بسم الله الرحمن الرحيم

King Abdulaziz University
Engineering College
Department of MENG
3rd Homework Assignment

Mechanical Vibrations
MENG 470
Spring 1425 H
Due Wed.: 19/1/1425 H

---

1) Give the amplitude, frequency, and period of oscillation for the signal illustrated in Figure P1.1.

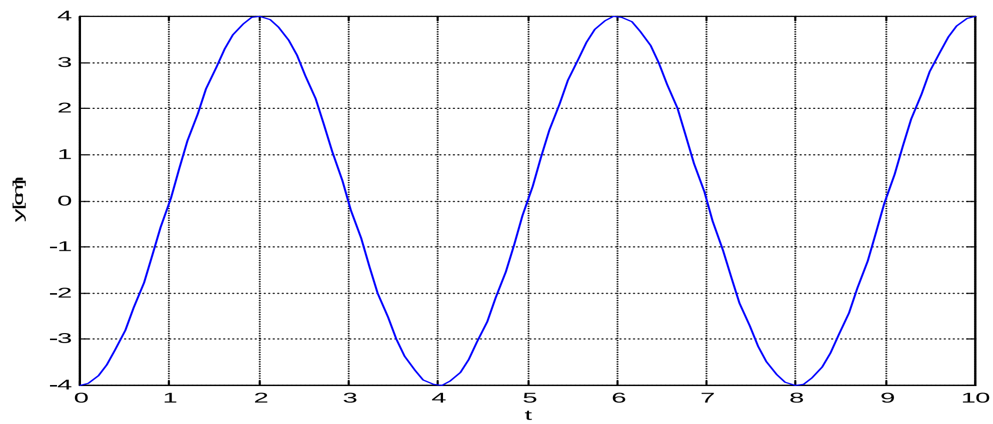


Figure 1

Sol.:

Amplitude = 4

Period = $\tau_n = \dfrac{1}{f_n} = 4$ sec

Frequency = $f_n = \dfrac{1}{\tau_n} = 0.25$ Hz

2) What is the natural frequency for the system illustrated in Figure 2 in terms of m, k_1, k_2, k_3 and k_4?

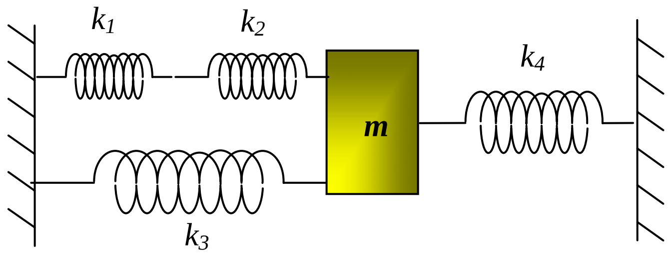


Figure 2

Sol.:

$$\frac{1}{k_{12}} = \frac{1}{k_1} + \frac{1}{k_1}$$

$$\therefore k_{12} = \frac{k_1 k_2}{k_1 + k_2}$$

$$k_{123} = k_{12} + k_3 = \frac{k_1 k_2}{k_1 + k_2} + k_3 = \frac{k_1 k_2 + k_1 k_3 + k_2 k_3}{k_1 + k_2}$$

$$k_{eq} = k_{123} + k_4 = \frac{k_1 k_2 + k_1 k_3 + k_2 k_3}{k_1 + k_2} + k_4 = \frac{k_1 k_2 + k_1 k_3 + k_2 k_3 + k_1 k_4 + k_2 k_4}{k_1 + k_2}$$

$$\because \omega_n = \sqrt{\frac{k_{eq}}{m}}$$

$$\therefore \omega_n = \sqrt{\frac{k_1 k_2 + k_1 k_3 + k_2 k_3 + k_1 k_4 + k_2 k_4}{m\left(k_1 + k_2\right)}}$$

3) Derive the equation of motion of the system shown in Figure 3, using the following methods: (a) Newton's second law of motion, (b) D'Alembert's principle, (c) principle of virtual work, and (d) principle of conservation of energy.

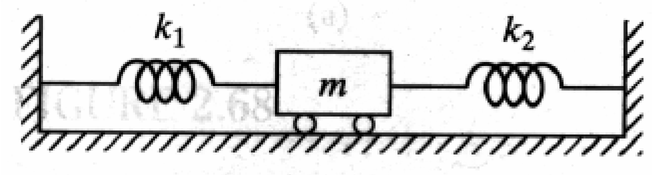
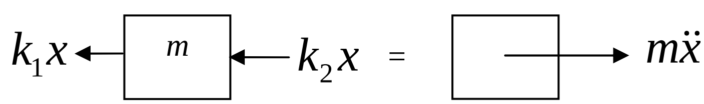


Figure 3

Sol.:

