بسم الله الرحمن الرحيم

King Abdulaziz University
Engineering College
Department of Production and Mechanical System Design

MENG 470 Mechanical Vibrations

First Exam Solution
Closed-book Exam
Monday: 8/2/1425 H
Time Allowed: 90 mins

Question 1	5	5
Question 2	5	5
Question 3	5	5
TOTAL	15	15

Instructions

1. There are totally 3 problems in this exam.
2. This is a closed book and closed notes Opportunity to Shine
3. Show all work for partial credit.
4. Assemble your work for each problem in logical order.
5. Justify your conclusion. I cannot read minds.

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Mechanical Vibrations	Closed Book Exam
MENG 470	Time: 1 Hour
First Exam	Monday: 8/2/1425 H

1. The single degree-of-freedom harmonic oscillator shown below in Figure 1 has a logarithmic decrement and static deflection equal to 1.435 and $0.765m$, respectively. The spring stiffness is $k = 525\,N/m$.

   (a) Show the characteristic roots of the system in the complex plane.

   (b) Indicate the free response type (i.e., undamped, underdamped, critically damped, or overdamped).

   (c) If the oscillator is subjected to a (vertical) harmonic force with magnitude f_o and frequency $0.477Hz$, determine the range of f_o such that the steady-state amplitude of vibration is less than $0.798cm$.

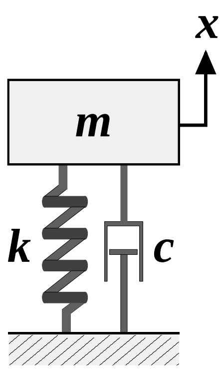


Figure 1

Sol.:

(a)

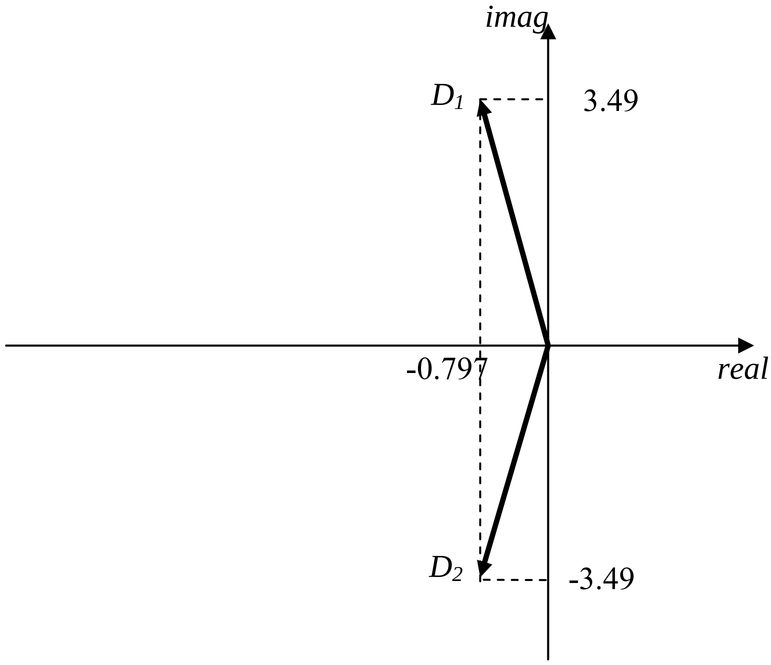
$$\omega_n = \sqrt{\frac{g}{\delta_{st}}} = \sqrt{\frac{9.81}{0.765}} = 3.581 \text{ rad/sec}$$

$$where \qquad \zeta = \frac{\delta}{\sqrt{(2\pi)^2 + \delta^2}} = \frac{1.435}{\sqrt{(2\pi)^2 + 1.435^2}} = 0.223$$

$$\omega = 2\pi f = 2\pi(0.477) = 2.997 \text{ rad/sec}$$

The characteristics roots are:

$$D_{1,2} = -\zeta\omega_n \pm \omega_n\sqrt{\zeta^2 - 1} = -(0.223)(3.581) \pm (3.581)\sqrt{(0.223)^2 - 1} = -0.797 \pm i\,3.49$$



(b) Since the dumping ratio = 0.22 (i.e. 0 < ζ < 1), then the system is underdamped.

