بسم الله الرحمن الرحيم

Mechanics of Materials
MENG270
Second Exam

Closed Book Exam
Time: 1 ½ hrs
Thursday: 12/9/1424 H

Student Name:								Number:	
Q1:	10 /10	Q2:	10 /10	Q3:	10/10	Q4:	10 /10	Total:	40 /40

1. An aluminum alloy beam with the cross section shown in Figure 1 experiences positive bending by an applied moment M. The allowable stress is 150 MPa. Determined:

a) The maximum moment that can be applied to the beam.
b) Stresses at point A, B, and C when maximum moment is applied.

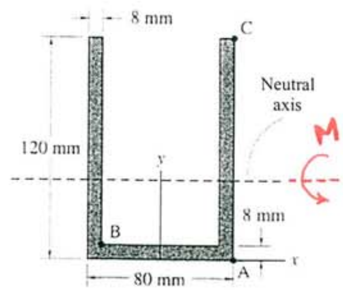


Figure 1

The centriod of the cross section is:

$$\bar{y} = \frac{\sum_i \bar{y}_i A_i}{\sum A_i} = \frac{4(64)(8) + 2(60)(8)(120)}{8(80-16+240)} \Longrightarrow \boxed{\bar{y} = 48.21 \text{ mm}}$$

The distances from the neutral axis to the centroids of the horizontal and vertical bars are: $d_{n1} = 48.21 - 4 = 44.21$ mm

$d_{n2} = 60 - 48.21 = 11.79$ mm

The area moment of inertia of the composite structure is:

$$I = I_1 + A_1 d_{n1}^2 + 2(I_2 + A_2 d_{n2}^2)$$
$$= \frac{64(8)^3}{12} + (64)(8)(44.2)^2 + 2\left[\frac{8(120)^3}{12} + 8(120)(11.79)^2\right] = 357.4 \text{ cm}^4$$

The distances from the neutral axis to A, B, and C are:

$d_{nA} = 48.21$ mm , $d_{nB} = 48-21-8 = 40.21$ mm

$d_{nC} = 120 - 48.21 = 71.79$ mm

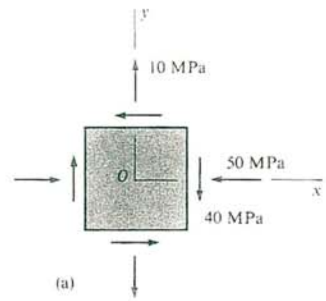
$\therefore$ Maximum Moment: $M_{max} = \frac{\sigma_{all} I}{d_{nC}} = \frac{150\times10^6(357.4\times10^{-8})}{(71.79\times10^{-3})} = 7468$ N.m

b) $\sigma_A = \frac{M_{max} d_{nA}}{I} = -\frac{7468(48.21\times10^{-3})}{357.4\times10^{-8}} = -100.7$ MPa.

$\sigma_B = \frac{M_{max} d_{nB}}{I} = -\frac{7468(40.21\times10^{-3})}{357.4\times10^{-8}} = -84$ MPa, $\sigma_c = 150$ MPa

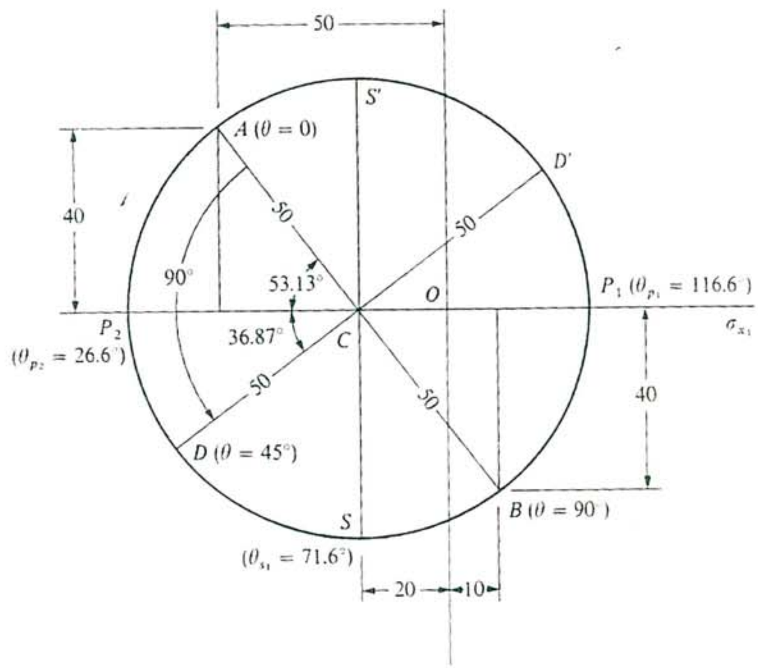
2. An element in plane stress is subjected to stresses as shown in Figure 2. Using Mohr's circle, determine:

