} = 95.492\ N-m$

$$\tau_{AB} = \frac{16 \times T}{\pi d^3} \quad (for\ solid\ shaft)$$

$$\tau_{AB} = \frac{16 \times 95.492}{\pi \times (25 \times 10^{-3})^3}$$

$\tau_{AB} = 31.12\ MPa$ *Answer*

Force acting on gear C

$$F_C = \frac{T}{r_B}$$

$$F_C = 1273 \quad N$$

Torque on gear C

$$T_C = F_C \times r_C = 1273 \times 125$$

$$T_C = 159125 \quad N-mm$$

$$\therefore T_{CD} = 159.125 \quad N-m$$

$$\tau_{CD} = \frac{16T}{\pi d^3}$$

$$\tau_{CD} = 51.867 \quad MPa \qquad Answer$$

Maximum shear stress is 51.867 MPa

$$F.S = \frac{\tau_{all}}{\tau_{\max}}$$

$$F.S = \frac{60}{51.867}$$

$$F.S = 1.15 \qquad Answer$$