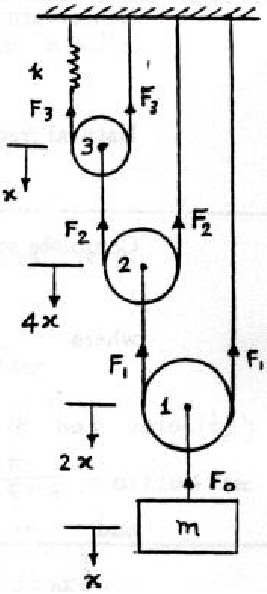
2.14 For a displacement of x of mass m, pulleys 1, 2 and 3 undergo displacements of 2x, 4x and 8x, respectively. The equation of motion of mass m can be written as



$$m\ddot{x} + F_0 = 0 \quad (1)$$

where $F_0 = 2F_1 = 4F_2 = 8F_3$ as shown in figure. Since $F_3 = (8x)\,k$, Eq. (1) can be rewritten as

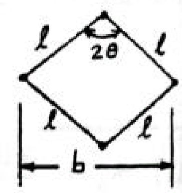
$$m\ddot{x} + 8F_3 = 8\,(8k) = 0 \quad (2)$$

from which we can find

$$\omega_n = \sqrt{\frac{64\,k}{m}} = 8\sqrt{\frac{k}{m}} \quad (3)$$

2.21 $b = 2l \sin\theta$

Neglect masses of links.



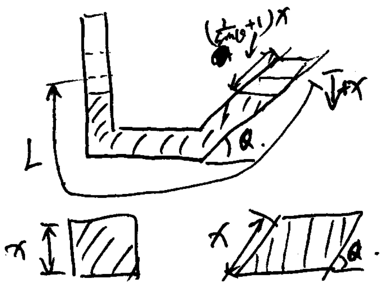
(a) $k_{eq} = k\left(\frac{4l^2 - b^2}{b^2}\right) = k\left(\frac{4l^2 - 4l^2\sin^2\theta}{4l^2\sin^2\theta}\right)$

$= k\left(\frac{\cos^2\theta}{\sin^2\theta}\right)$

$\omega_n = \sqrt{\frac{k_{eq}}{m}} = \sqrt{\frac{k\,g\,\text{cosec}^2\theta}{W}}$ (from solution of problem 1.8)

(b) $\omega_n = \sqrt{\frac{k\,g}{W}}$ since $k_{eq} = k$.

2.32.



Equation of motion.

$m\ddot{x} = \Sigma F_x$

i.e., $(LA\rho)\ddot{x} = -A\,x(1+\sin\theta)\,\rho g$

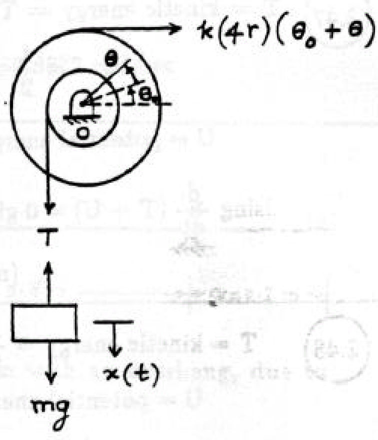
i.e., $\ddot{x} + \frac{g(1+\sin\theta)}{L}x = 0$.

A = cross-section area of tube

ρ = density of mercury

$\therefore\ W_n = \sqrt{\frac{g}{L}(1+\sin\theta)}$

(2.45)



Equation of motion:

$$\text{Mass m:}\quad m\,g - T = m\,\ddot{x} \tag{1}$$

$$\text{Pulley } J_0:\quad J_0\,\ddot{\theta} = T\,r - k\,4\,r\,(\theta + \theta_0)\,4\,r \tag{2}$$

where θ_0 = angular deflection of the pulley under the weight, mg, given by:

$$m\,g\,r = k\,(4\,r\,\theta_0)\,4\,r \quad \text{or} \quad \theta_0 = \frac{m\,g}{16\,r\,k} \tag{3}$$

Substituting Eqs. (1) and (3) into (2), we obtain

$$J_0\,\ddot{\theta} = (m\,g - m\,\ddot{x})\,r - k\,16\,r^2\,(\theta + \frac{m\,g}{16\,r\,k}) \tag{4}$$

