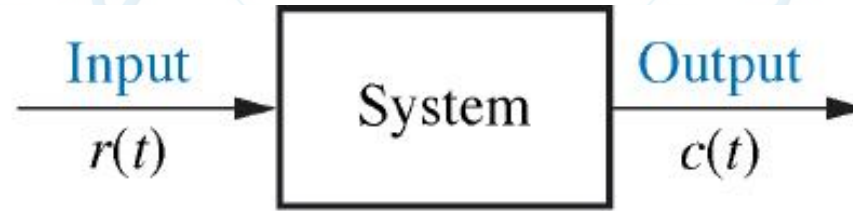


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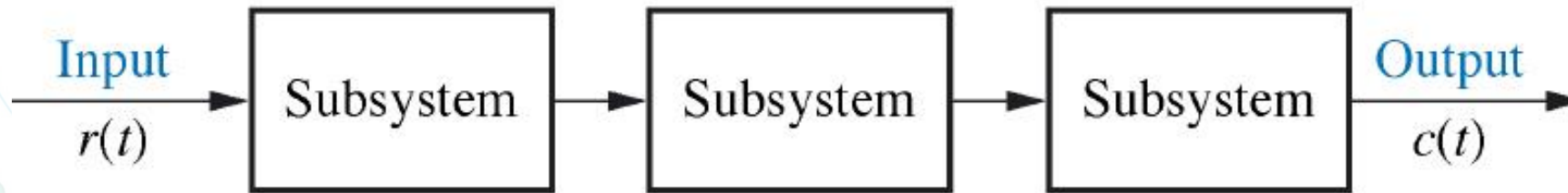
Transfer Functions & Block Diagrams

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Transfer Function



(a)



(b)

Note: The input, $r(t)$, stands for *reference input*.
The output, $c(t)$, stands for *controlled variable*.

Transfer Function

The output $c(t)$ is related to the input $r(t)$ by

$$a_n \frac{d^n c}{dt^n} + a_{n-1} \frac{d^{n-1} c}{dt^{n-1}} + \dots + a_0 c = b_m \frac{d^m r}{dt^m} + b_{m-1} \frac{d^{m-1} r}{dt^{m-1}} + \dots + b_0 r$$

where a 's and b 's are system parameters

Using Laplace transform with zero initial conditions

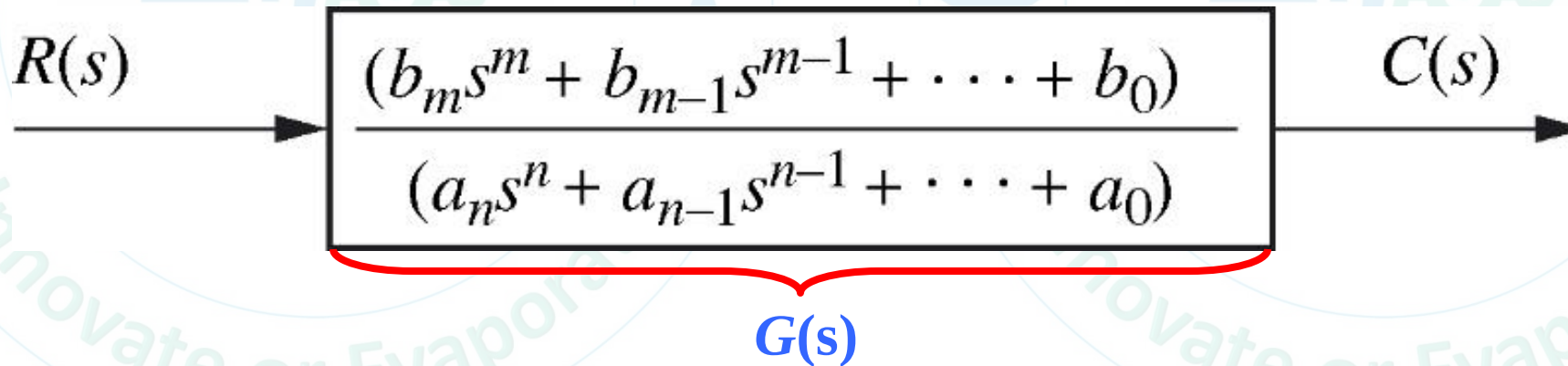
$$(a_n s^n + a_{n-1} s^{n-1} + \dots + a_0) C(s) = (b_m r^m + b_{m-1} r^{m-1} + \dots + b_0) R(s)$$

