بسم الله الرحمن الرحيم

King Abdulaziz University	**Mechanical Vibrations**
Engineering College	**MENG 470**
Department of MENG	**Spring 1425 H**
3rd Homework Assignment	**Due Wed.: 19/1/1425 H**

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1) Consider the ODE of motion of 1-DOF system under damped free vibration:

$$\ddot{x} + 2\zeta\omega_n\dot{x} + \omega_n^2 = 0 \quad x(0) = 1;\ \dot{x}(0) = 0$$

a. For w $_n$ = 2, z = 0.1

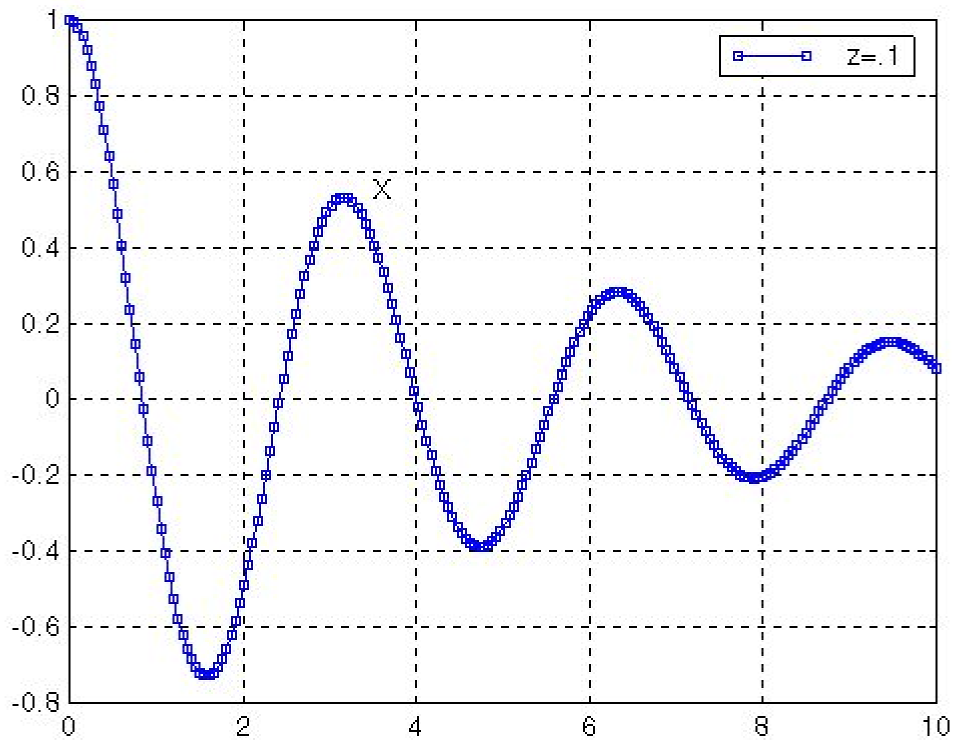
   Plot the solution for 0 £ t £ 10

   Label the coordinates

b. For w $_n$ = 2 plot the solution (0 £ t £ 10) for z = 0.1, 0.4, and 0.99 on the same figure. Label coordinates and denote each curve by its value of z
c. Repeat (b) for z = 0.1, 0.4, 0.99, 2, and 5.

Sol. :