Using $x = r\,\theta$ and $\ddot{x} = r\,\ddot{\theta}$, Eq. (4) becomes

$$(J_0 + m\,r^2)\,\ddot{\theta} + (16\,r^2\,k)\,\theta = 0$$

(2.62) (a) $\omega_n = \sqrt{\frac{g}{l}}$

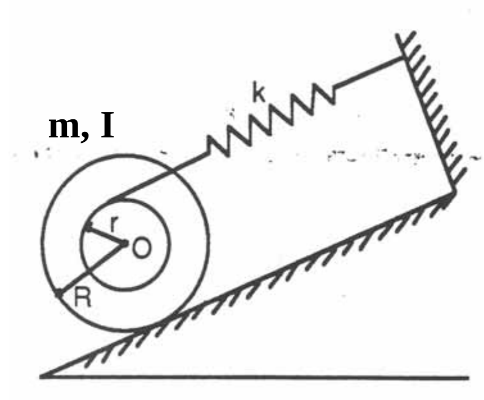
(b) $m l^2 \ddot{\theta} + k a^2 \sin\theta + m g l \sin\theta = 0$; $m l^2 \ddot{\theta} + (k a^2 + m g l)\,\theta = 0$

$\omega_n = \sqrt{\frac{k a^2 + m g l}{m l^2}}$

(c) $m l^2 \ddot{\theta} + k a^2 \sin\theta - m g l \sin\theta = 0$; $m l^2 \ddot{\theta} + (k a^2 - m g l)\,\theta = 0$

$\omega_n = \sqrt{\frac{k a^2 - m g l}{m l^2}}$

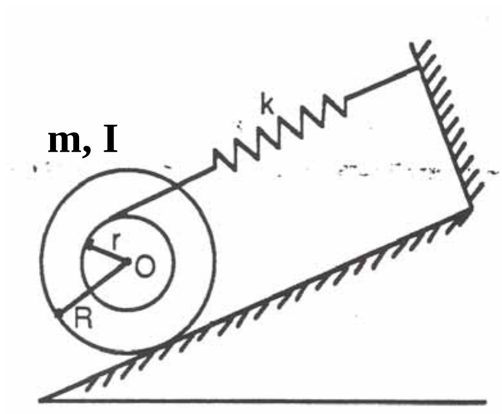
configuration (b) has the highest natural frequency.

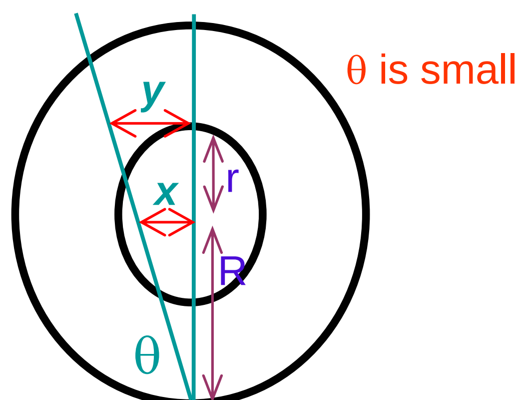


Q1. Find the equation of motion in three different ways:

- Newtonian Approach
- Conservation of Energy

Q2. Find the natural frequency of the system using the fact that
The max KE and max PE are equal





Wheel's centre moves by x & Spring attachment point moves by y

$$\theta = \frac{x}{R} = \frac{y}{R+r}$$

Spring extension y $= (R+r)\theta = \dfrac{R+r}{R}x$

Max strain energy: $V_{\max} = \dfrac{1}{2}ky^2 = \dfrac{1}{2}k\,[x(R+r)/R]^2$

Max kinetic energy: $T_{\max} = (T_{TRANSLATION})_{\max} + (T_{ROTATION})_{\max}$

$$= \frac{1}{2}m(\dot{x}^2)_{\max} + \frac{1}{2}I(\dot{\theta}^2)_{\max}$$

$$= \frac{1}{2}m(\omega_n x)^2 + \frac{1}{2}I(\omega_n\theta)^2 = \frac{1}{2}m(\omega_n x)^2 + \frac{1}{2}I\omega_n^2\frac{x^2}{R^2}$$

$T_{\max} = V_{\max}$ gives: $\omega_n = \sqrt{\dfrac{k(R+r)^2}{I+mR^2}}$