Mechanics of Materials		MENG 270
Fall 2003	Exam 3	Time allowed: 90min

Name.				Computer No.	
Q.1(a)	Q.1 (b)	Q.2	Q.3	Q.4	Total

Problem No.1 (a) [5Points]

An air vessel is 500 mm average diameter and 10 mm thickness, the length being 2 meters. Find the stresses induced in the material and change in diameter & length when charged to 10 N/mm^2 internal pressure.
Take E= 200 kN/ mm^2 and Poisson's ratio is 0.3

Given:

$d = 50$ mm $t = 10$ mm
$l = 2$ m $p = 10$ N/ mm^2
$E = 200$ kN/ mm^2 $\upsilon = 0.3$
$\delta d = ???$ $\delta l = ???$

Solution:

Stresses in the material

Hoop Stress $\sigma_1 = \dfrac{pr}{t} = \dfrac{10 \times 250}{10} = 250 \quad N/mm^2$ ***Answer***

Longitudinal stress $\sigma_2 = \dfrac{pr}{2t} = \dfrac{10 \times 250}{2 \times 10} = 125 \quad N/mm^2$ ***Answer***

Change in the diameter

$\delta d = \varepsilon_1 d$

$$\varepsilon_1 = \frac{\sigma_1}{E} - \nu \frac{\sigma_2}{E}$$

$$\varepsilon_1 = \frac{250}{200 \times 1000} - 0.3 \times \frac{125}{200 \times 1000}$$

$$\varepsilon_1 = 0.0010625$$

$\delta d = 0.531$ mm ***Answer***

Change in Length

$\delta l = \varepsilon_2 l$

$$\varepsilon_2 = \frac{\sigma_2}{E} - \nu \frac{\sigma_1}{E}$$

$$\varepsilon_2 = \frac{125}{200 \times 1000} - 0.3 \times \frac{250}{200 \times 1000} = 0.00025$$

$\delta l = o.5$ mm ***Answer***

Mechanics of Materials		*MENG 270*
Fall 2003	*Exam 3*	*Time allowed: 90min*

Name	*Computer No.*

Problem No 1(b) [5 Points]

A spherical pressure vessel of 900 mm outer diameter is fabricated from steel having ultimate strength of σ_u = 400 MPa, knowing that a factor of safety of 4 is desired and the gauge pressure can reach 3.5 MPa, determine the smallest wall thickness that should be used.

***Given**:*

d_o = 900 mm r_o = 450 mm

σ_u = 400 MPa p = 3.5 MPa

F.S = 4

t = ???

Solution:

For spherical shell

$$\sigma_1 = \sigma_2 = \frac{pr}{2t}$$

$$r = r_o - t$$

$$\sigma_1 = \frac{p(r_0 - t)}{2t}$$

$$\therefore t = \frac{p\,r_o}{2\sigma_1 + p}$$

$$\sigma_{all} = \frac{\sigma_u}{F.S} = \frac{400}{4} = 100\ MPa$$

$$\therefore t = \frac{3.5 \times 450}{2 \times 100 + 3.5}$$

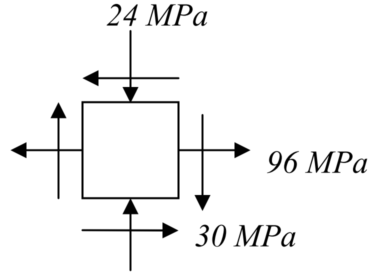
t = 7.74 mm ***Answer***

Mechanics of Materials		MENG 270
Fall 2003	Exam 3	Time allowed: 90min

Name	Computer No.

Problem No 2 [10 Points]

The state of plane stress occurs in an aluminum member made of an alloy with tensile yield strength of 210 MPa. Determine factor of safety with respect to yield using (a) maximum shearing stress criterion (b) maximum distortion energy criterion



Given:

$\sigma_x = 96\ MPa \qquad \sigma_y = -24\ MPa \qquad \tau_{xy} = 30\ MPa$

$\sigma_Y = 210\ MPa$

F.S = ??? Considering;

(a) Maximum shearing stress criterion

(b) Maximum distortion energy criterion

Solution:

For plane stress condition the principal stresses are determined by the equation;

