

King Abdulaziz University  
Engineering College  
Department of Production and Mechanical System Design



MENG 470 Mechanical Vibrations

Final Exam  
Closed-book Exam  
Wednesday: 24/11/1425 H  
Time Allowed: 120 mins

Name:	ID No.:
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Question 1		25
Question 2		25
Question 3		25
Question 4		25
TOTAL		100

قال الله تعالى : (تلك الدار الآخرة نجعلها للذين لا يريدون علوا في الأرض ولا فسادا والعاقبة للمتقين)

Instructions

1. This is a closed book and closed notes Opportunity to Shine
2. There are five questions in this exam. Answer ONLY four of them.
3. Show all work for partial credit.
4. Assemble your work for each problem in logical order.
5. Justify your conclusion. I cannot read minds.

Q1. Indicate whether each of the following statements is **true** or **false**:

1. The amplitude of an undamped system will not change with time.
2. A system vibrating in air can be considered a damped system.
3. The equation of motion of a single degree of freedom system will be the same whether the mass moves in a horizontal plane or an inclined plane.
4. When a mass vibrates in a vertical direction, its weight can always be ignored in deriving the equation of motion.
5. The principle of conservation of energy can be used to derive the equation of motion of both damped and undamped systems.
6. The damped frequency can in some cases be larger than the undamped natural frequency of the system.
7. The damped frequency can be zero in some cases.
8. The natural frequency of vibration of a torsional system is given by  $\sqrt{k_T / J}$ , where  $k_T$  and  $J$  denote the torsional spring constant and the polar mass moment of inertia, respectively.
9. The undamped natural frequency of a system is given by  $\sqrt{g / \delta_{st}}$  where  $\delta_{st}$  is the static deflection of the mass.
10. For an undamped system, the velocity leads the displacement by  $\pi / 2$ .
11. The motion diminishes to zero in both underdamped and overdamped cases.
12. The logarithmic decrement can be used to find the damping ratio.
13. In torsional vibration, the displacement is measured in terms of linear coordinate
14. The phase angle of the response depends on the system parameter  $m$ ,  $c$ ,  $k$ , and  $\omega$ .
15. During beating, the amplitude of the response builds up and then diminishes in a regular pattern.
16. The  $Q$ -factor can be used to estimate the damping in a system.
17. The amplitude ratio attains its maximum value at resonance in the case of viscous damping.
18. Damping reduces the amplitude ratio for all values of the forcing frequency.
19. The unbalance in a rotating machine causes vibration.
20. The normal modes can also be called principal modes.
21. The generalized coordinates are linearly dependent.
22. Principal coordinates can be considered as generalized coordinates.
23. The vibration of a system depends on the coordinate system.
24. The nature of coupling depends in the coordinate system.
25. The magnification factor is the ratio of maximum amplitude and static deflection.
26. The response will be harmonic if excitation is harmonic.

27. The principal (or modal) coordinates avoid both static and dynamic coupling.
28. The use of principal (or modal) coordinates can NOT be used to find the response of the system.
29. The mass, stiffness, and damping matrices of a two degree of freedom system are always NOT symmetric.
30. The characteristics of a two degree of freedom system are used in the design of dynamic vibration absorber.
31. A semidefinite system can NOT have nonzero natural frequencies.
32. During free vibration, different degrees of freedom oscillate with different amplitudes.
33. The modal eigenvectors of a system are the physical not-normalized modes of vibration.
34. The vibration of a system under external forces is called damped vibration.
35. When a two degree of freedom system is subjected to a harmonic force, the system vibrates at the frequency of applied force.
36. When the forcing frequency is equal to one of the natural frequencies of the system, a phenomenon known as *beating* occurs.
37. For an underdamped multidegree of freedom system, all the eigenvalues can be complex.
38. The amplitudes and phase angles are determined from the boundary conditions of the system.
39. A definite system has at least one rigid body motion.
40. The elastic coupling is also known as dynamic coupling while the inertia coupling is also known as static coupling.
41. The equations of motion of a system will be coupled when principal (or principle) coordinates are used.
42. The vibration of a system under initial conditions only is called forced vibration.
43. The number of degrees of freedom of a vibrating system depends only on number of masses.
44. The equations of motion of a two degree of freedom system are in general coupled.
45. The stiffness matrix of a system is always symmetric and positive definite.
46. For a multidegree of freedom system, one equation of motion can be written for each degree of freedom.
47. Lagrange's equation cannot be used to derive the equations of motion of a multidegree of freedom system.
48. The mass, stiffness, and damping matrices of a multidegree of freedom are always symmetric.
49. A multidegree of freedom system can have six of the natural frequencies equal to zero
50. The mass matrix of a system is always symmetric and positive definite.

