



*Problem:* Determine the support reaction forces at the two ends of the bar shown above, given the following,

$$P = 6.0 \times 10^4 \text{ N}, \quad E = 2.0 \times 10^4 \text{ N / mm}^2,$$

$$A = 250 \text{ mm}^2, \quad L = 150 \text{ mm}, \quad \Delta = 1.2 \text{ mm}$$

*Solution:*

We first check to see if or not the contact of the bar with the wall on the right will occur. To do this, we imagine the wall on the right is removed and calculate the displacement at the right end,

$$\Delta_0 = \frac{PL}{EA} = \frac{(6.0 \times 10^4)(150)}{(2.0 \times 10^4)(250)} = 1.8 \text{ mm} > \Delta = 1.2 \text{ mm}$$

Thus, contact occurs.

The global FE equation is found to be,

$$\frac{EA}{L} \begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \end{Bmatrix} = \begin{Bmatrix} F_1 \\ F_2 \\ F_3 \end{Bmatrix}$$

The load and boundary conditions are,

$$F_2 = P = 6.0 \times 10^4 \text{ N}$$

$$u_1 = 0, \quad u_3 = \Delta = 1.2 \text{ mm}$$

FE equation becomes,

$$\frac{EA}{L} \begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix} \begin{Bmatrix} 0 \\ u_2 \\ \Delta \end{Bmatrix} = \begin{Bmatrix} F_1 \\ P \\ F_3 \end{Bmatrix}$$

The 2<sup>nd</sup> equation gives,

$$\frac{EA}{L} [2 \quad -1] \begin{Bmatrix} u_2 \\ \Delta \end{Bmatrix} = \{P\}$$

that is,

$$\frac{EA}{L} [2] \{u_2\} = \left\{ P + \frac{EA}{L} \Delta \right\}$$

Solving this, we obtain

$$u_2 = \frac{1}{2} \left( \frac{PL}{EA} + \Delta \right) = 1.5 \text{ mm}$$

and

$$\begin{Bmatrix} u_1 \\ u_2 \\ u_3 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 1.5 \\ 1.2 \end{Bmatrix} \text{ (mm)}$$

To calculate the support reaction forces, we apply the 1<sup>st</sup> and 3<sup>rd</sup> equations in the global FE equation.

The 1<sup>st</sup> equation gives,

$$F_1 = \frac{EA}{L} \begin{bmatrix} 1 & -1 & 0 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \end{Bmatrix} = \frac{EA}{L} (-u_2) = -5.0 \times 10^4 \text{ N}$$

and the 3<sup>rd</sup> equation gives,

$$\begin{aligned} F_3 &= \frac{EA}{L} \begin{bmatrix} 0 & -1 & 1 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \end{Bmatrix} = \frac{EA}{L} (-u_2 + u_3) \\ &= -1.0 \times 10^4 \text{ N} \end{aligned}$$

***Check the results.!***