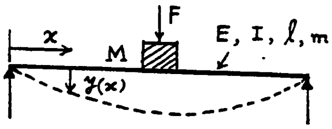
(2.74) Let m_{eff} = effective part of mass of beam (m) at middle. Thus vibratory inertia force at middle is due to $(M + m_{eff})$. Assume a deflection shape: $y(x,t) = Y(x) \cos(\omega_n t - \phi)$ where $Y(x)$ = static deflection shape due to load at middle given by:



$$Y(x) = Y_0 \left(3 \frac{x}{\ell} - 4 \frac{x^3}{\ell^3}\right) \; ; \; 0 \le x \le \frac{\ell}{2}$$

where Y_0 = maximum deflection of the beam at middle = $\dfrac{F \ell^3}{48 E I}$

Maximum strain energy of beam = maximum work done by force F = $\frac{1}{2} F Y_0$.

Maximum kinetic energy due to distributed mass of beam:

$$= 2 \left\{ \frac{1}{2} \frac{m}{\ell} \int_0^{\frac{\ell}{2}} \dot{y}^2(x,t) \big|_{max} dx \right\} + \frac{1}{2} \left(\dot{y}_{max}\right)^2 M$$

$$= \frac{m \omega_n^2}{\ell} \int_0^{\frac{\ell}{2}} Y^2(x) \, dx + \frac{1}{2} \omega_n^2 Y_{max}^2 M$$

$$= \frac{m \omega_n^2}{\ell} \int_0^{\frac{\ell}{2}} Y_0^2 \left(\frac{9 x^2}{\ell^2} + 16 \frac{x^6}{\ell^6} - 24 \frac{x^4}{\ell^4}\right) dx + \frac{1}{2} Y_0^2 M \omega_n^2$$

$$= \frac{m \omega_n^2 Y_0^2}{\ell} \left[\frac{9}{\ell^2} \frac{x^3}{3} + \frac{16}{\ell^6} \frac{x^7}{7} - \frac{24}{\ell^4} \frac{x^5}{5}\right] \Big|_0^{\frac{\ell}{2}} + \frac{1}{2} Y_0^2 M \omega_n^2$$

$$= \frac{1}{2} Y_0^2 \omega_n^2 \left(\frac{17}{35} m + M\right)$$

This shows that $m_{eff} = \dfrac{17}{35} m = 0.4857 \, m$

(2.75) For small angular rotation of bar PQ about P,

$$\frac{1}{2} (k_{12})_{eq} (\theta \ell_3)^2 = \frac{1}{2} k_1 (\theta \ell_1)^2 + \frac{1}{2} k_2 (\theta \ell_2)^2$$

$$(k_{12})_{eq} = \frac{k_1 \ell_1^2 + k_2 \ell_2^2}{\ell_3^2}$$

Since $(k_{12})_{eq}$ and k_3 are in series,

$$k_{eq} = \frac{(k_{12})_{eq} \, k_3}{(k_{12})_{eq} + k_3} = \frac{k_1 k_3 \ell_1^2 + k_2 k_3 \ell_2^2}{k_1 \ell_1^2 + k_2 \ell_2^2 + k_3 \ell_3^2}$$

T = kinetic energy = $\frac{1}{2} m \dot{x}^2$, U = potential energy = $\frac{1}{2} k_{eq} x^2$

If $x = X \cos \omega_n t$,

$T_{max} = \frac{1}{2} m \omega_n^2 X^2$, $U_{max} = \frac{1}{2} k_{eq} X^2$

$T_{max} = U_{max}$ gives $\omega_n = \sqrt{\dfrac{k_1 k_3 \ell_1^2 + k_2 k_3 \ell_2^2}{m (k_1 \ell_1^2 + k_2 \ell_2^2 + k_3 \ell_3^2)}}$

2.84 For pendulum, $\omega_n = \sqrt{g/l}$ in vaccum $= 0.5$ Hz $= \pi$ rad/sec

$$l = g/\pi^2 = 9.81/\pi^2 = 0.9940 \text{ m}$$

$\omega_d = \omega_n\sqrt{1-\zeta^2}$ in viscous medium $= 0.45$ Hz $= 0.9\pi$ rad/sec

$$\zeta^2 = \frac{\omega_n^2 - \omega_d^2}{\omega_n^2} = \left(\frac{1-0.81}{1}\right) =$$

$\zeta = 0.43589$

Equation of motion: $m l^2 \ddot{\theta} + c_t \dot{\theta} + m g l \theta = 0$

$$c_{ct} = 2(m l^2)\,\omega_n = 2(1)(0.994)^2(\pi) = 6.2080$$

Since $\zeta = \dfrac{c_t}{c_{ct}}$ $\qquad c_t = 2.7$ N-m-sec/rad.