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_{1,2} = \frac{96 - 24}{2} \pm \sqrt{\left(\frac{96 + 24}{2}\right)^2 + 30^2}$$

$$\sigma_1 = 103.1\ MPa$$

$$\sigma_2 = -31.1 MPa$$

(a) Maximum shear stress theory

According to maximum shear stress criterion for plane stress condition,

$$\tau_{max} = \frac{1}{2}\left|\sigma_1 - \sigma_2\right|$$

$$\therefore \tau_{max} = \frac{1}{2}\left|103.1 - 31.1\right| = 67.1 MPa$$

And

$$\tau_{all} = \frac{\sigma_Y}{2} = \frac{210}{2} = 105\ MPa$$

$$\therefore\ F.S = \frac{105}{67.1} = 1.564$$ ***Answer***

(b) Maximum Distortion Energy Theory

According to maximum distortion energy criterion for plane stress condition we have;

$$\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2 = \left(\frac{\sigma_Y}{F.S}\right)^2$$

$$\therefore\ (103.1)^2 - (103.1)(-31.1) + (-31.1)^2 = \left(\frac{210}{F.S}\right)^2$$

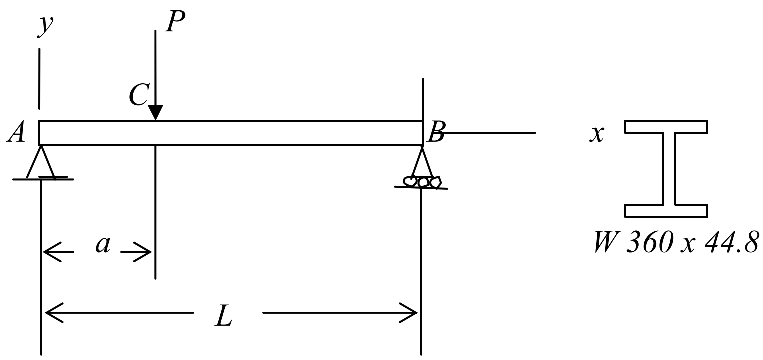
$$\therefore\ F.S = 1.762$$ ***Answer***

Mechanics of Materials		MENG 270
Fall 2003	Exam 3	Time allowed: 90min

Name	Computer No.

Problem No 3 [10 Points]

Determine equation of elastic curve for the beam and loading shown in figure. Knowing that a=1.5 m, L= 5 m and P =150 kN, determine (a) the slope at support A and (b) deflection at point C.
Take E = 200 GPa.



Given:

(a) Elastic curve equation between AB = ???
Knowing that a = 1.5 m L = 5 m P = 150 kN E = 200 GPa
find;
(b) Deflection at point C (c) Slope at support A

Solution:

Free body diagram is shown in Figure 3-a

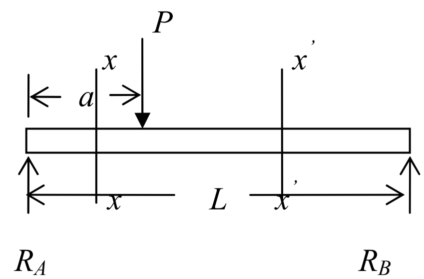


FIG: 3-a

$$\sum M_B = 0$$

$$R_A = \frac{Pb}{L} \quad (let \;\; L - a = b)$$

$$\sum M_A = 0$$

$$R_B = \frac{Pa}{L}$$

Free Body diagram for the section at x-x where x<a is shown in Figure 3-b

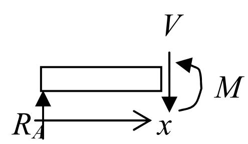


FIG: 3-b

$$M_1 = \frac{Pb}{L} x \quad for\, 0 < x < a$$

The differential equation of the deflection curve of bent beam is

$$EI \frac{d^2 y}{dx^2} = M$$

$$\therefore EI\frac{d^2y}{dx^2} = \frac{Pb}{L}x \qquad (1)$$

The integration of equation (1) gives

$$EI\frac{dy}{dx} = EI\theta_1 = \frac{Pb}{L}\frac{x^2}{2} + C_1 \qquad (2)$$

The integration of equation (2) yields

$$EI\,y_1 = \frac{Pbx^3}{6L} + C_1x + C_2 \qquad (3)$$

In the region to the right of the force P at the section x'-x' where a<x<L, the F.B.D is shown in Figure 3-c ;

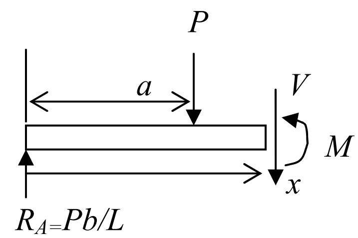


FIG: 3-c

The bending moment equation is

$$M_2 = \frac{Pb}{L}x - P(x-a)$$

$$\therefore EI\frac{d^2y}{dx^2} = \frac{Pb}{L}x - P(x-a) \qquad (4)$$

The integration of equation (4) yields

$$EI\frac{dy}{dx} = EI\,\theta_2 = \frac{Pbx^2}{2L} - \frac{P(x-a)^2}{2} + C_3 \qquad (5)$$