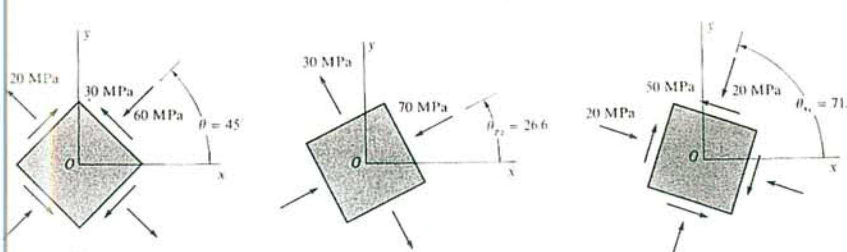
a) The stresses acting on an element oriented at an angle $\theta = 45^\circ$.
b) The principal stresses.
c) The angle of the principal stresses.
d) The maximum and minimum shear stresses.
e) The angle of the maximum shear stress.



(a)

Figure 2





3. The simply supported beam shown in Figure 3a has a span length L=3 ft. The cross section is rectangular with width 1 in. and height 4 in. (Figure 3b). The total uniform load on the beam (including its weight) is q=160 Ib/in. Calculate the following:

a) The reaction loads acting at point A and point B.
b) The bending moment M and shear force V at the cross section through point C.
c) The normal stresses acting at point C.
d) The shear stresses acting at point C.
e) On an element, show the directions of the normal and shear stresses.

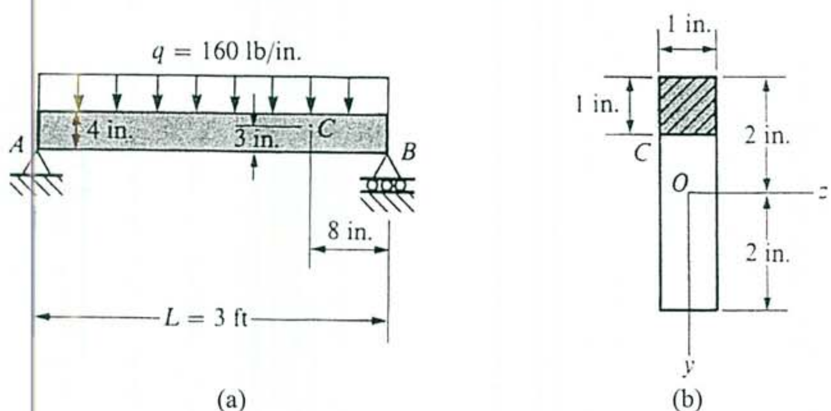


Figure 3

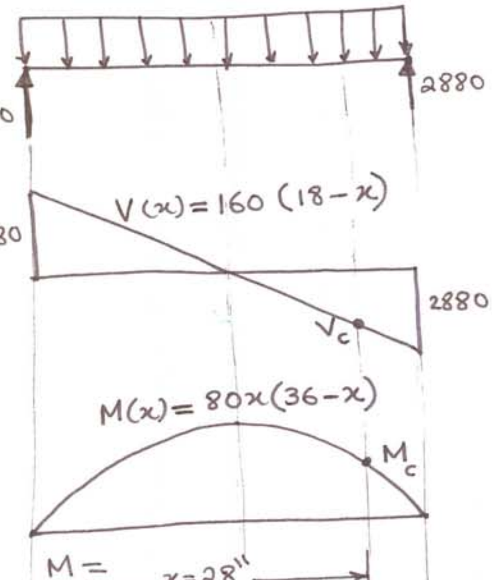
a)

$\Sigma F_x = 0 \Rightarrow A_x = 0 \qquad \Sigma F_y = 0 \Rightarrow A_y + B_y = 160(3\times 12) = 5760 \text{ Ib}$

