بسم الله الرحمن الرحيم

King Abdulaziz University
Engineering College
Department of Production and Mechanical System Design

MENG 470 Mechanical Vibrations

First Exam
Closed-book Exam
Monday: 8/2/1425 H
Time Allowed: 60 mins

Name:	Sec. No.:	ID No.:

Question 1		10
Question 2		10
Question 3		10
Question 4		10
TOTAL		40

Instructions

1. There are totally 4 problems in this exam.
2. This is a closed book and closed notes Opportunity to Shine
3. Show all work for partial credit.
4. Assemble your work for each problem in logical order.
5. Justify your conclusion. I cannot read minds.

Q1. Figure.1 shows the free response of a vibration system to an initial displacement. Determine all possible characteristics which describe the vibration of the system. The mass is 5 kg.

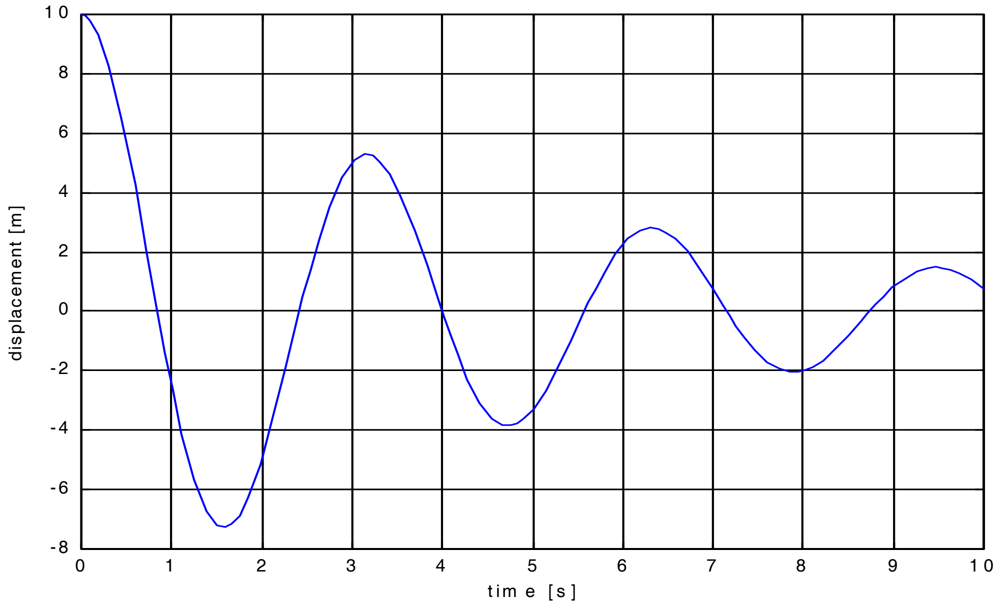


Figure.1

Solution:

Expect a 2nd order model. (Could be a higher order DE too!)

$$\frac{m}{k}\ddot{x} + \frac{c}{k}\dot{x} + x = 0 \qquad \text{or} \qquad \frac{1}{\omega_n^2}\ddot{x} + \frac{2\zeta}{\omega_n}\dot{x} + x = 0$$

Initial conditions: at t=0, $x(0) = 10$ (the x value)

$\dot{x}(0) = v(0) = 0$ (the slope of the plot)

Logarithmic decrement:

Peak 1 occurs at t= 0 with value of x_i = 10.0

Peak 2 occurs at t= 3.2 with value of x_{i+T}= 5.4

Peak 3 occurs at t =6.3 with value of x_{i+2T} = 2.8

$$\delta = \ln\frac{x_i}{x_{i+T}} = \ln\frac{x_{i+T}}{x_{i+2T}} = \frac{1}{2}\ln\frac{x_i}{x_{i+2T}} = \frac{2\pi\zeta}{\sqrt{1-\zeta^2}}$$

$\delta = \ln \frac{x_i}{x_{i+T}} = \ln (10 \ / \ 5.4)$ where $x_i = x(0)$ and x_{i+T} were used.