2.85 From Eq. (2.85),

$$\ln\left(\frac{x_j}{x_{j+1}}\right) = \ln(18) \Rightarrow \frac{2\pi\zeta}{\sqrt{1-\zeta^2}} = 2.8904$$

$$\zeta = \left\{\frac{(2.8904)^2}{(2.8904)^2 + 4\pi^2}\right\}^{\frac{1}{2}} = 0.4179$$

(a) If damping is doubled, $\zeta_{new} = 0.8358$

$$\ln\left(\frac{x_j}{x_{j+1}}\right) = \frac{2\pi\,\zeta_{new}}{\sqrt{1-\zeta_{new}^2}} = \frac{2\pi(0.8358)}{\sqrt{1-(0.8358)^2}} = 9.5656$$

$$\therefore \quad \frac{x_j}{x_{j+1}} = 14265.362$$

(b) If damping is halved, $\zeta = 0.2090$

$$\ln\left(\frac{x_j}{x_{j+1}}\right) = \frac{2\pi\,\zeta_{new}}{\sqrt{1-\zeta_{new}^2}} = \frac{2\pi(0.2090)}{\sqrt{1-(0.2090)^2}} = 1.3428$$

$$\therefore \quad \frac{x_j}{x_{j+1}} = 3.8296$$

2.88 Equation (2.92) can be expressed as $\delta = \frac{1}{m}\ln\left(\frac{x_0}{x_m}\right)$

For half cycle, $m = \frac{1}{2}$ and hence $\delta = 2\ln\left(\frac{x_0}{x_{\frac{1}{2}}}\right) = 2\ln\left(\frac{1}{0.15}\right)$

$= 3.7942$

Necessary damping ratio ζ_0 is

$$\zeta_0 = \frac{\delta}{\sqrt{(2\pi)^2 + \delta^2}} = \frac{3.7942^2}{\sqrt{4\pi^2 + 3.7942^2}}$$

$= 0.5169$

(a)
If $\zeta = \frac{3}{4}\zeta_0 = 0.3877$, the overshoot can be determined by finding δ from Eq. (2.85):

$$\delta = \frac{2\pi\zeta}{\sqrt{1-\zeta^2}} = \frac{2\pi\,(0.3877)}{\sqrt{1-0.3877^2}} = 2.6427 = 2\ln\left(\frac{x_0}{x_{\frac{1}{2}}}\right)$$

$$\ln\left(\frac{x_0}{x_{\frac{1}{2}}}\right) = 1.32135$$

$$x_{\frac{1}{2}} = x_0 / e^{1.32135} = 0.266775\, x_0$$

$\therefore$ Overshoot is 26.6775%

(b)
If $\zeta = \frac{5}{4}\zeta_0 = 0.6461$, δ is given by

$$\delta = \frac{2\pi\zeta}{\sqrt{1-\zeta^2}} = \frac{2\pi\,(0.6461)}{\sqrt{1-(0.6461)^2}} = 5.3189 = 2\ln\left(\frac{x_0}{x_{\frac{1}{2}}}\right)$$

$$\frac{x_0}{x_{\frac{1}{2}}} = 14.2888 \quad , \qquad x_{\frac{1}{2}} = 0.0700 \; x_0$$

$\therefore$ overshoot = 7%

(2.97) $k = 5000$ N/m , $\quad c_c = 0.2$ N-s/mm $= 200$ N-s/m

$$= 2\sqrt{k\,m} = 2\sqrt{5000\; m}$$

$$m = 2 \text{ kg}$$

$$\omega_n = \sqrt{k/m} = \sqrt{5000/2} = 50 \text{ rad/sec}$$

Logarithmic decrement $= \delta = \dfrac{2\pi\zeta}{\sqrt{1-\zeta^2}} = 2.0$

i.e., $\zeta = \dfrac{c}{c_c} = 0.3033$ and $c = 0.3033\,(0.2) = 60.66$ N-s/m

Assuming $x_0 = 0$ and $\dot{x}_0 = 1$ m/s,

$$x(t) = e^{-\zeta\omega_n t}\,\frac{\dot{x}_0}{\omega_n\sqrt{1-\zeta^2}}\,\sin\sqrt{1-\zeta^2}\;\omega_n t$$

For x_{max}, $\omega_n t \approx \pi/2$ and $\sin\sqrt{1-\zeta^2}\,\omega_n t \approx 1$

$$\therefore \quad x_{max} \simeq e^{-0.3033\,(\pi/2)}\,\frac{1}{50\sqrt{1-0.3033^2}}\,(1) = 0.01303 \text{ m}$$