(c) f = 0.477 Hz , $X_{\max}$ = 0.00798 m,

$\omega_n = 3.581$ rad/sec, $k = 525$ N/m, and $\zeta = 0.22$

$$m = \frac{k}{\omega_n^{\,2}} = \frac{525}{3.581^2} = 40.94 \quad kg$$

$$c = 2\zeta\omega_n m = 2(0.223)(3.581)(40.94) = 65.39 \text{ rad.kg/sec}$$

$$\omega = 0.477(2\pi) = 2.997 \text{ rad/sec}$$

$$\left|(f_0)_{\max}\right| = X\sqrt{(k - m\omega^2) + (c\omega)^2}$$

$$\left|(f_0)_{\max}\right| = 0.00798\sqrt{(525 - 40.94(2.997^2))^2 + \left((65.39)(2.997)\right)^2} = 2 \text{ N}$$

OR: $r = \dfrac{\omega}{\omega_n} = \dfrac{2.997}{3.581} = 0.837$

$$\left|(f_0)_{\max}\right| = kX\sqrt{(1 - r^2) + (2\zeta r)^2} = 525(0.00798)\sqrt{(1 - (0.837)^2) + (2(0.223)(0.837))^2} = 2 \text{ N}$$

The range of f_0 is as following: $-2 \text{ N} < f_0 < 2 \text{ N}$

2. Consider the system shown below, which consists of a uniform rigid bar *OA* that is welded to a uniform disk. The composite system is pinned to ground at point *0*. Using the parameters indicated in the Figure 2, determine a) the equation of motion for the system in terms of the angular coordinate $\theta(t)$ and b) the critical damping constant.

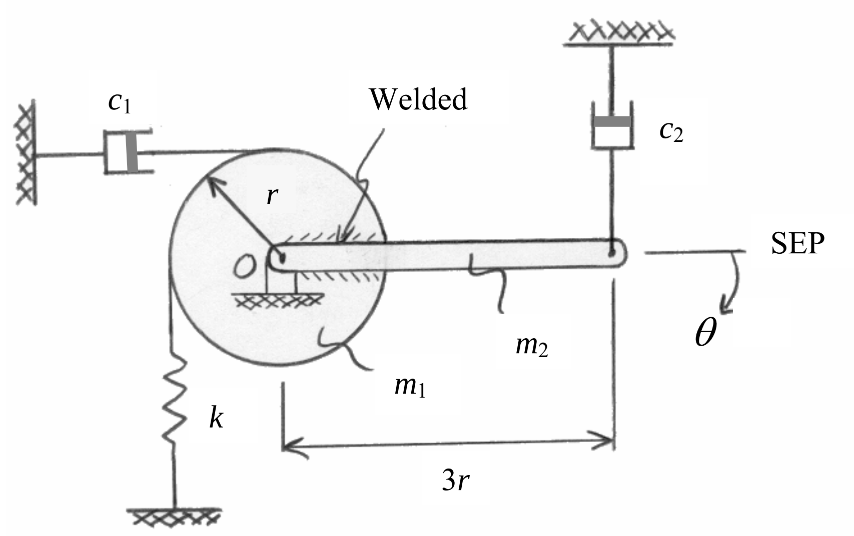


Figure 2

Sol.:

$$F_{c1} = c_1 r\dot{\theta} \quad , \quad F_{c2} = 3c_2 r\dot{\theta} \quad , \quad F_k = kr\theta$$

$$J_0 = \frac{1}{2}m_1 r^2 + \frac{1}{3}m_2 (3r)^2$$

$$\sum M_0 = -J_0\ddot{\theta}$$

$$kr^2\theta + c_2 (3r)^2\dot{\theta} + c_1 r^2\dot{\theta} = -(\frac{1}{2}m_1 r^2 + 3m_2 r^2)\ddot{\theta}$$

$$(\frac{1}{2}m_1 + 3m_2)r^2\ddot{\theta} + (c_1 + 9c_2)r^2 \dot{\theta} + kr^2 \theta = 0$$

$$(\frac{1}{2}m_1 + 3m_2)\ddot{\theta} + (c_1 + 9c_2)\dot{\theta} + k \theta = 0$$

$$m_{eq} = \frac{1}{2}m_1 + 3m_2$$

$$c_{eq} = c_1 + 9c_2$$

$$k_{eq} = k$$

$$c_{cr} = 2\sqrt{m_{eq}k_{eq}} = 2\sqrt{\left(\frac{m_1}{2} + 3m_2\right)k}$$

3. Also late for a dinner party, Ahmed is driving his MAZDA on *old Makkah road*. Since Ahmed is a poor composer, he cannot afford to replace the bad shocks on his vehicle. They have lost most of their damping properties and behave essentially like springs.