where $C(s)$ & $R(s)$ are the Laplace transforms of $c(t)$ & $r(t)$

Transfer Function

$$\rightarrow \frac{C(s)}{R(s)} = G(s) = \frac{(b_m r^m + b_{m-1} r^{m-1} + \dots + b_0)}{(a_n s^n + a_{n-1} s^{n-1} + \dots + a_0)}$$

where $G(s)$ = transfer function of the system



$$\rightarrow C(s) = G(s) R(s)$$

Poles & Zeros of Transfer Function

$$\frac{C(s)}{R(s)} = G(s) = \frac{(b_m r^m + b_{m-1} r^{m-1} + \dots + b_0)}{(a_n s^n + a_{n-1} s^{n-1} + \dots + a_0)} = \frac{N(s)}{D(s)}$$

Definitions

Zeros = roots of $N(s)$, *i.e.* the values at which $N(s)=0$. At these values the numerator becomes ZERO

Poles = roots of $D(s)$, *i.e.* the values at which $D(s)=0$. At these values the denominator vanishes & the system output (for any finite input) goes to infinity.

Transfer fⁿ from DE

Example.1:

Find the transfer function represented by:

$$\frac{dc(t)}{dt} + 2c(t) = r(t)$$

Solution with zero initial conditions

$$\rightarrow sC(s) + 2C(s) = R(s)$$

or $G(s) = \frac{C(s)}{R(s)} = \frac{1}{s + 2}$

Response from T. fⁿ

Example.2:

Find the time response of :

$$\frac{dc(t)}{dt} + 2c(t) = r(t)$$

For a unit step input $r(t)$ (i.e. $r(t)=u(t)$).

Solution as the transfer function is

$$G(s) = \frac{C(s)}{R(s)} = \frac{1}{s+2}$$

As $r(t)=u(t) \rightarrow R(s)=1/s$

Response from T. fⁿ

$$\rightarrow C(s) = G(s)R(s) = \frac{1}{s(s+2)}$$

or by expanding into partial fraction

$$C(s) = \frac{1}{2s} - \frac{1}{2(s+2)}$$

Taking the inverse Laplace transform

$$c(t) = \frac{1}{2} - \frac{1}{2}e^{-2t}$$

Response from T. fⁿ

With MATLAB

```
>> syms t s
```

```
>> ilaplace(1/(s*(s+2)),s,t)
```

```
ans =
```

```
-1/2*exp(-2*t)+1/2
```