$\Sigma M_A = 0 \Rightarrow B_y = \dfrac{5760(18)}{36} \Rightarrow \boxed{B_y = 2880 \text{ Ib}} \ \& \ \boxed{A_y = 2880 \text{ Ib}}$

b) $V_c = 160(18-28) \Rightarrow \boxed{V_c = -1600 \text{ Ib}}$

$M_c = 80x(36-x)\Big|_{x=28} \Rightarrow \boxed{M_c = 17{,}920 \text{ Ib.in}}$



c) $I = \dfrac{bh^3}{12} = \dfrac{1}{12}(1")(4")^3 = 5.333 \text{ in}^4$

$\sigma_x = \dfrac{My}{I} = \dfrac{17{,}920(-1)}{5.333} = -3{,}360 \text{psi}$

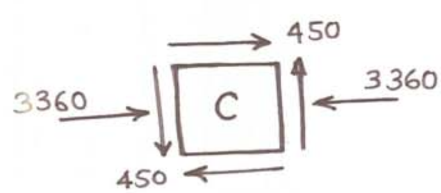
$\therefore \boxed{\sigma_x = -3{,}360 \text{ psi}}$

d) $Q = (1)(1)(1.5) \Rightarrow \boxed{Q = 1.5 \text{ in}^3}$

$\tau_c = \dfrac{VQ}{Ib} = \dfrac{(1600)(1.5)}{(5.333)(1)}$

$\boxed{\tau_c = 450 \text{ psi}}$

e)



4. Using the SINGULARITY function, determine the shear force and bending moment equations for the beam shown in Figure 4 and plot the results.

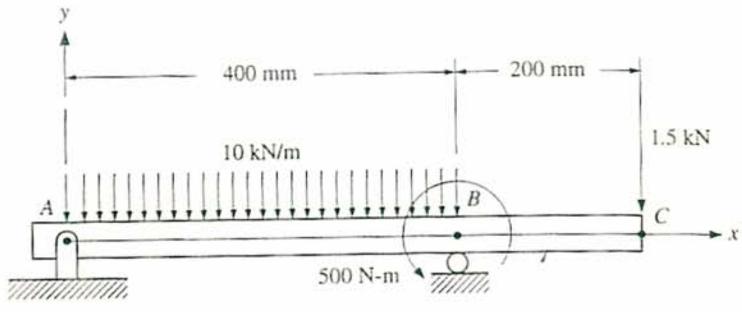


Figure 4

$\sum F_y = 0$,

$A_y + B_y = 5.5$ KN

$\sum M_A = 0$,

$-4*200 + B_y*400 - 1.5*600 + 500 = 0$

Hence, $B_y = 3$ KN and $A_y = 2.5$ KN .

$V = 2.5 < x >^0 - 10 < x >^1 + 10 < x - 0.4 >^1 + 3 < x - 0.4 >^0$

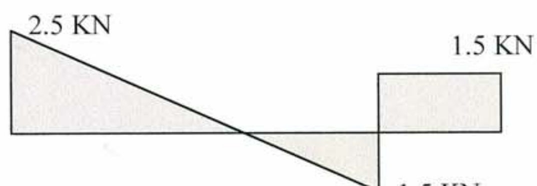
Now $M = \int V dx$,

Then, $M = 2.5 < x >^1 - 5 < x >^2 + 5 < x - 0.4 >^2 + 3 < x - 0.4 >^1 - 0.5 < x - 0.4 >^0$

For $0 \le x \le 0.4$,

$V = 2.5 - 10x$

$M = 2.5x - 5x^2$



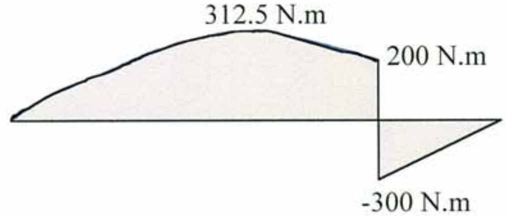
Shear Force Diagram

For $0.4 \le x \le 0.6$,

$V = 2.5 - 10x + 10(x - 0.4) + 3 = 1.5$ KN

$M = 2.5x - 5x^2 + 5(x - 0.4)^2 + 3(x - 0.4) - 0.5$

or, $M = 1.5x - 0.9$



Bending Moment Diagram

مع تمنياتي لكم بالتوفيق

وشهركم مبارك

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