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King Abdulaziz University
Engineering College
Department of MENG
2nd Homework Assignment

Mechanical Vibrations
MENG 470
Spring 1425 H
Due Wed.: 12/1/1425 H

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1. Determine the Fourier series for the rectangular wave shown in the Figure. 1. and then express the series in the exponential form.

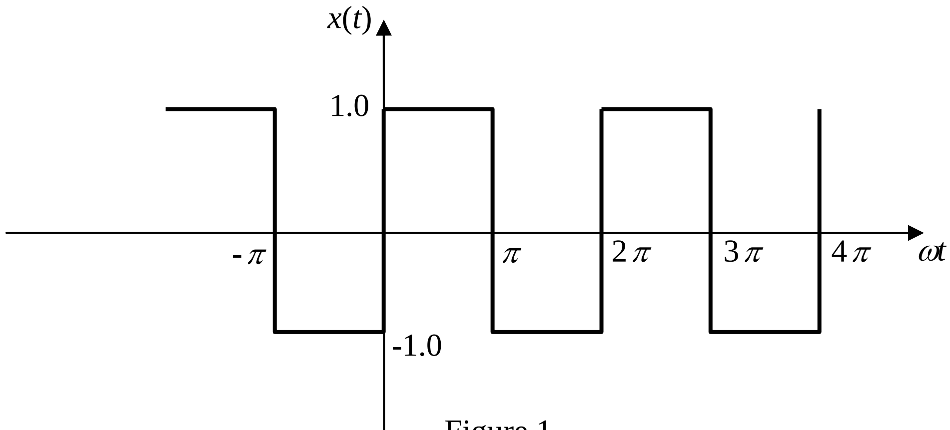
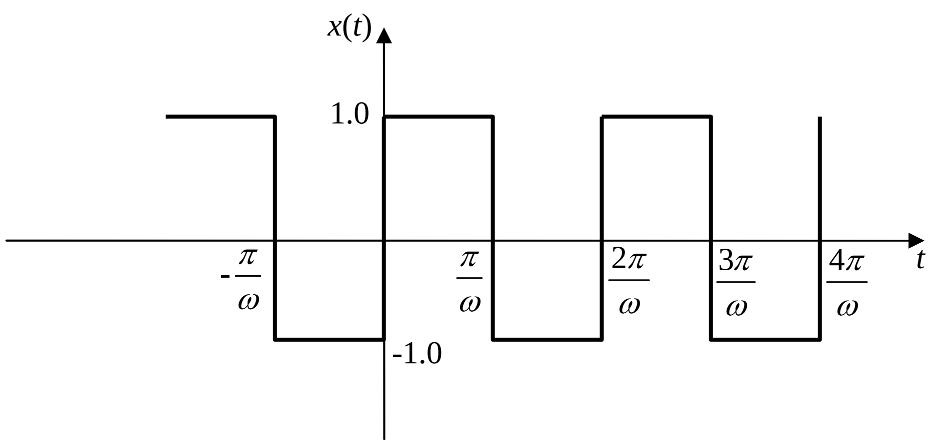


Figure 1

Sol.:



$x(t)$ is odd function. Then, $a_0 = 0$, and $a_n = 0$.

$$b_n = \frac{2}{\tau}\int_{-\tau/2}^{\tau/2} x(t)\sin(n\omega t)dt \qquad \text{where, } \tau = \frac{2\pi}{\omega}$$

$$b_n = \frac{\omega}{\pi}\left[\int_{-\pi/\omega}^{0}(-1)\sin(n\omega t)dt + \int_{0}^{\pi/\omega}(+1)\sin(n\omega t)dt\right]$$

$$= \frac{\omega}{\pi}\left[\frac{\cos n\omega t}{n\omega}\Bigg|_{-\pi/\omega}^{0} - \frac{\cos n\omega t}{n\omega}\Bigg|_{0}^{\pi/\omega} \right] = \frac{2}{n\pi}\left(1-\cos(n\pi)\right)$$

$$b_n = \begin{cases} 0 \ \text{ for } \ n \ \text{ even.} \\ \dfrac{4}{n\pi} \ \text{ for } n \ \text{ odd.} \end{cases}$$

$$x(t) = \frac{4}{\pi}\left(\sin \omega t + \frac{1}{3}\sin 3\omega t + \frac{1}{5}\sin 5\omega t + ... \right)$$

2. Consider the triangular wave shown in Figure 2.

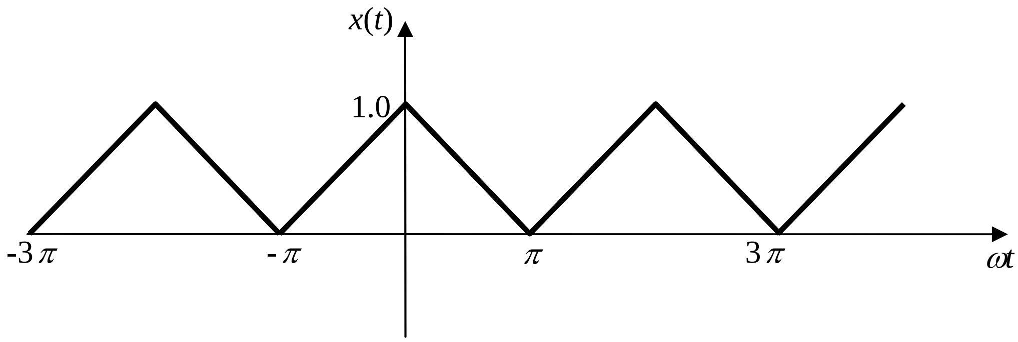


Figure 2

a) Determine the Fourier series in terms of sin and cos functions.
b) Determine the Fourier series in terms of the exponential function.
c) Compare the two results.
d) Plot the frequency spectrum.