The integration of equation (5) gives

$$EIy_2 = \frac{Pbx^3}{6L} - \frac{P(x-a)^3}{6} + C_3x + C_4 \qquad (6)$$

Apply boundary conditions:

At $x = 0$, $y = 0$

$\therefore$ *From equation (3)* $C_2 = 0$ *(7)*

At $x = a$, $\theta_1 = \theta_2$

$\therefore$ *Comparing equations (2) & (5) and substituting values of constants of integration we get;*

$C_1 = C_3$ *(8)*

At $x = a$ $y_1 = y_2$

Therefore comparing equations (3) & (6) and substituting values of constants of integration we get,

$$\frac{Pba^3}{6L} + C_1a = \frac{Pba^3}{6L} + C_3a + C_4$$

Since $C_1 = C_3$ *, we have* $C_4 = 0$ *(9)*

At $x = L$, $y = 0$

Equation (6) gives

$$C_3 = \frac{Pb}{6L}\left(b^2 - L^2\right) \qquad (10)$$

The values of constants of integration may now be substituted in equations (3)&(6) to give;

$$EIy_1 = \frac{Pb}{6L}\left[x^3 - \left(L^2 - b^2\right)x\right] \qquad for \;\; 0 \langle x \langle a \qquad (11)$$

$$EIy_2 = \frac{Pb}{6L}\left[x^3 - \left(L^2 - b^2\right)x - \frac{L}{b}\left(x - a\right)^3\right] \qquad for \;\; a < x < L \qquad (12)$$

Equations (11) & (12) are valid for in the region indicated
Similarly from equations (2) ,the slope at support A is ,

$$\theta_A = -\frac{Pb(L^2 - b^2)}{6EIL} \qquad (13)$$

Deflection at point C
At point C, x = a, equation (11) gives

$$y = -\frac{Pa^2b^2}{3EIL} \qquad (14)$$

Substituting the given values in equation (14) we get

$$y = -\frac{150 \times 1000 \times 1.5^2 \times 3.5^2}{3 \times 200 \times 10^9 \times 121.1 \times 10^{-6} \times 5} \qquad (I = 121.1x10^{-6}\ m^4 \;\; from\ Appendix\ C\ )$$

y = - .01138 m = -11 .38 mm ***Answer***

Slope at Support A
Substituting the given values in equation (13) we get

$$\theta_A = \frac{150 \times 1000 \times 3.5 \times \left(5^2 - 3.5^2\right)}{6 \times 200 \times 10^9 \times 121.1 \times 10^{-6} \times 5} = 9.21 \times 10^{-3} \;\; rad$$ ***Answer***

<u>BY SINGUARITY METHOD</u>
Referring to figure 3-c

$$M = \frac{Pb}{L}\langle x \rangle^1 - P\langle x - a \rangle^1$$

$$\therefore EI\frac{d^2y}{dx^2} = \frac{Pb}{L}\langle x \rangle^1 - P\langle x - a \rangle^1$$

Integrating we have

$$EI\frac{dy}{dx} = EI\theta = \frac{Pb}{2L}\langle x \rangle^2 - \frac{P}{2}\langle x - a \rangle^2 + C_1 \qquad (1)$$

Integrating equation (1) we get

$$EIy = \frac{Pb}{6L}\langle x \rangle^3 - \frac{P}{6}\langle x - a \rangle^3 + C_1x + C_2 \qquad (2)$$

Apply boundary conditions
At x= 0 y=0
$\therefore$ Equation (2) gives $C_2=0$

At x=L   y=0

∴ From equation (2) we get

$$0=\frac{Pb}{6L}\langle L\rangle^3-\frac{P}{6}\langle L-a\rangle^3+C_1 L$$

$$\therefore C_1=\frac{P}{6L}\langle L-a\rangle^3-\frac{PbL}{6}$$

Substituting values of constants C_1 and C_2 in equations (1) and (2) we get

$$EI\theta=\frac{Pb}{2L}\langle x\rangle^2-\frac{P}{2}\langle x-a\rangle^2+\frac{P}{6L}\langle L-a\rangle^3-\frac{PbL}{6} \qquad (3)$$

$$EIy=\frac{Pb}{6L}\langle x\rangle^3-\frac{P}{6}\langle x-a\rangle^3+\frac{Px}{6L}\langle L-a\rangle^3-\frac{PbLx}{6} \qquad (4)$$

Slope at support A

At x=0 $\theta=\theta_A$

Substituting the given values in equation (3) we have

$$\theta_A=\frac{150\times1000\times3.5}{6\times5\times200\times10^9\times121.1\times10^{-6}}\left[0-0+\frac{1}{3.5}(3.5)^3-5^2\right]$$