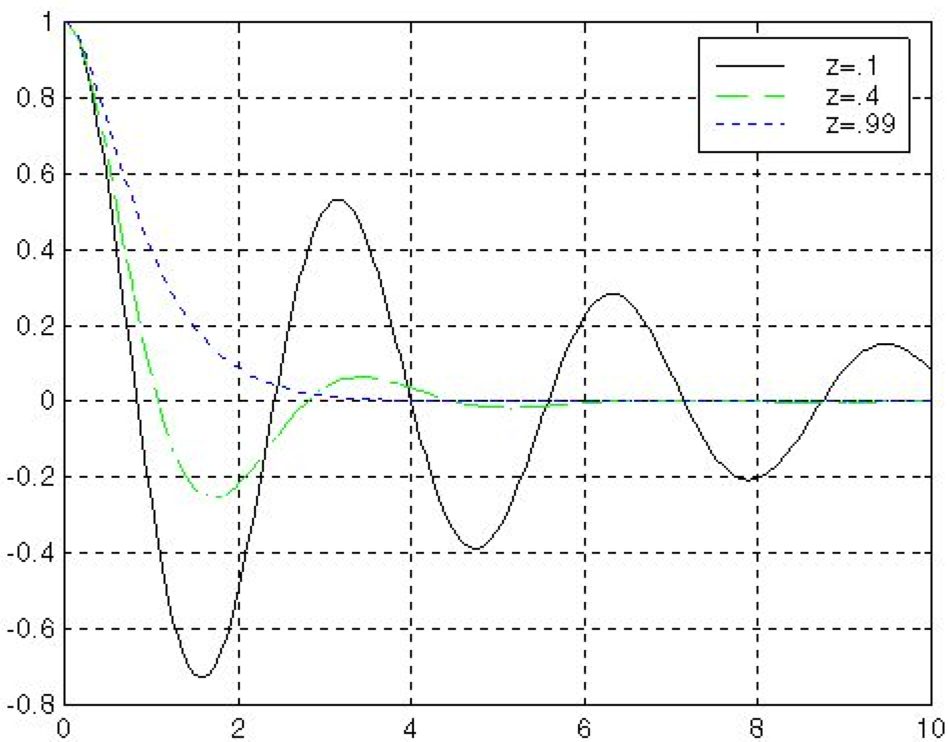
```
x0=1; xdot0=0; wn=2;

z=.1;

t=linspace(0,10,200);

wd=wn*sqrt(1-z.^2);

x=exp(-z*wn*t).*(((xdot0+z.*wn*x0)./wd).*sin(wd.*t) +
x0.*cos(wd.*t));

plot(t,x,'marker','square','markersize',4)

legend('z=.1')

grid on

text(3.5,.55,'X')
```



b)

```
x0=1; xdot0=0.; wn=2;

z=[.1 .4 .99];

t=linspace(0,10,200);

for n=1:3

wd=wn*sqrt(1-z(n).^2);

% x=x0.*exp(-2.*wn.*z(n).*t).*cos(wd.*t);

x=exp(-z(n).*wn.*t).*(((xdot0+z(n).*wn.*x0)./wd).*sin(wd.*t) +
x0.*cos(wd.*t));

if n==1

sym ='k-';

elseif n==2

sym = 'g--';
```

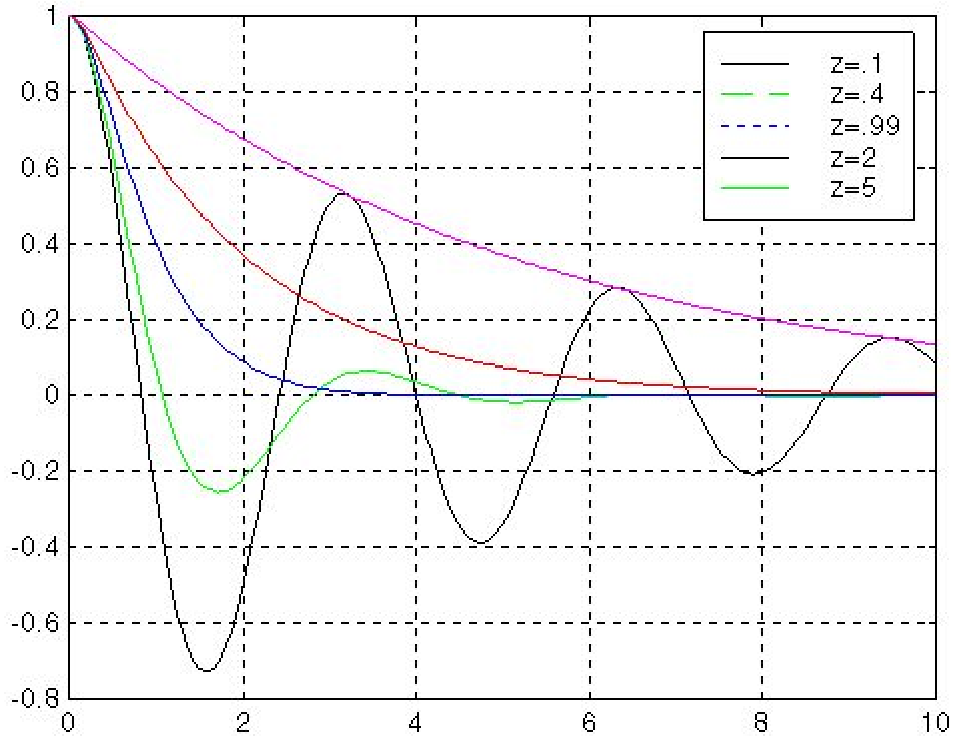
```
else
sym = 'b:';
end
plot(t,x,sym)
legend('z=.1','z=.4','z=.99')
grid on
hold on
end
```