Amanat Jeddah has halted old Makkah road maintenance for some reason. As a result, the road becomes very bumpy now. If the road surface can be approximately described by $y(t) = y_0 \sin(\omega t)$,

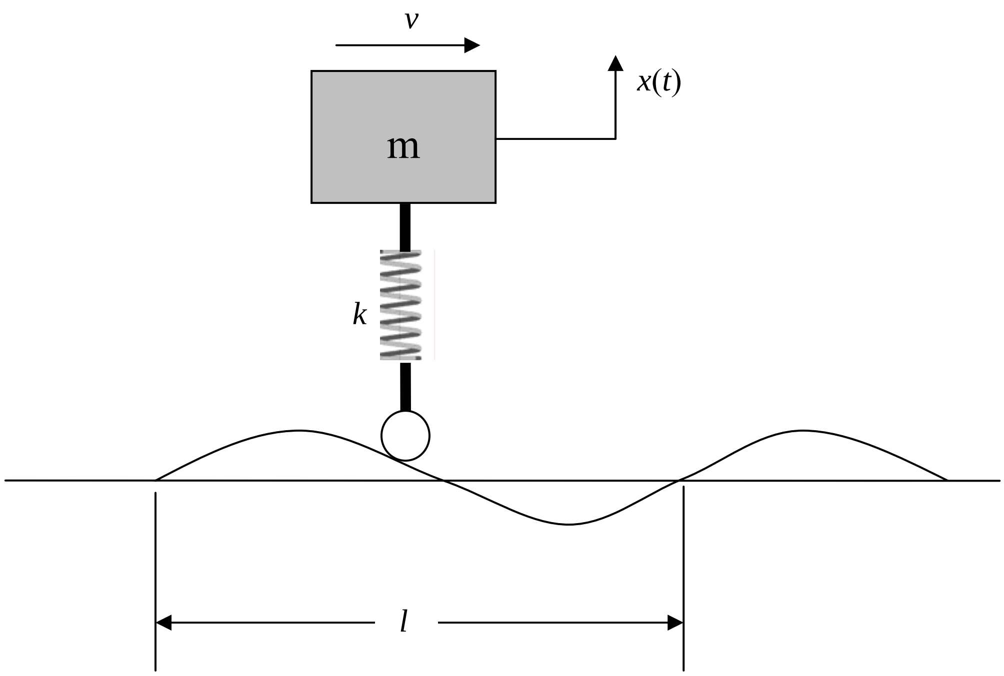
(a) What is the most unfavorable speed for which Ahmed can travel?

(b) If Ahmed replaces the shocks on his MAZDA, is the most unfavorable speed less than, equal to, or greater than the most unfavorable speed found in part (a). Why?

Sol.:

a) Since the car have lost most of its damping properties:

$$\zeta <<< 1 \Rightarrow c \approx 0$$



Road elevation: $y(t) = Y \sin(\omega t)$, $\omega = \omega(v, l)$

Data: *m, k, l, v, Y*

$m\ddot{x} + kx = ky = kY \sin(\omega t)$

Steady State Response: $x_{ss} = X \sin(\omega t - \Phi)$

$$\left|\frac{X}{Y}\right| = \frac{1}{\left|1 - r^2\right|}$$

a) $v = \dfrac{l\,\omega}{2\pi}$, Thus $v_{unfavorable} = \left.\dfrac{l\,\omega}{2\pi}\right|_{\omega=\omega_n} = \dfrac{l\,\omega_n}{2\pi}$

b) From Figure 3.15 page 240 (Rao) and knowing that ω_n is fixed for different value of ζ.

Then, the speed after replacing the shocks is less than before, because if ζ increases the peak value is shifted to the left.

مع دعواتي لكم بالتوفيق

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