$\theta_A = 9.21 \times 10^{-3}$ rad ***Answer***

Deflection at point C

From equation (4)

$$y=\frac{Pb}{6EIL}\left[\langle x\rangle^3-L\langle x-a\rangle^3+\frac{x}{b}\langle L-a\rangle^3-L^2 x\right]$$

At C , x=1.5

$$\therefore y_c=\frac{150\times1000\times3.5}{6\times200\times10^9\times121.1\times10^{-6}\times5}\left[1.5^3-0+\frac{1.5}{3.5}\times(3.5)^3-5^2\times1.5\right]$$

y_c =.01138 m

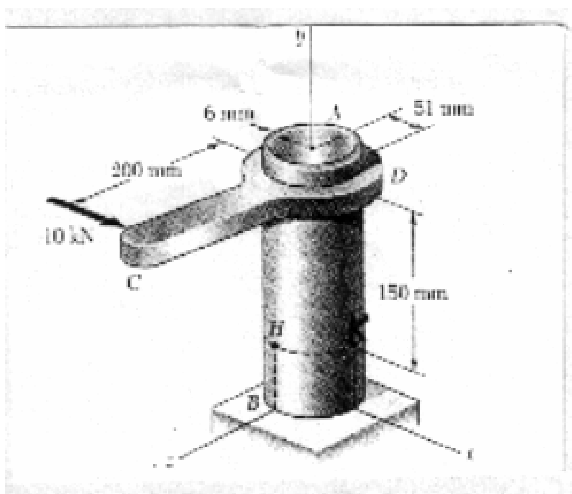
y_c = 11.38 mm ***Answer***

Mechanics of Materials *MENG 270*
Fall 2003 *Exam 3* *Time allowed: 90min*

Name	*Computer No.*

Problem No. 4 [10 Points]

The steel pipe AB has 102 mm outer diameter and 6 mm wall thickness. Knowing that arm CD is rigidly attached to the pipe, determine the principal stresses and maximum shearing stress at point K.



Given:

D_o = *102 mm* *t =6mm* *P= 10kN*

Principal stresses at point K = ???

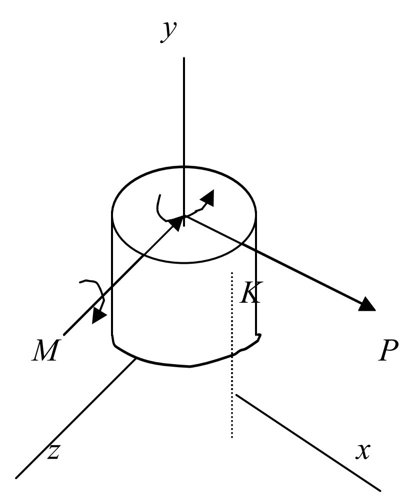
Solution:

Replace the P by an equivalent force-couple system at the center C of the transverse section containing point K:

P = 10 kN *T = (10)x(200) = 2000 kN-mm*

M = 10 x 150 kN-mm

Stresses σ_x **,** σ_y **,** τ_{xy} ***at point K***



$$\sigma_x = 0 \qquad \sigma_y = -\frac{Mc}{I}$$

T

$$I = \frac{\pi}{64}\left(D_o^4 - D_i^4\right)$$

Substituting the given values in above equations

$$\sigma_y = -\frac{10\times150\times1000\times51}{\frac{\pi}{64}\times\left(102^4 - 90^4\right)} = -36.55 \quad MPa$$

$$\tau_{xy} = \frac{T\,r}{J}$$

$$J = \frac{\pi}{32}\left(D_o^4 - D_i^4\right)$$

$$\therefore \tau_{xy} = \frac{10\times1000\times200\times51}{\frac{\pi}{32}\left(102^4 - 90^4\right)} = 24.36 \; MPa$$

We note that shearing force P does not cause any shearing stress at point K

Principal Stresses

$$\sigma_{\max,\min} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\therefore \sigma_{\max,\min} = \frac{-36.55}{2} \pm \sqrt{\left(\frac{-36.55}{2}\right)^2 + 24.36^2}$$

$$\sigma_{\max,\min} = -18.2 \pm 30.45$$

$\sigma_{\max} = 12.25 \quad MPa$ ***Answer***

$\sigma_{\min} = -48.7 \; MPa$ ***Answer***

$$\tau_{\max} = \sqrt{\left(\frac{-36.55}{2}\right)^2 + 24.36^2}$$

$\boldsymbol{\tau}_{\max}$ = *30.45 MPa* ***Answer***