$= 0.616$

Find the damping ratio, ζ:

$$\zeta = \frac{\delta}{\sqrt{\delta^2 + 4\pi^2}}$$

$$= \frac{0.616}{\sqrt{0.616^2 + 4\pi^2}} = 0.0976$$

Damped Period:

$T_d =$ (2nd peak time - 1st peak time) = 3.2 s

Damped natural frequency

$$\omega_d = \frac{2\pi}{T_d}$$

$$= \frac{2\pi}{3.2} = \text{1.96 rad/s}$$

Undamped natural frequency

$$\omega_n = \omega_d \frac{1}{\sqrt{1-\zeta^2}}$$

$$= 1.96 \frac{1}{\sqrt{1-0.0976^2}} = \text{1.97 rad/s}$$

Therefore, if this is a mass-spring-damper system:

$$\frac{1}{\omega_n^2} = \frac{m}{k} \quad \Rightarrow \quad k = m\omega_n^2 = (5kg)(1.97 rad/s)^2 = 19.5 N/m$$

$$\frac{2\zeta}{\omega_n} = \frac{c}{k} \qquad \Rightarrow \quad c = \frac{2\ \zeta\ k}{\omega_n} = \frac{2(0.0976)(19.5N/m)}{(1.97 rad/s)} = 1.93N - s/m$$

Equation:

$$5\frac{d^2x}{dt^2} + 1.93\frac{dx}{dt} + 19.5x = 0$$

ICs: $x(0) = 10.0$ and $\dot{x}$ (0) = v(0)= 0.0

Q2. The system shown in Figure.2 has a natural frequency of 5 hz for the following data:
m = 10 kg, J_0=5 kg-m^2, r_1 = 10 cm, r_2 = 25 cm. When the system is disturbed by giving it an initial displacement, the amplitude of free vibration is reduced by 80 percent in 10 cycles.
Determine the values of k and c.

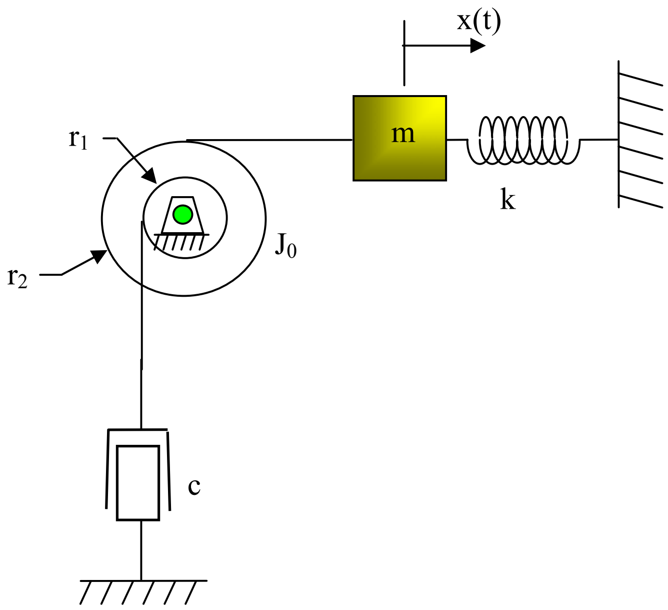


Figure.2

Solution: Expect model to be 1-DOF Damped Free Response.

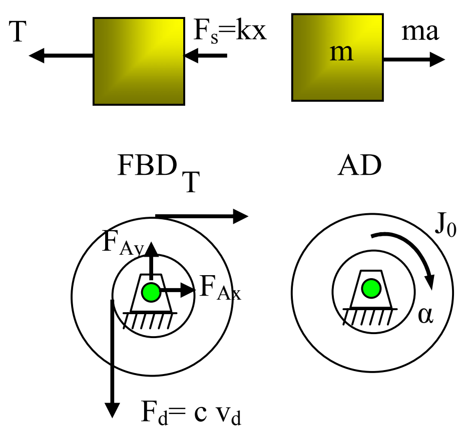
Using Newton's 2nd Law:

for block:

$$\Sigma F = ma$$

$$-T - kx = ma$$

$$ma + kx + T = 0$$



for pulley:

$$\Sigma M_A = J\alpha$$

$$F_d r_1 - T r_2 = -J_0 \alpha$$

$$T = \frac{J_0}{r_2}\alpha + \frac{r_1}{r_2} c v_d$$

from kinematics:

$$v_d = r_1\omega \quad \text{and} \quad \dot{x} = v = r_2\omega \quad \rightarrow \quad v_d = \frac{r_1}{r_2}v$$

and

$$a = r_2\alpha \quad \rightarrow \quad \alpha = \frac{a}{r_2}$$

Combining these equations:

$$ma + kx + \left\{\frac{J_0}{r_2}\left(\frac{a}{r_2}\right) + \frac{r_1}{r_2}c\left(\frac{r_1}{r_2}v\right)\right\} = 0$$

therefore from comparing the standard equation form

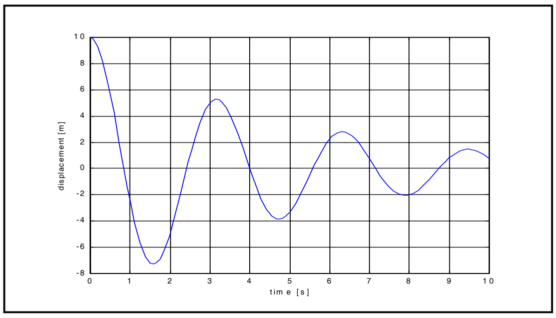
$$\ddot{x} + 2\zeta\omega_n\dot{x} + \omega_n^2 x = 0$$

$$\left(m + \frac{J_0}{r_2^{\,2}}\right)\ddot{x} + \frac{r_1^{\,2}}{r_2^{\,2}}c\dot{x} + kx = 0$$

$$\omega_n = \sqrt{\frac{k}{m_{eq}}} = \sqrt{\frac{k}{m + \frac{J_0}{r_2^{\,2}}}}$$

$$\frac{c_{eq}}{m_{eq}} = 2\zeta\omega_n \quad \Rightarrow \quad \zeta = \frac{r_1^{\,2}c\,\omega_n}{2r_2^{\,2}k}$$

<u>Since the system's vibration is reduced by 80% over 10 cycles, the logarithmic decrement can be found.</u>



$$\delta = \frac{1}{10}\ln\left(\frac{x_i}{x_{i+10T}}\right)$$

where $x_i = 100\% x_0$ and $x_{i+10T} = 20\% x_0$

so

$$\delta = \frac{1}{10}\ln\left(\frac{100}{20}\right) = 0.1609$$

The damping ratio is found as

$$\zeta = \frac{\delta}{\sqrt{\delta^2 + 4\pi^2}} = \frac{0.1609}{\sqrt{0.1609^2 + 4\pi^2}} = 0.0256$$

The natural frequency was given as f_d = 5 hz and since the damping ratio is quite small ($\zeta < 0.2$)

$$\omega_n \approx \omega_d = 2\pi f_d = 10\pi \ rad/s$$

<u>Finally we can solve for k and c:</u>

$$\omega_n^2 = \frac{k{r_2}^2}{m{r_2}^2 + J_0}$$

$$k = \frac{\omega_n^2\left(m{r_2}^2 + J_0\right)}{{r_2}^2}$$

$$= \frac{(10\pi rad/s)^2\left((10kg)(0.25m)^2 + (5kg\cdot m^2)\right)}{(0.25m)^2} * \frac{1N}{1kg\cdot m/s^2}$$