Answers:

	True	False
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Q2. Consider the system shown in Figure 1 where  $m_1 = 30$  kg,  $m_2 = 2$  kg,  $k = 15$  N/m,  $l = 2$  m,  $f(t) = 10 \sin(5t)$  N.

- What's the degree of the system?
- Write the equation of motion of the system in matrix form.
- Is the system statically or dynamically coupled or both.
- Find the natural frequencies and corresponding mode shapes.
- Calculate the normalized eigenvectors of the system.
- Write down the equation of motion in matrix form.
- Decouple the coupled equations using modal transformation.
- Recover the physical degrees of freedom from the modal degree of freedom.

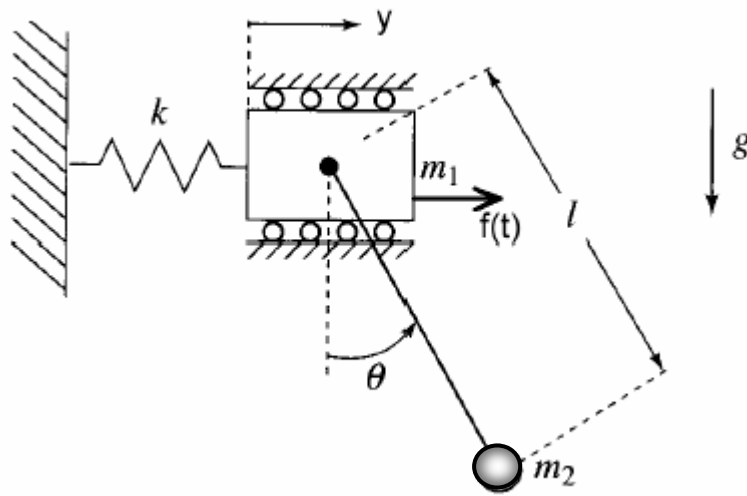


Figure 1

Q3. Consider a cable shown in Figure 2 that has one end fixed and the other end free to slide along a smooth vertical guide. The free end cannot support a transverse force so that we have:

$$\frac{\partial w(L,t)}{\partial x} = 0$$

The cable length  $L=100\text{m}$  is made out of steel with a uniform density  $\rho=7.8 \times 10^3 \text{ kg/m}^3$ , and constant cross sectional area  $A=7.854 \times 10^{-5} \text{ m}^2$ ; and it is under tension of  $T=10,000 \text{ N}$ .

Calculate the natural frequencies and mode shape of the cable. Plot the first four mode shapes (Normalized the mode shapes so that its maximum amplitude is one).

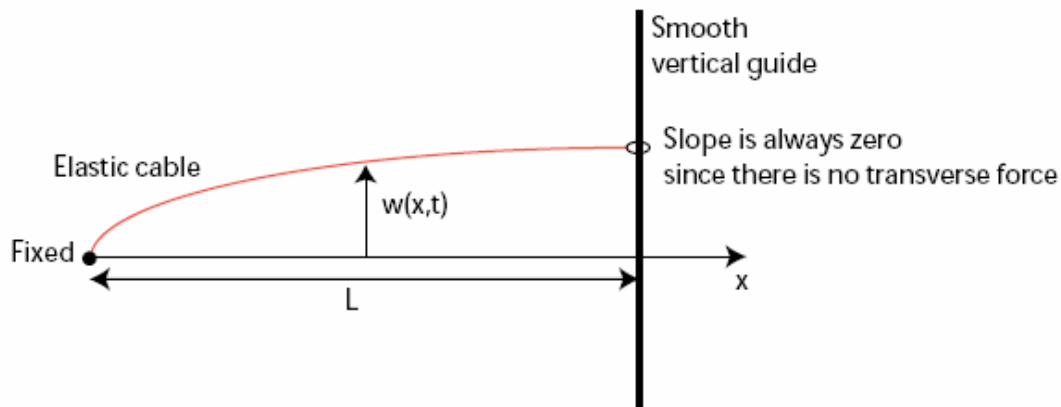


Figure 2

- Q.4 Consider the system shown in Figure 3 and determine the following:
- The degree of freedom.
  - The kinetic energy of the system in terms of  $\dot{x}$ .
  - The potential energy of the system in terms of  $x$ .
  - The equation (or equations) of motion.
  - The natural frequency (or frequencies).