c)

```
x0=1; wn=2;

z=[.1 .4 .99 2 5];

t=linspace(0,10,200);

sym = ['k' 'g' 'b' 'r' 'm'];

for n=1:5

if z(n)<1

wd=wn*sqrt(1-z(n).^2);

x=exp(-z(n).*wn.*t).*(((xdot0+z(n).*wn.*x0)./wd).*sin(wd.*t) +
x0.*cos(wd.*t));

plot(t,x,sym(n))

grid on

hold on

else

r1=-wn*z(n) + wn*sqrt(z(n)^2 - 1);

r2=-wn*z(n) - wn*sqrt(z(n)^2 - 1);

a=r2/(r2-r1);

b=-r1/(r2-r1);

x=a*exp(r1*t) + b*exp(r2*t);

plot(t,x,sym(n))

legend ('z=.1','z=.4','z=.99','z=2','z=5')

grid on

hold on

end
```

```
end
```



2) A vibrating system consisting of a mass of 2.267 kg and a spring of stiffness 17.5 N/cm is viscously damped such that the ratio of any two consecutive amplitudes is 1.00 and 0.98. Determine:

a. The natural frequency of the damped system.
b. The logarithmic decrement.
c. The damping factor
d. The damping coefficient.

Sol.:

$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{1750}{2.267}} = 27.78$ rad/s

$\delta = \ln\left(\frac{X_1}{X_2}\right) = \ln\left(\frac{1}{0.98}\right) = 2.02x10^{-2}$

$\zeta = \sqrt{\frac{\delta^2}{4\pi^2 + \delta^2}} = 3.215x10^{-3}$

$C = 2\zeta m\omega_n = 0.405$ kg/s

3) A vibrating system consists of a mass of 4.534 kg, a spring of stiffness 35.0 N/cm, and a dashpot with a damping coefficient of 0.1243 N/cm/s. Find:

a. The damping factor.
b. The logarithmic decrement.
c. The ratio of any two consecutive amplitudes.

Sol.:

a. $\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{3500}{4.534}} = 27.78$ rad/s

$\zeta = \frac{c}{2m\omega_n} = 4.934x10^{-2}$

b. $\delta = \frac{2\pi\zeta}{\sqrt{1-\zeta^2}} = 0.3104$

c. $\frac{x_1}{x_2} = e^{\delta} = 1.364$

4) A vibrating system has the following constants: m = 17.5 kg, k = 70.0 N/cm, and c = 0.70 N/cm/s. Determine:

a. The damping ratio.
b. The natural frequency of damped oscillation.
c. The logarithmic decrement.
d. The ratio of any two consecutive amplitudes.

Sol.:

a. $\zeta = \dfrac{C}{2\sqrt{k\ m}} = 0.1$

b. $\omega_d = \sqrt{\dfrac{k}{m}\left(1-\zeta^2\right)} = 19.9$ rad/s

c. $\delta = \dfrac{2\pi\zeta}{\sqrt{1-\zeta^2}} = 0.6315$

d. $\dfrac{x_1}{x_2} = e^{\delta} = 1.88$

5) Set up the differential equation of motion for the system shown in Figure 1. Determine the expression for:

a. The critical damping coefficient
b. The natural frequency of damped oscillation.