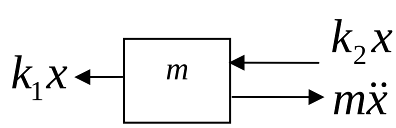
a) Newton's Second Law of motion:

$$F(t) = -k_1 x - k_2 x = m\ddot{x}$$
$$m\ddot{x} + (k_1 + k_2)x = 0$$



b) D'Alembert Principle:

$$F(t) - m\ddot{x} = 0$$
$$(k_1 + k_2)x - (-m\ddot{x}) = 0$$
$$m\ddot{x} + (k_1 + k_2)x = 0$$



c) Principle of Virtual Work:

When mass m is given a virtual displacement δx

Virtual work done by the spring force = $-(k_1 + k_2)x\delta x$

Virtual work done by the inertia force = $-(m\ddot{x})\delta x$

The total virtual work done by all forces = 0

$$-m\ddot{x}\delta x - (k_1 + k_2)x\delta = 0$$
$$m\ddot{x} + (k_1 + k_2)x = 0$$

d)

$$T = \frac{1}{2} m\dot{x}^2$$

$$U = \frac{1}{2}(k_1 + k_2)x^2$$

$$T + U = Const.$$

$$\frac{d}{dt}(T + U) = 0$$

$$m\ddot{x} + (k_1 + k_2)x = 0$$

4) Draw the free-body diagram and derive the equation of motion using Newton's second law of motion for the system shown in Figure 4.

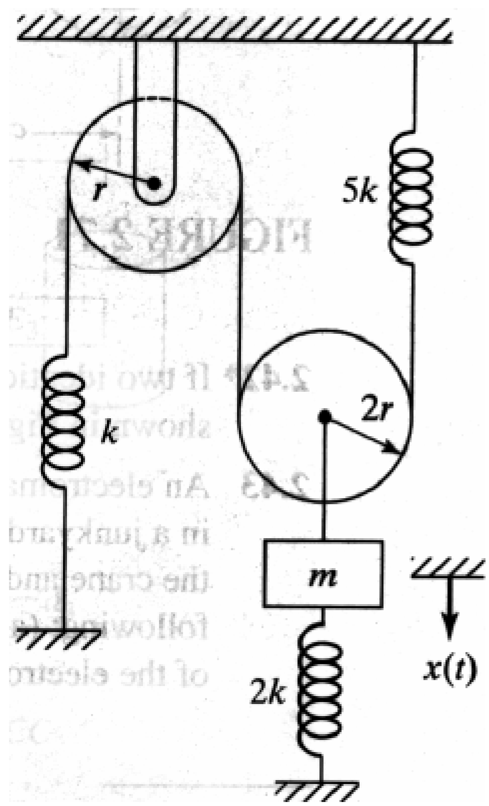
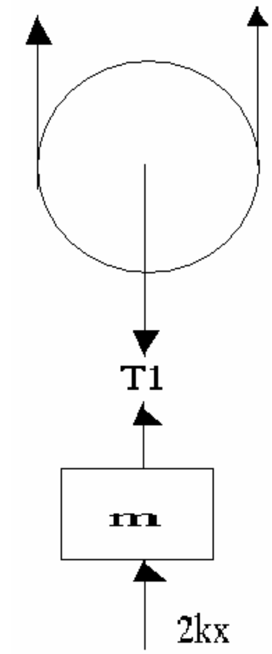


Figure 4

Sol.:



Consider the springs connected to the pulleys (by rope) to be in series. Then:

$$\frac{1}{k_{eq}} = \frac{1}{k} + \frac{1}{5k}$$

$$\therefore k_{eq} = \frac{5}{6}k$$

Let the displacement of mass m be x.

Then the extension of the rope (Spring Connected to the pulleys) $= 2x$.

From the free body diagram , the equation of motion m becomes:

$$m\ddot{x} + 2kx + k_{eq} = 0$$

$$\therefore m\ddot{x} + \frac{11}{3}kx = 0$$

4) Draw the free-body diagram and derive the equation of motion using Newton's second law of motion for the system shown in Figure 5.

