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Mechanical Vibrations
MENG 470
First Exam

Closed Book Exam
Time: 2 Hours
Tuesday: 9/6/1425 H

1) A harmonic motion has an amplitude of *0.20 cm* and a period of *0.15 s*. Determine the maximum velocity and acceleration.

$A = 0.02\,m$

& $\tau = 0.15\,s \Rightarrow \omega = \frac{2\pi}{\tau} = \frac{2\pi}{0.15} \Rightarrow \omega = 41.5\ rad/s$

$\dot{x} = \omega A \cos \omega t \Rightarrow \dot{x}_{max} = \omega A = 41.5(0.02\,m)$

$\dot{x}_{max} = 0.84\ m/s$

$\ddot{x} = -\omega^2 A \sin \omega t \Rightarrow \ddot{x}_{max} = \omega^2 A = (41.5)^2(0.02\,m)$

$\Rightarrow \ddot{x}_{max} = 35\ m/s^2$

2) A harmonic motion has a frequency of *10 cps* (*cps = cycles per second=Hz*) and its maximum velocity is *4.57 m/s*. Determine its amplitude, its period, and its maximum acceleration.

$f = 10\ Hz$ & $\omega A = 4.57\ m/s$

$\omega = 2\pi f = 2\pi(10) = 62.8\ rad/s$

$\therefore A = \frac{4.57}{62.8} \Rightarrow A = 0.07\ m$

$\tau = \frac{1}{f} = 0.1\ sec$

$\ddot{x}_{max} = \omega^2 A = (62.8)^2(0.07) \Rightarrow \ddot{x}_{max} \cong 276.1\ m/s^2$

3) A machine of mass m=500 kg is mounted on a simply supported steel beam of length l = 2m having a rectangular cross-section (*depth = 0.1 m, width = 1.2 m*) and Young's modulus E=2.06 e^{11} N/m^2. To reduce the vertical deflection of the beam, a spring of stiffness k is attached at the mid-span, as shown below in Figure 1. Determine the value of k needed to reduce the deflection of the beam to one-third of its original value. Assume that the mass of the beam is negligible.

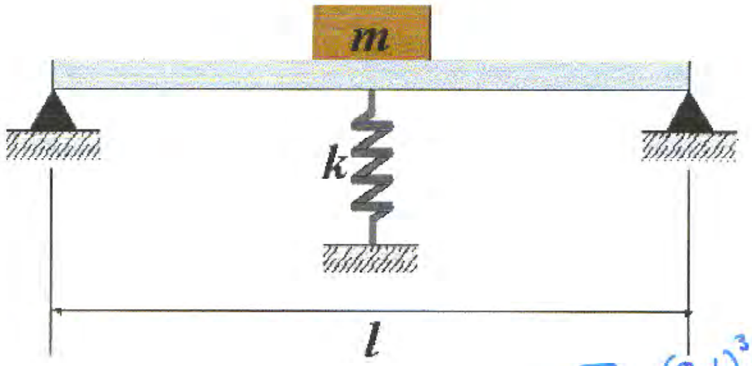


Figure 1

$$I = \frac{(0.1)^3(1.2)}{12} = 10^{-4} m^4$$

For simply supported beam,
for load at middle, $k_1 = \frac{48EI}{l^3} = \frac{48(2.06\times10^{11})(10^{-4})}{(2)^3}$

$$k_1 = 12.36\times10^7 \ N/m$$

$$\delta_1 = \text{original deflection} = \frac{mg}{k_1} = \frac{500(9.81)}{12.36\times10^7}$$

$$\Rightarrow \delta_1 = 396.84 \times 10^{-7} \ m$$

New deflection $= \frac{mg}{k_{eq}} = \frac{\delta_1}{3}$; $k_{eq} = \frac{3mg}{\delta_1}$

where, $k_{eq} = k + k_1$. $\quad k_{eq} = \frac{3(500)(9.81)}{396.84\times10^{-7}} = 370.8 \ KN/m$

$$\therefore k = k_{eq} - k_1 = 370.8 - 123.6$$

$$\Rightarrow \boxed{K = 247.2 \ KN}$$

4) Write the equation of motion for the spring-mass system shown in Figure 2. Let its displacement $x(t)$ be measured from:

a) the position for which both springs are unstretched. What is the natural frequency of the system?

b) the **static equilibrium position** of the system.