Response from T. fⁿ

The Transfer Function

$$G(s) = \frac{C(s)}{R(s)} = \frac{1}{s+2}$$

has Poles at $s=-2$, No Zeros

MATLAB Code

```
>> n=1;  
>> d=[1 2];  
>> [z,p,k]=tf2zp(n,d)
```

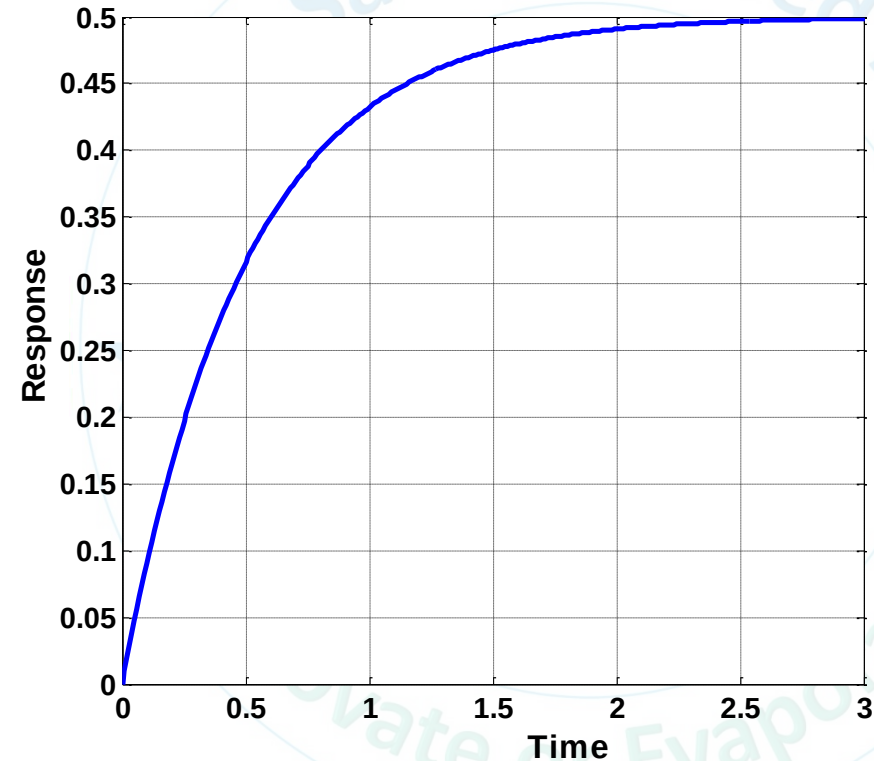
OUTPUT

```
z =  
Empty matrix: 0-by-1  
p = -2  
k = 1
```

Response from T. fⁿ

MATLAB Time Response

```
>> t=0:0.01:3;  
>> plot(t,(1/2-  
1/2*exp(-2*t)))  
>> grid  
>> xlabel('Time')  
>> ylabel('Response')
```