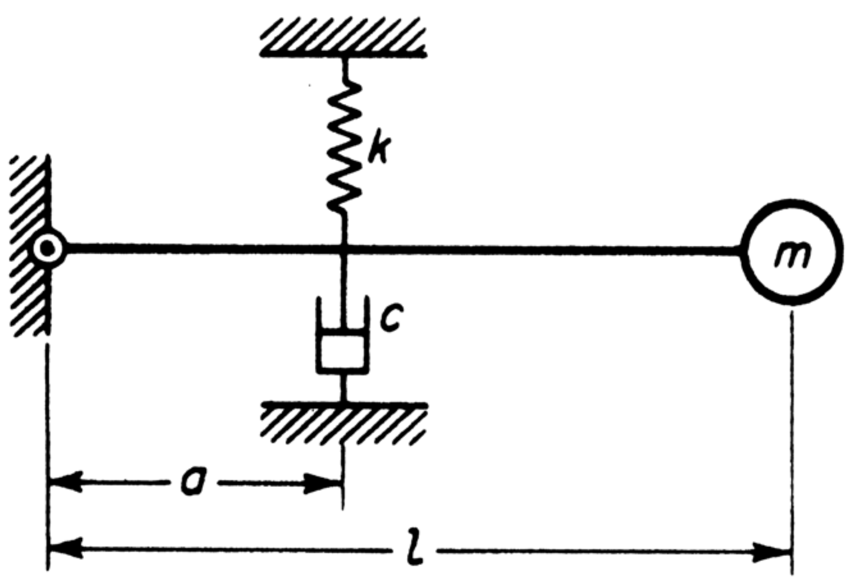


Figure 1

Sol.:

$$\sum M_o = I_o\ddot{\theta}$$

$$-ka\theta - Ca\dot{\theta} = ml^2\ddot{\theta}$$

$$\ddot{\theta} + \frac{c}{m}\left(\frac{a}{l}\right)^2\dot{\theta} + \frac{k}{m}\left(\frac{a}{l}\right)^2\theta = 0$$

$$\omega_n = \sqrt{\frac{k}{m}}\left(\frac{a}{l}\right)$$

$$2\zeta\omega_n = \frac{c}{m}\left(\frac{a}{l}\right)^2$$

$$\zeta = \frac{\frac{c}{m}\left(\frac{a}{l}\right)^2}{2\omega_n} = \frac{\frac{c}{m}\left(\frac{a}{l}\right)^2}{2\left(\sqrt{\frac{k}{m}}\left(\frac{a}{l}\right)\right)} = \frac{c}{2\sqrt{km}}\left(\frac{a}{l}\right)$$

$$\omega_d = \omega_n\sqrt{1-\zeta^2} = \frac{a}{l}\sqrt{\frac{k}{m} - \left(\frac{ca}{2l\sqrt{km}}\right)^2}$$

6) Write the differential equation of motion for the system shown in Figure 2 and determine the natural frequency of damped oscillation and the critical damping coefficient.

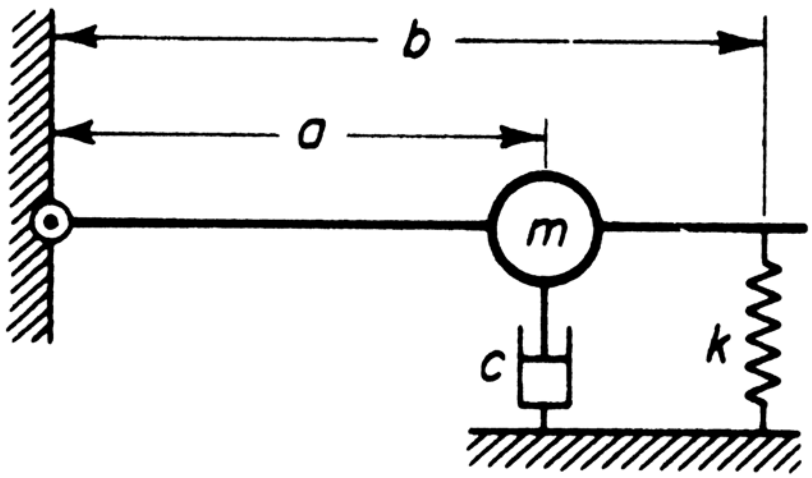


Figure 2

Sol.:

$$\sum M_o = I_o\ddot{\theta}$$

$$-kb^2\theta - ca^2\dot{\theta} = ma^2\ddot{\theta}$$

$$\ddot{\theta} + \frac{c}{m}\dot{\theta} + \frac{k}{m}\frac{b^2}{a^2}\theta = 0 \qquad \text{E.O.M}$$

$$\omega_n = \frac{b}{a}\sqrt{\frac{k}{m}}$$

$$\omega_d = \sqrt{\frac{kb^2}{ma^2} - \left(\frac{c}{2m}\right)^2}$$

$$c_{cr} = \frac{2b}{a}\sqrt{km}$$

7) A spring-mass system with viscous damping is displaced from the equilibrium position and released. If the amplitude diminished by 5% each cycle, what fraction of the critical damping does the system have?

