بسم ا الرحمن الرحيم

King Abdulaziz University
Engineering College
MENG Department

Mechanical Vibrations
MENG470
Homework Assignment 9

Due Saturday: 19/3/1425 H

Q1. For the string system shown in Figure (1), assume that its length is ℓ, mass per unit length is *m* and tension is *T*. One end of the string is fixed and the other end is connected to a mass *M* which moves in a frictionless slot. The mass *M* is also connected to a spring of stiffness *k*.

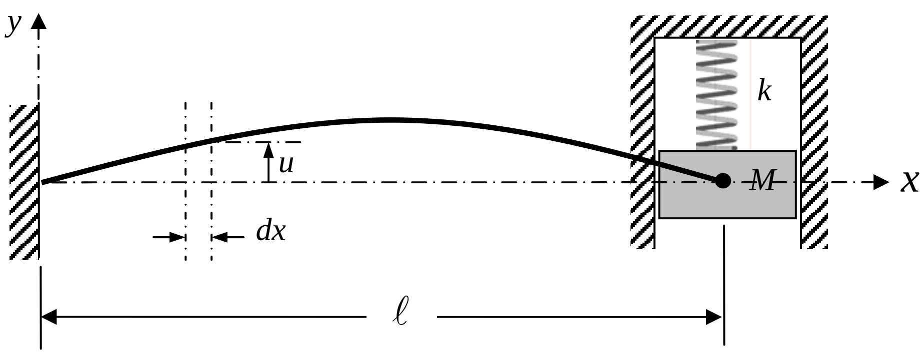

Figure (1)

a) Draw all the forces acting on an element of length *dx*.
b) Write the equilibrium equation of the forces in the dynamic case.
c) Derive the equation of motion (wave equation) of the free transverse vibration of the string. Assume that the string is uniform and the tension is constant.
d) Derive the associated boundary conditions.
e) Solve EXACTLY the wave equation using the method of separation of variables.
f) Derive the characteristics equation of the system.
g) Extract the first three natural frequencies of the string and draw their mode shapes. Assume m=1, M=1, k=1, T=1, and ℓ =1.

Q2. Determine the natural frequencies of a torsional system consisting of a uniform shaft of mass moment of inertia J_s with a disk of inertia J_0 attached to each end. Check the fundamental frequency by reducing the uniform shaft to a torsional spring with end masses.

Q3. Consider a rod in axial vibration attached to a spring of stiffness *k* at *x*=0 and free at x= ℓ as shown in Figure (2). The rod is subjected to the force per unit length *f(x,t),* its mass per unit length is *m(x)* and its axial stiffness is *EA(x)*, where *E* is the modulus of elasticity and *A(x)* the cross-sectional area.

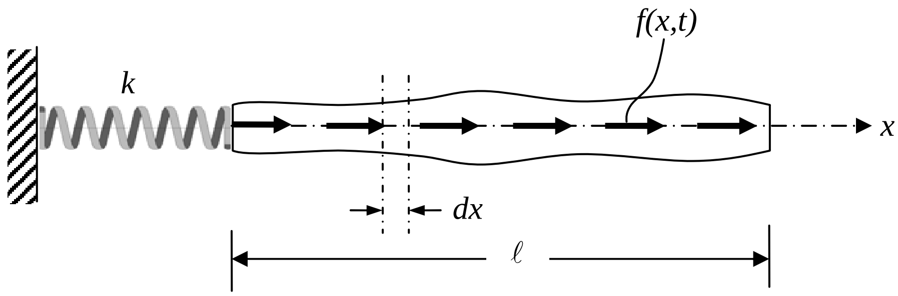

Figure (2)

a) Draw all the forces acting on an element of length *dx*.
b) Write the equilibrium equation of the forces in the dynamic case.
c) Derive the equation of motion for the rod in the axial vibration.
d) Derive the associated boundary conditions.
e) If the rod is uniform and under free vibration, what's the characteristics equation?
f) Plot the three lowest modes of vibration. Assume *m*=1, *E*=1, *A*=1, *k*=1, and ℓ =1.

Q4. A uniform beam of length ℓ, mass per unit length *m*, Young's modulus *E*, area of cross section A, and area of moment of inertia *I* is subjected to transverse vibration. The beam is fixed at *x*=0 and the other end is connected to a mass *M*. The mass is also connected to a damper of damping constant *c* as shown in Figure (3).

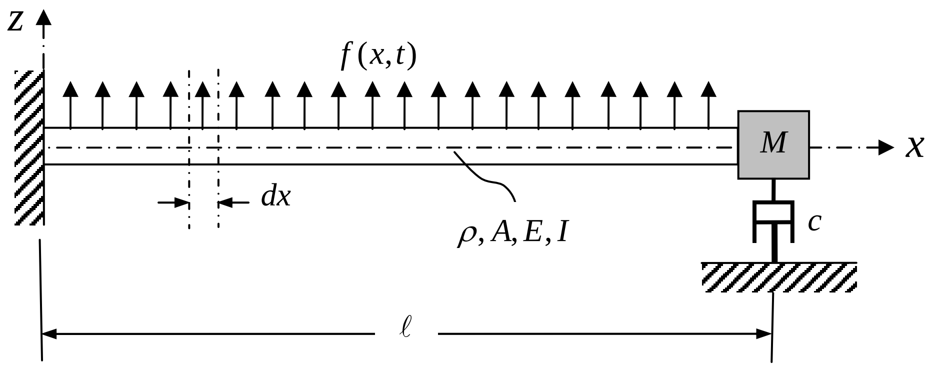

Figure (3)

a) Draw a free body diagram corresponding to the beam differential element of length *dx* and show all the loads that act on it.
b) Write the force equation of motion in the *z* direction.
c) Show that $V = \frac{\partial M}{\partial x}$ using the moment equation of motion about y axis.
d) Derive the partial differential equation for the bending vibration of the beam.
e) Express the boundary conditions of the beam.
f) Extract the first three natural frequencies of the beam and draw their mode shapes. Assume all system parameters to be equal to 1.