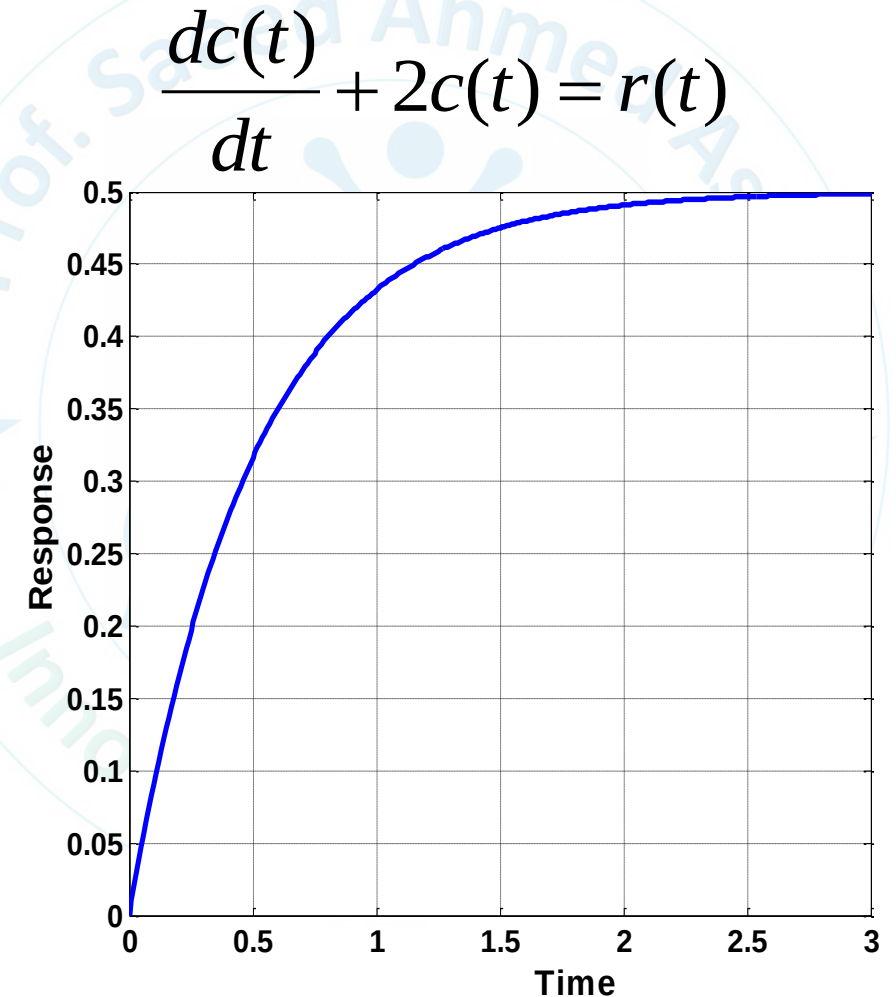
Response from T. fⁿ

MATLAB Time Response

```
%*****
%*****Step Input*****
%*****
syms s t c C ct

eqn=sym('D(c)(t)+2*c(t)=1');
lteqn=laplace(eqn,t,s)
neweqn=subs(lteqn,{'laplace(c(t),t,s)','c(0)'},{C,0})
cs=solve(neweqn,C)
ct=ilaplace(cs,s,t)
%*****

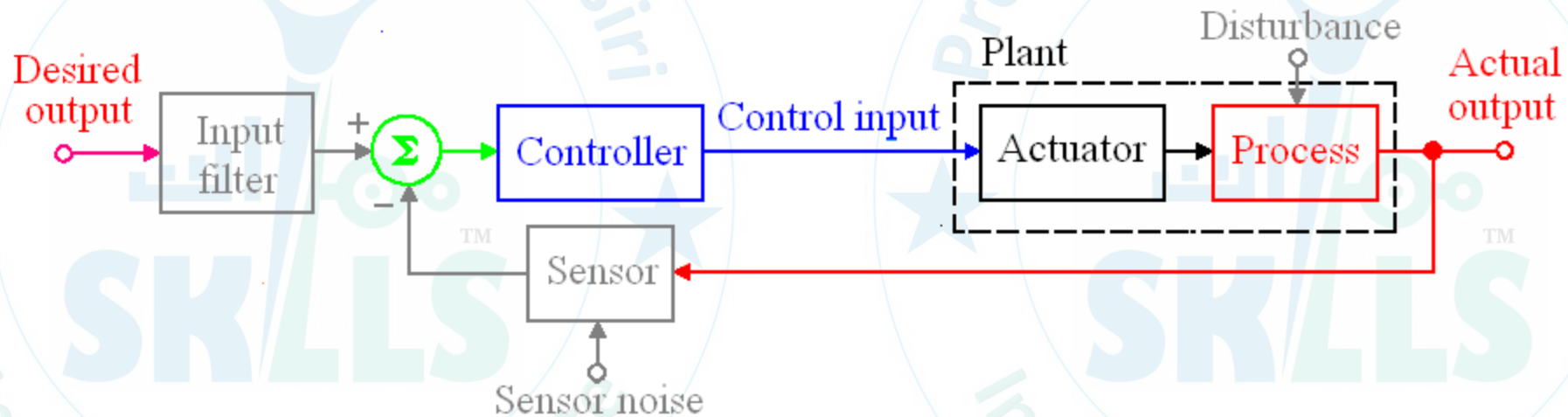
ezplot(ct,[0 3])
grid
xlabel('Time - s')
ylabel('Response - c(t)')
%*****
```



Block Diagram

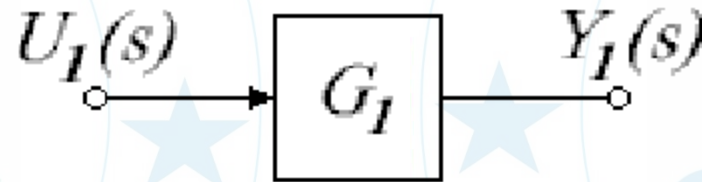
- A graphical tool can help us to **visualize the model** of a system and **evaluate the mathematical relationships between their elements**, using their transfer functions.
- In many control systems, the system of equations can be written so that their components do not interact **except by having the input of one part be the output of another part.**
- In these cases, it is very easy to draw a block diagram that represents the mathematical relationships in similar manner to that used for the component block diagram.