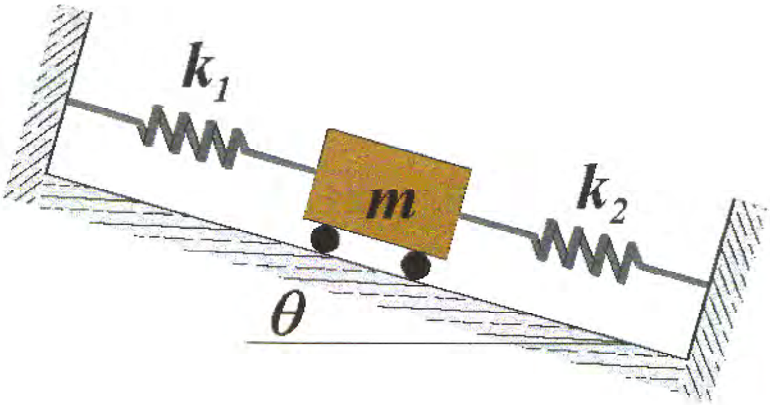
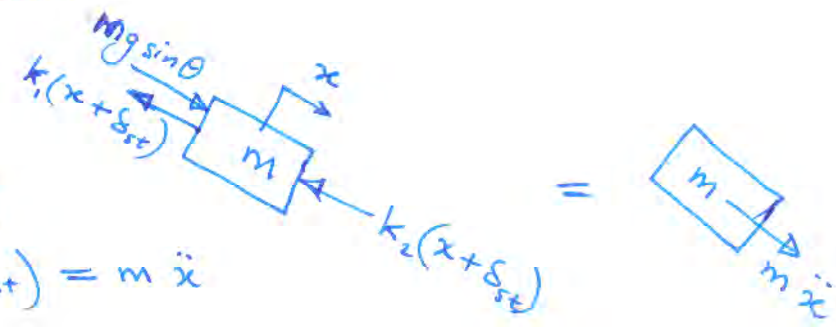


Figure 2

a) Equation of motion:

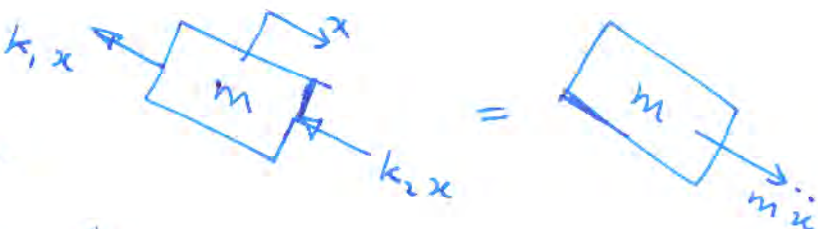


$$mg\sin\theta - k_1(x+\delta_{st}) - k_2(x+\delta_{st}) = m\ddot{x}$$

But, $\delta_{st}(k_1+k_2) = W\sin\theta$ (From static equilibrium)

$$\therefore\ m\ddot{x} + (k_1+k_2)x = 0 \quad \& \quad \omega_n = \sqrt{\frac{k_1+k_2}{m}}$$

b) Equation of Motion:



$$-k_1 x - k_2 x = m\ddot{x}$$

$$\therefore\ m\ddot{x} + (k_1+k_2)x = 0$$

a) The nature of damping is Viscous damping.

b) $\tau_d = 0.2$ sec

$f_d = 5$ Hz

$\omega_d = 31.416$ rad/sec

c) $$\frac{x_i}{x_{i+1}} = e^{\xi \omega_n \tau_d} \Rightarrow \ln\left(\frac{x_i}{x_{i+1}}\right) = \frac{2\pi\xi}{\sqrt{1-\xi^2}}$$

$$\ln\left(\frac{x_i}{x_{i+1}}\right) = \ln 2 = 0.6931$$

$$\therefore \frac{2\pi\xi}{\sqrt{1-\xi^2}} = 0.6931 \Rightarrow \boxed{\xi = 0.1096}$$

$$\because \omega_d = \omega_n \sqrt{1-\xi^2}$$

$$\therefore \omega_n = \frac{\omega_d}{\sqrt{1-\xi^2}} = \frac{31.416}{\sqrt{0.98798}} = 31.61 \text{ rad/sec}$$

$$\therefore \boxed{\omega_n = 31.61 \text{ rad/sec}}$$

$$\because \omega_n^2 = \frac{k}{m} \Rightarrow k = m\omega_n^2 = \frac{500}{9.81}(31.61)^2$$

$$\Rightarrow \boxed{k = 50.92 \text{ kN/m}}$$

Hence $c = 2m\omega_n\xi = 2\left(\frac{500}{9.81}\right)(31.61)(0.1096)$

$$\Rightarrow \boxed{c = 353.11 \text{ N.s/m}}$$