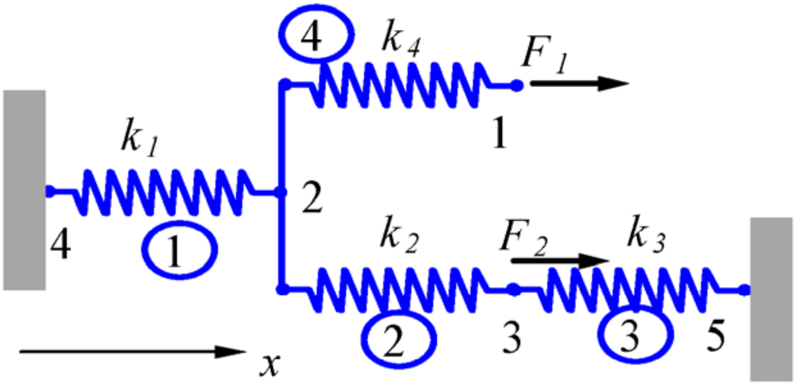
Example 1.2



Problem: For the spring system with arbitrarily numbered nodes and elements, as shown above, find the global stiffness matrix.

Solution:

First we construct the following

Element Connectivity Table

Element	*Node i (1)*	*Node j (2)*
1	4	2
2	2	3
3	3	5
4	2	1

which specifies the *global* node numbers corresponding to the *local* node numbers for each element.

Then we can write the element stiffness matrices as follows

$$\mathbf{k}_1 = \begin{bmatrix} k_1 & -k_1 \\ -k_1 & k_1 \end{bmatrix} \quad (u_4, u_2) \qquad \mathbf{k}_2 = \begin{bmatrix} k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix} \quad (u_2, u_3)$$

$$\mathbf{k}_3 = \begin{bmatrix} k_3 & -k_3 \\ -k_3 & k_3 \end{bmatrix} \quad (u_3, u_5) \qquad \mathbf{k}_4 = \begin{bmatrix} k_4 & -k_4 \\ -k_4 & k_4 \end{bmatrix} \quad (u_2, u_1)$$

Finally, applying the superposition method, we obtain the global stiffness matrix as follows

$$\mathbf{K} = \begin{bmatrix} k_4 & -k_4 & 0 & 0 & 0 \\ -k_4 & k_1 + k_2 + k_4 & -k_2 & -k_1 & 0 \\ 0 & -k_2 & k_2 + k_3 & 0 & -k_3 \\ 0 & -k_1 & 0 & k_1 & 0 \\ 0 & 0 & -k_3 & 0 & k_3 \end{bmatrix} \quad (u_1, u_2, u_3, u_4, u_5)$$

The matrix is *symmetric*, *banded*, but *singular*.