Reminder: Component Block Diagram



Block Diagram

- It represents the **mathematical relationships** between the elements of the system.



$$U_1(s) G_1(s) = Y_1(s)$$

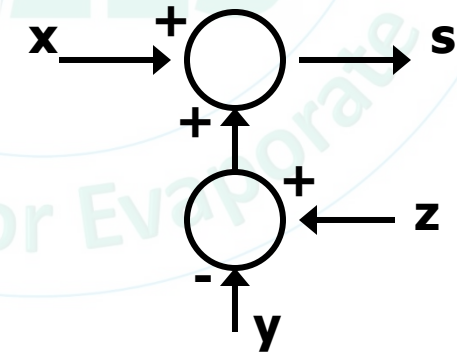
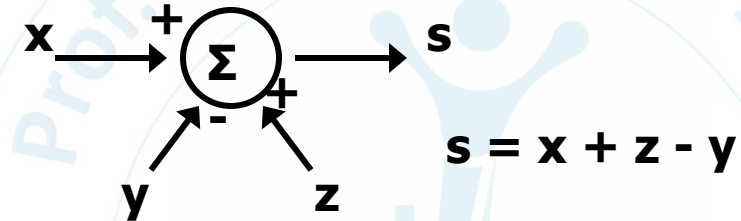
- The **transfer function** of each component is placed **in box**, and the **input-output relationships** between components are indicated by **lines and arrows**.

Block Diagram Algebra

- Using block diagram, we can ***solve the equations by graphical simplification***, which is often easier and more informative than algebraic manipulation, even though the methods are in every way equivalent.
- It is convenient to think of ***each block as representing an electronic amplifier*** with the transfer function printed inside.
- The interconnections of blocks include summing points, where any number of signals may be added together.

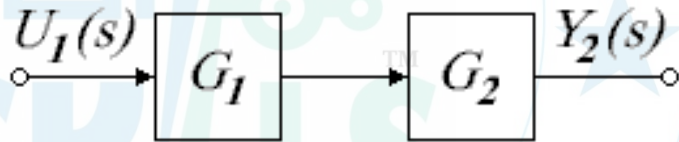
Block Diagrams

- A line is a signal
- A block is a gain
- A circle is a sum
- Due to h.f. noise, use proper blocks: num deg $\leq$ den deg
- Try to use just horizontal or vertical lines
 - Use additional “ Σ ” to help



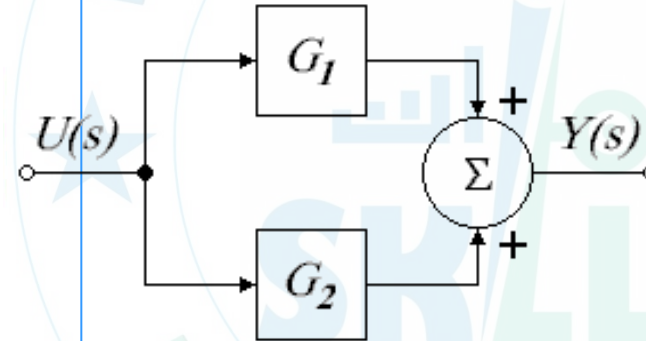
1st & 2nd Elementary Block Diagrams

- Block in series:



$$\frac{Y_2(s)}{U_1(s)} = G_1 G_2$$

- Blocks in parallel with their outputs added:

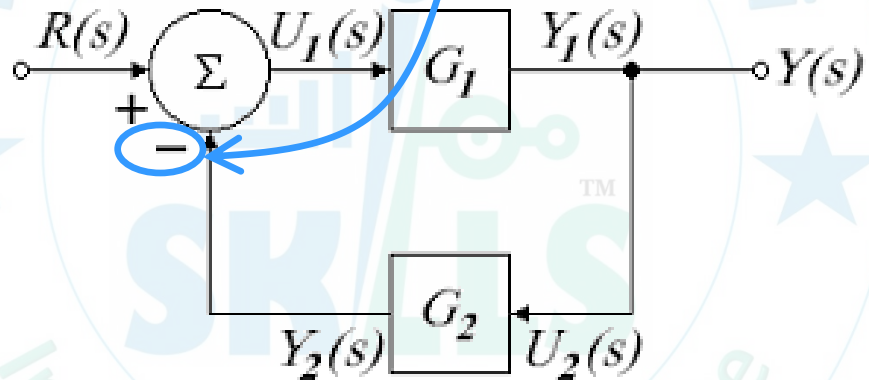


$$\frac{Y_2(s)}{U_1(s)} = G_1 + G_2$$

3rd Elementary Block Diagram



- **Single-loop negative feedback**

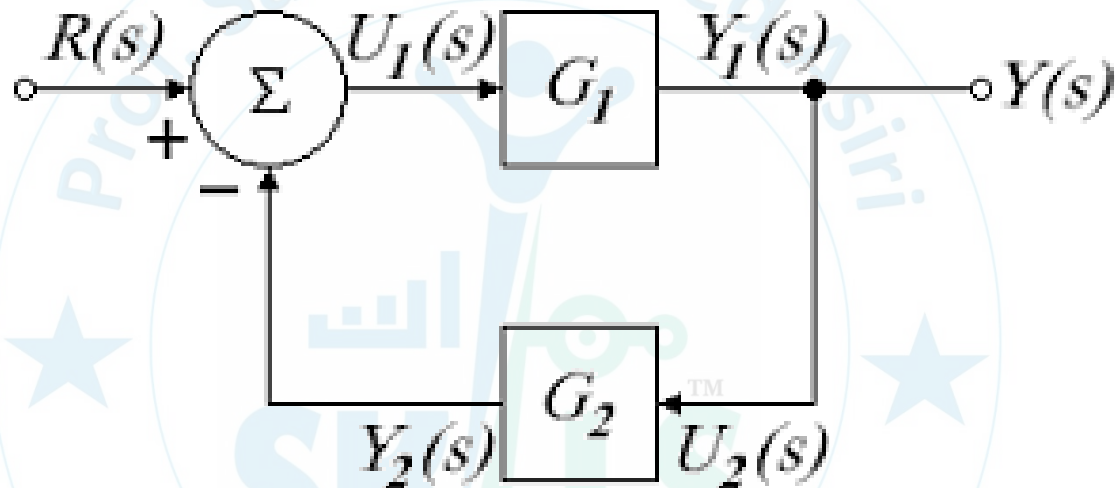


Two blocks are connected in a feedback arrangement so that each feeds into the other:

- The overall transfer function is given by:

$$\frac{Y(s)}{R(s)} = \frac{G_1}{1 + G_1 G_2}$$

Feedback Rule

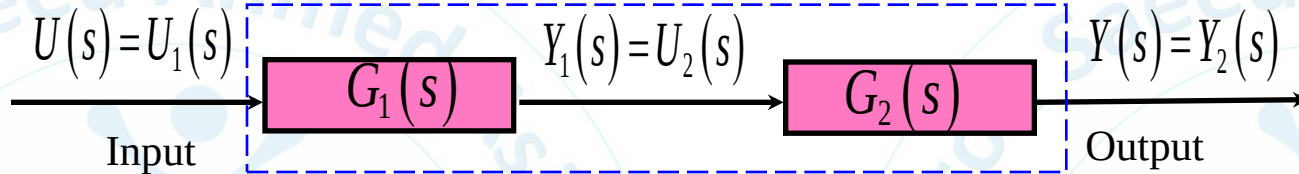


$$\frac{Y(s)}{R(s)} = \frac{G_1}{1 + G_1 G_2}$$

The gain of a single-loop negative feedback system is given by the forward gain divided by the sum of 1 plus the loop gain

