2.108

(a) $m = 10$ kg, $c = 150$ N-s/m, $k = 1000$ N/m

$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{1000}{10}} = 10$ rad/s

$\zeta = \frac{c}{2 m \omega_n} = \frac{150}{2(10)(10)} = 0.75$

$\omega_d = \omega_n \sqrt{1-\zeta^2} = 10\sqrt{1-0.75^2} = 6.61438$ rad/s

(under-damped)

(b) $m = 10$ kg, $c = 200$ N-s/m, $k = 1000$ N/m

$\omega_n = \sqrt{\frac{k}{m}} = 10$ rad/s

$\zeta = \frac{c}{2 m \omega_n} = \frac{200}{2(10)(10)} = 1.0$

$\omega_d = 10\sqrt{1-1.00^2} = 0$

(critically-damped)

(c) $m = 10$ kg, $c = 250$ N-s/m, $k = 1000$ N/m

$\omega_n = \sqrt{\frac{k}{m}} = 10$ rad/s

$\zeta = \frac{c}{2 m \omega_n} = \frac{250}{2(10)(10)} = 1.25$

$\omega_d =$ not applicable

(over-damped)

2.109

(a) Underdamped system: Response: Eq. (2.70)

$$X_0 = \left\{ x_0^2 + \left(\frac{\dot{x}_0 + \zeta \omega_n x_0}{\omega_d} \right)^2 \right\}^{1/2} \qquad (E.1)$$

Using $x_0 = 0.1$, $\dot{x}_0 = 10$, $\zeta = 0.75$, $\omega_n = 10$, $\omega_d = 6.61438$,

Eq. (E.1) gives $X_0 = 1.62832$ m.

$$\phi_0 = \tan^{-1}\left(- \frac{\dot{x}_0 + \zeta \omega_n x_0}{\omega_d x_0} \right)$$

$$= \tan^{-1}\left(- \frac{10 + 0.75(10)(0.1)}{6.61438(0.1)} \right) = -86.47908^\circ = -1.50935 \text{ rad}$$

Eq. (2.70) gives:

$$x(t) = 1.62832\, e^{-7.5t} \cos(6.61438\, t + 1.50935) \text{ m}$$

(b) Critically damped system: Response: Eq. (2.80)

$$x(t) = \{ x_0 + (\dot{x}_0 + \omega_n x_0)\, t \}\, e^{-\omega_n t}$$

$$= \{0.1 + (10 + 10 * 0.1)\, t\}\, e^{-10t}$$

$$= (0.1 + 11\, t)\, e^{-10t} \quad m$$

(c) overdamped system: Response: Eq. (2.81)

Using $\sqrt{\zeta^2 - 1} = \sqrt{1.25^2 - 1} = 0.75$, we obtain

$$C_1 = \frac{x_0\, \omega_n \{\zeta + \sqrt{\zeta^2 - 1}\} + \dot{x}_0}{2\, \omega_n \sqrt{\zeta^2 - 1}} \qquad \text{Eq. (2.82)}$$

$$= \frac{0.1\,(10)\,\{1.25 + 0.75\} + 10}{2\,(10)\,(0.75)} = 0.8$$

$$C_2 = \frac{-x_0\, \omega_n \{\zeta - \sqrt{\zeta^2 - 1}\} - \dot{x}_0}{2\, \omega_n \sqrt{\zeta^2 - 1}} \qquad \text{Eq. (2.82)}$$

$$= \frac{-0.1\,(10)\,\{1.25 - 0.75\} - 10}{2\,(10)\,(0.75)} = -0.7$$

Eq. (2.81) gives

$$x(t) = C_1\, e^{(-\zeta + \sqrt{\zeta^2 - 1})\, \omega_n t} + C_2\, e^{(-\zeta - \sqrt{\zeta^2 - 1})\, \omega_n t}$$

$$= 0.8\, e^{(-1.25 + 0.75)(10)\, t} - 0.7\, e^{(-1.25 - 0.75)(10)\, t}$$

$$= 0.8\, e^{-5t} - 0.7\, e^{-20t} \quad m$$

2.110 Energy dissipated in a cycle of motion, $x(t) = X \sin \omega_d t$, is given by

$$\Delta W = \pi\, c\, \omega_d\, X^2 \qquad \text{(E.1)}$$

(a) $\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{1000}{10}} = 10$ rad/s

$$\zeta = \frac{c}{2\, m\, \omega_n} = \frac{50}{2\,(10)\,(10)} = 0.25$$

$$\omega_d = \omega_n \sqrt{1 - \zeta^2} = 10 \sqrt{1 - 0.25^2} = 9.682458 \text{ rad/s}$$

For $X = 0.2$ m, Eq. (E.1) gives

$$\Delta W = \pi (50)(9.682458)(0.2^2) = 60.83682 \text{ Joules}$$

(b) $\omega_n = \sqrt{\frac{k}{m}} = 10$ rad/s

$$\zeta = \frac{c}{2 m \omega_n} = \frac{150}{2(10)(10)} = 0.75$$

$$\omega_d = \omega_n \sqrt{1-\zeta^2} = 10 \sqrt{1-0.75^2} = 6.614378 \text{ rad/s}$$

For $X = 0.2$ m, Eq. (E.1) gives

$$\Delta W = \pi (150)(6.614378)(0.2^2) = 124.678385 \text{ Joules}$$

(2.112) $m = 20$ kg, $k = 10000$ N/m, $\frac{4 \mu N}{k} = \frac{150-100}{4}$ mm $= 12.5 \times 10^{-3}$ m

$$\mu = \frac{(12.5 \times 10^{-3})(10\,000)}{4(20 \times 9.81)} = 0.1593$$

Time elapsed $= 4 \tau_n = 4 \times \frac{2\pi}{\omega_n} = 8\pi \sqrt{\frac{m}{k}} = 1.124$ sec