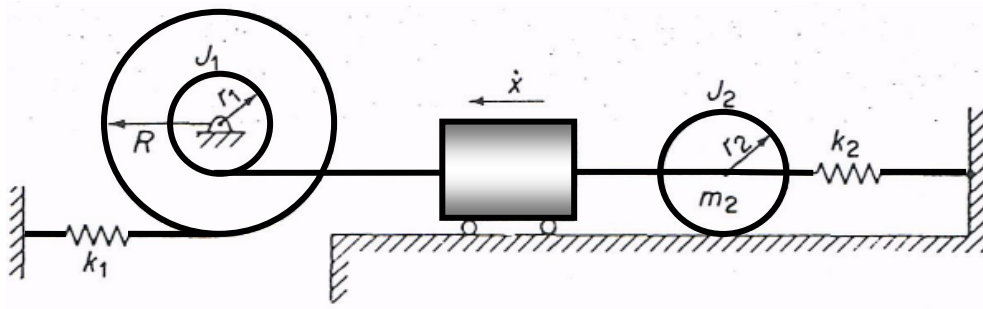


Figure 3

Q5. The system shown in Figure 4 has the following parameters:

$$m = 1 \text{ kg}, I_G = 2 \text{ kg.m}^2, k_1 = k_2 = 100 \text{ N/m}, r = 2 \text{ m}, M_2(t) = 100 \cos \omega t$$

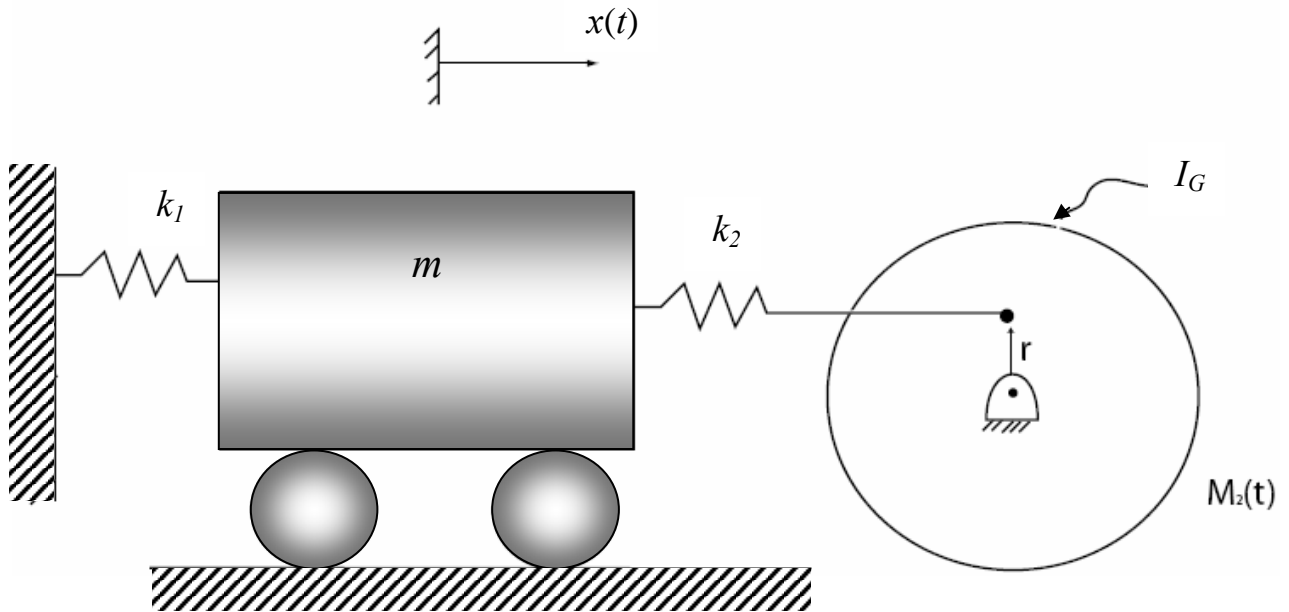


Figure 3

Figure1: A two degree of freedom system with translation and rotation.

- Derive the equations of motion.
- Find the natural frequencies for the system  $\omega_1$  and  $\omega_2$ .
- Find the “mass normalized” eigenvectors ( $U$ ).
- Find  $U^T M U$  and  $U^T K U$ .
- Decouple the equations of motion into modal coordinates and find the transient and steady state solution, or modal displacements, for each modal coordinate ( $\eta_1$  and  $\eta_2$ ).

Use the following initial conditions:  $x(0) = 0$ ,  $\dot{x}(0) = 4$ ,  $\theta(0) = 0$ ,  $\dot{\theta}(0) = 0$  ..

- Use the solution in modal coordinates to write the physical displacement of  $m$  and physical rotation of  $IG$ , or the position vector.  $X = [x \ \theta]^T$ .