Sol.:

$$\delta = \ln\left(\frac{X_1}{X_2}\right) = \ln\left(\frac{1}{0.95}\right) = 5.129x10^{-2}$$

$$\zeta = \sqrt{\frac{\delta^2}{4\pi^2 + \delta^2}} = 8.163x10^{-3}$$

8) A rigid Uniform bar of mass *m* and length *l* is pinned at *O* and supported by a spring and viscous damper, as sown in Figure 3. Measuring θ from the static equilibrium position, Determine:

a. The equation for small θ (the moment of inertia of the bar about *O* is $ml^2/3$).
b. The equation for the undamped natural frequency.
c. The expression for critical damping. Use Virtual Work and D'Alembert's Principles.

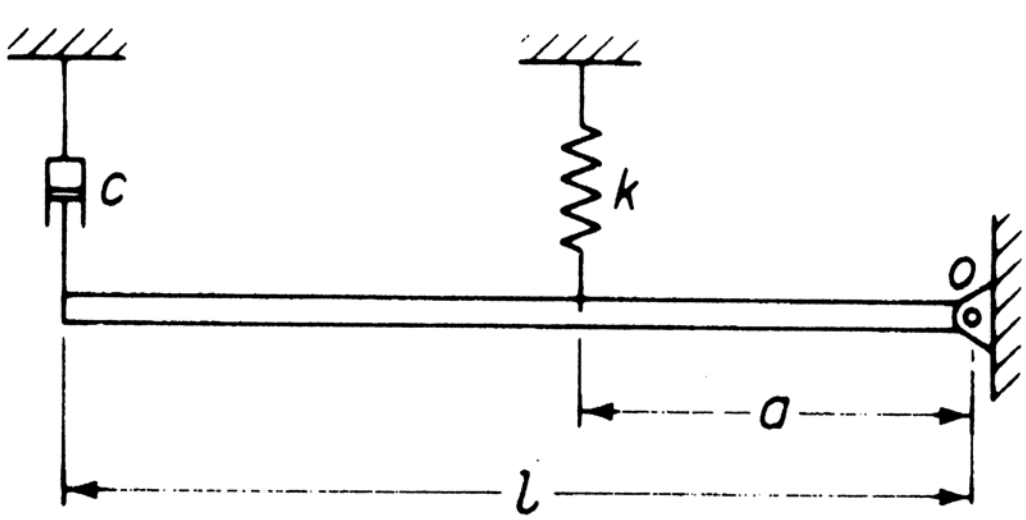


Figure 3

Sol.:

a. Using Newton's 2nd law, and considering linear vibrations, (i.e. θ is very small, so $\sin\theta = \theta$, and $\cos\theta = 1$), the equation of mation is:

$$\sum M_o = I_o \ddot{\theta} \qquad \mathrm{I}_o = ml^2$$

$$\ddot{\theta} + \frac{3c}{m}\dot{\theta} + \frac{3ka^2}{ml^2}\theta = 0$$

In case of using the principle of Virtual Work,

$$\delta W = -\frac{ml^2}{3}\ddot{\theta}\ \delta\theta - cl\dot{\theta}\ \delta\theta - ka\theta\ a\delta\theta = 0$$

Hence, $\ddot{\theta} + \frac{3c}{m}\dot{\theta} + \frac{3ka^2}{ml^2}\theta = 0$

b.

$$2\zeta\omega_{\mathrm{n}} = \frac{3c}{m}$$

$$\omega_n^2 = \frac{3ka^2}{ml^2}$$

$$\omega_n = \frac{a}{l}\sqrt{\frac{3k}{m}}$$

$$c_{cr} = \frac{3a}{2l}\sqrt{3km}$$

$$\zeta = \frac{3c}{2m\omega_{\mathrm{n}}}$$

c.

$$\omega_d = \omega_{\mathrm{n}}\sqrt{1-\zeta^2} = \frac{a}{l}\sqrt{\frac{3k}{m}\left(1 - \frac{3}{4km}\left(\frac{cl}{a}\right)^2\right)}$$