= 88826 N/m

and

$$\zeta = \frac{{r_1}^2 c\omega_n}{2{r_2}^2 k}$$

$$c = \frac{2{r_2}^2 k\zeta}{{r_1}^2\omega_n}$$

$$= \frac{2(0.25m)^2(88826N/m)(0.0256)}{(0.10m)^2(10\pi \ rad/s)}$$

= 28.8 N/(m/s)

therefore the model's equation of motion is:

$$\frac{1}{\omega_n^2}\ddot{x} + \frac{2\zeta}{\omega_n}\dot{x} + x = 0$$

$$\frac{1}{(10\pi)^2}\ddot{x} + \frac{2(0.0256)}{10\pi}\dot{x} + x = 0$$

$$\ddot{x} + 1.608\dot{x} + 987x = 0$$

Q3. An unknown mass, *m*, attached at the end of an unknown spring, *k*, has a natural frequency of 95 Hz. When a 0.5 kg mass is added to *m*, the natural frequency is lowered to 75 Hz. Determine the mass, *m* (kg), and the spring constant, *k* (N/m).

Solution:

$$f_1 = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = 95 \text{ Hz} \quad \text{hence} \quad k = (95\times 2\pi)^2 m$$

$$f_2 = \frac{1}{2\pi}\sqrt{\frac{k}{(m+0.5)}} = 75 \text{ Hz} \quad \text{hence} \quad k = (75\times 2\pi)^2 (m+0.5)$$

Simultaneous solution yields:

m = 0.8272 kg ; k = 294.7 kN/m

Q4. Figure.3 represents a simplified arrangement for a spring-supported vehicle traveling over a rough road. Determine an equation for the amplitude of motion for *m* as a function of road speed. What is the worst road speed?

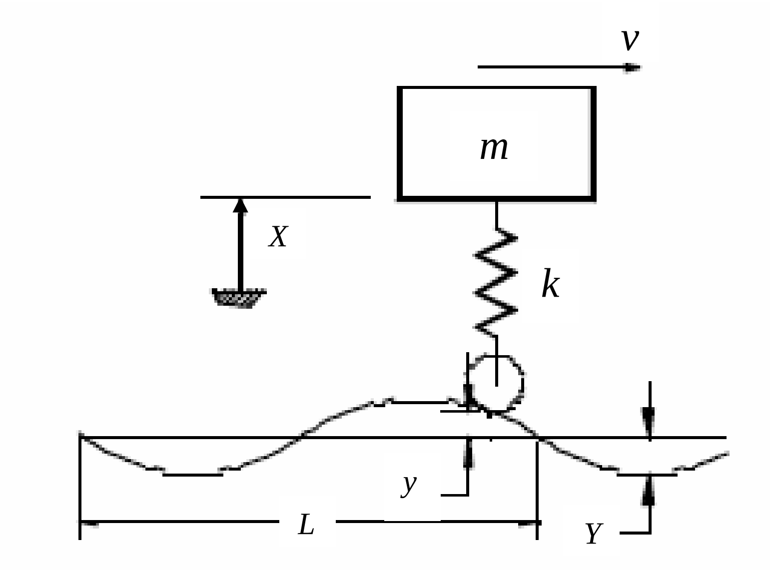


Figure.3

Solution:

$$\left|\frac{X}{Y}\right| = \frac{\sqrt{\omega_n^4 + 4\zeta^2\omega_n^2\omega^2}}{\sqrt{\left(\omega_n^2 - \omega^2\right)^2 + 4\zeta^2\omega_n^2\omega^2}} \quad \text{and} \quad \phi = \tan^{-1}\left(\frac{2\zeta\omega^3}{\omega_n\left(\omega_n^2 - \omega^2\right) + 4\zeta^2\omega_n\omega^2}\right)$$

For this problem:

$$\omega = \frac{2\pi v}{L} \quad \text{and} \quad \omega_n = \sqrt{\frac{k}{m}}$$

Most unfavorable speed is when:

$$\frac{2\pi v_{WORST}}{L} = \omega = \omega_R = \omega_n\sqrt{1 - 2\zeta^2}$$

hence $$v_{WORST} = \frac{\omega_n L\sqrt{1 - 2\zeta^2}}{2\pi} = f_n L\sqrt{1 - 2\zeta^2}$$