Sol.:

$x(t)$ is even function. Then, $b_n = 0$.

$$x(t) = \begin{cases} \dfrac{1}{\pi}(\pi + t) \quad -\pi \le t \le 0. \\ \dfrac{1}{\pi}(\pi - t) \quad 0 \le t \le \pi. \end{cases}$$

$$\frac{1}{2}a_0 \equiv \text{average value} = \frac{1}{2}$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{0} \frac{1}{\pi}(\pi + t)\cos(n\omega t)\, d(\omega t) + \frac{1}{\pi}\int_{0}^{\pi} \frac{1}{\pi}(\pi - t)\cos(n\omega t)\, d(\omega t)$$

$$= \frac{2}{\pi}\frac{\sin n\omega t}{n}\Bigg|_{0}^{n\omega=\pi} - \frac{2}{\pi^2}\left(\frac{\cos n\omega t}{n^2} + \omega t\, \frac{\sin n\omega t}{n}\Bigg|_{0}^{n\omega=\pi} \right)$$

$$a_n = \begin{cases} 0 & \text{for } n \text{ even.} \\ \dfrac{4}{n^2\pi^2} & \text{for } n \text{ odd.} \end{cases}$$

$$x(t) = \frac{1}{2} + \frac{4}{\pi^2}\left(\cos\omega t + \frac{1}{3^2}\cos(3\omega n) + \frac{1}{5^2}\cos(5\omega n) + ...\right)$$

3. For the sawtooth curve shown in the Figure. 3.

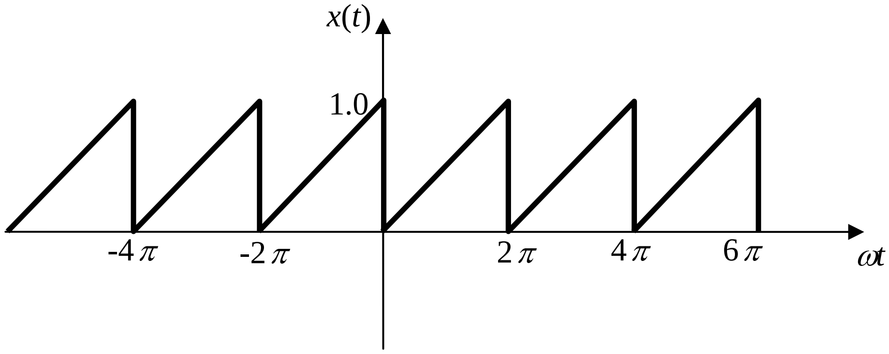


Figure 3

a) Determine the Fourier series in terms of sin and cos functions.
b) Express the Fourier series in the exponential form.
c) Compare the results.

Sol.:

$$x(t) = \frac{\omega t}{2\pi} \qquad 0 \le \omega t \le 2\pi$$

$$c_n = \frac{1}{2\pi}\int_0^{2\pi} \frac{\omega t}{2\pi} e^{-in\omega t} d(\omega t)$$

$$c_n = \frac{1}{(2\pi)^2}\left[\frac{e^{-in\omega t}}{(-in)^2}(-in\omega t - 1)\right]_0^{2\pi}$$

$$c_n = \frac{1}{(2\pi)^2 n^2}\left[-1 + (1 + i\,2\pi n)e^{-i\,2\pi n}\right] = \frac{i}{2\pi}\;\frac{1}{n}$$

$$x(t) = \sum_{-\infty}^{\infty} c_n e^{i\omega nt} = ...-\frac{1}{3}\frac{i}{2\pi}e^{-3i\omega t} - \frac{1}{2}\frac{i}{2\pi}e^{-2i\omega t} - \frac{i}{2\pi}e^{-i\omega t} + \text{c}_0 + \frac{i}{2\pi}e^{i\omega t} + \frac{1}{2}\frac{i}{2\pi}e^{2i\omega t} + \frac{1}{3}\frac{i}{2\pi}e^{3i\omega t} + ...$$

$$x(t) = c_0 + \frac{i}{2\pi}\left[\left(e^{i\omega t} - e^{-i\omega t}\right) + \frac{1}{2}\left(e^{2i\omega t} - e^{-2i\omega t}\right) + \frac{1}{3}\left(e^{3i\omega t} - e^{-3i\omega t}\right) + ...\right]$$

$$x(t) = \frac{1}{2} - \frac{1}{\pi}\left[\sin\omega t + \frac{1}{2}\sin 2\omega t + \frac{1}{3}\sin 3\omega t + ...\right]$$

4. A harmonic motion has a frequency of 10 cps and its maximum velocity is 4.57 m/s. Determine the following:

a) The period.
b) The amplitude.
c) The maximum acceleration.

Sol.:

a) $\omega = 2\pi f = 2\pi(10) = 62.83 \;\; r/s$

$\tau = \dfrac{1}{f} = 0.1 \text{ sec}$

b) $\dot{x}_{\text{max}} = \omega A = 4.57 \text{ m/s}$ $\quad\Rightarrow\quad$ A=0.07274 m

c) $\ddot{x}_{\text{max}} = \omega^2 A = 287.1 \text{ m/s}^2$