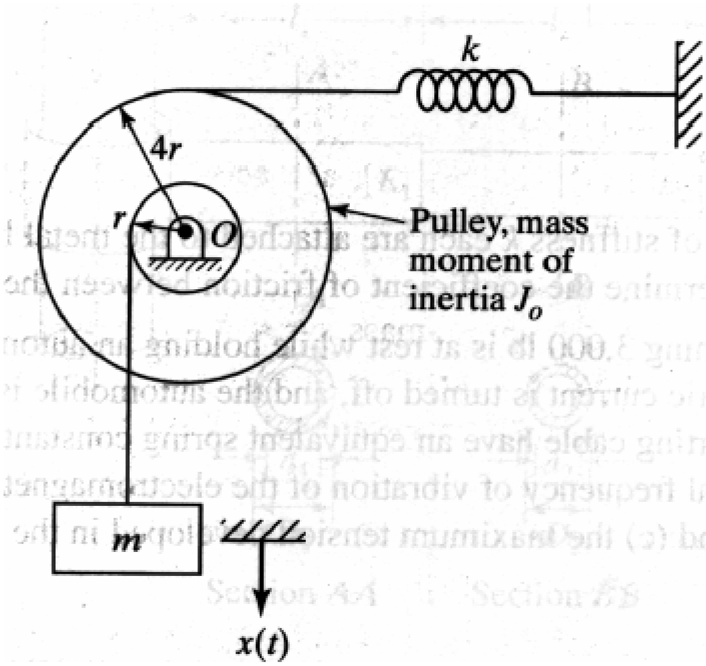


Figure 5

Sol.:

Equation of Motions:

Mass m : $mg - T = m\ddot{x}$ (1)

Pulley J_0 : $J_0\theta = Tr - k4r(\theta - \theta_0)4r$ (2)

Where θ_0 is the angular deflection of the pulley under the weight, mg, given by:

$mgr = k(4r\theta_0)4r$ or $\theta_0 = \dfrac{mg}{16rk}$ (3)

Subs. Eqs. (1) and (3) into (2):

$$J_0\ddot{\theta} = (mg - m\ddot{x})r - k16r^2(\theta + \frac{mg}{16rk}) \quad (4)$$

Using $x = r\theta$ and $\ddot{x} = r\ddot{\theta}$, Eq. (4) becomes:

$$J_0 + mr^2)\ddot{\theta} + (16r^2k)\theta = 0$$

Alternative Method (Conservation of Energies):

$$\text{Kinetic Energy} = T = T_{mass} + T_{pulley}$$

$$= \frac{1}{2} m\dot{x}^2 + \frac{1}{2} J_0 \dot{\theta}^2 = \frac{1}{2}(mr^2 + J_0)\,\dot{\theta}^2$$

$$\text{Potential Energy} = U = \frac{1}{2} k x_s{}^2$$

$$= \frac{1}{2} k(4r\theta)^2$$

$$= \frac{1}{2} k(16r^2)\theta^2$$

Using $\frac{d}{dt}(T+U) = 0$, gives:

$$(mr^2 + J_0)\ddot{\theta} + (16r^2 k)\theta = 0$$

6) Derive the equation of motion of the system shown in Figure 6, using the following methods:

(a) Newton's second law of motion,
(b) D' Alembert's principle.
(c) Principle of virtual work.

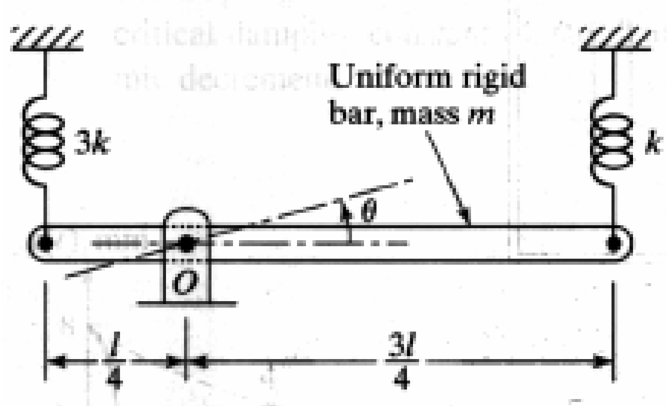


Figure 6

Sol.:

let θ be measured from static equilibrium position so that the force of gravity need not be considered.

a) Newton′s second law of motion

$$J_0\ddot{\theta} = -3k\left(\theta\frac{l}{4}\right)\frac{l}{4} - k\left(\theta\frac{3l}{4}\right)\left(\frac{3l}{4}\right)$$

$$J_0\ddot{\theta} + \frac{3}{4}kl^2\theta = 0$$

b) D′Alembert′s Principle

$$M(t) - J_0\ddot{\theta} = 0$$

$$-3k\left(\theta\frac{l}{4}\right)\frac{l}{4} - k\left(\theta\frac{3l}{4}\right)\left(\frac{3l}{4}\right) - J_0\ddot{\theta} = 0$$

$$J_0\ddot{\theta} + \frac{3}{4}kl^2\theta = 0$$

c) Principle of Virtual Work

virtual work done by the spring force:

$$\delta W_s = -3k\left(\theta\frac{l}{4}\right)\left(\frac{l}{4}\delta\theta\right) - k\left(\theta\frac{3l}{4}\right)\left(\frac{3l}{4}\delta\theta\right)$$

the virtual work done by the moment of inertia:

$$\delta W_m == -\left(J_0\ddot{\theta}\right)\delta\theta$$

Setting the total of virtual work done by all forces and moments equal to zero,we obtain:

$$J_0\ddot{\theta} + \frac{3}{4}kl^2\theta = 0$$