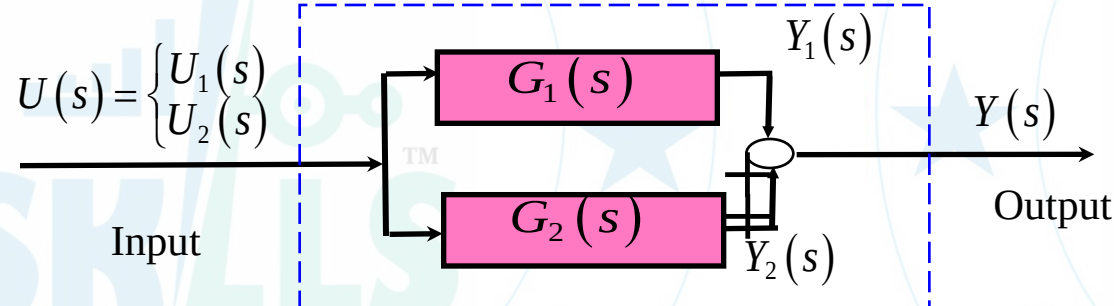
System Connections

Cascaded System



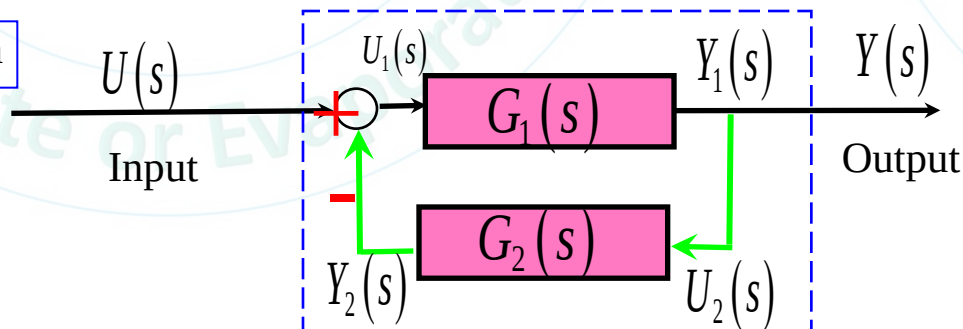
$$G(s) = G_2(s)G_1(s)$$

Parallel System



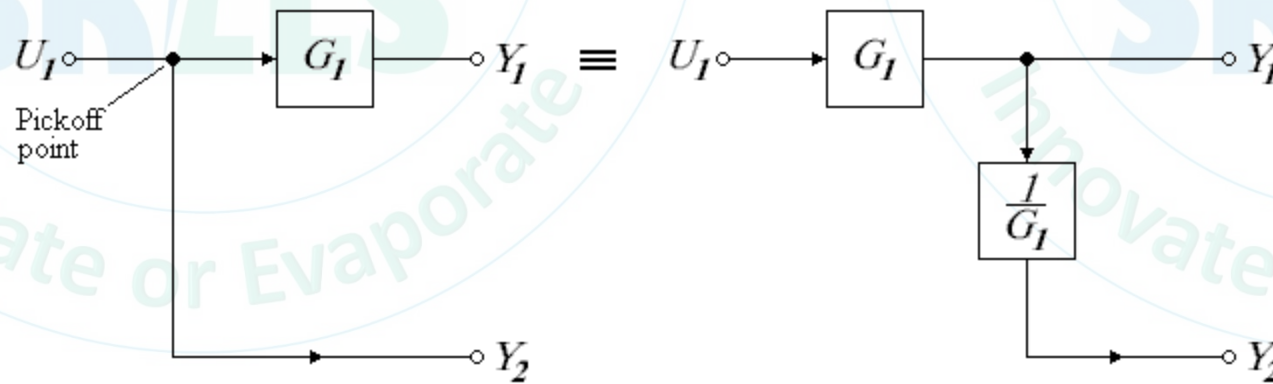
$$G(s) = G_1(s) + G_2(s)$$

Feedback System

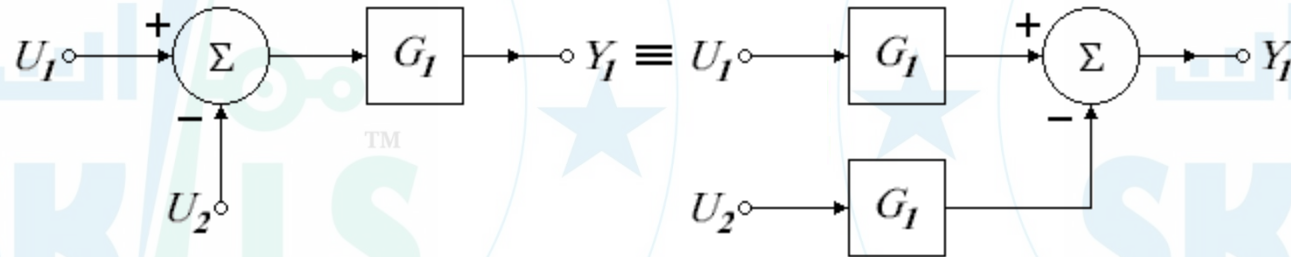


$$G(s) = \frac{G_1(s)}{1 + G_1(s)G_2(s)}$$

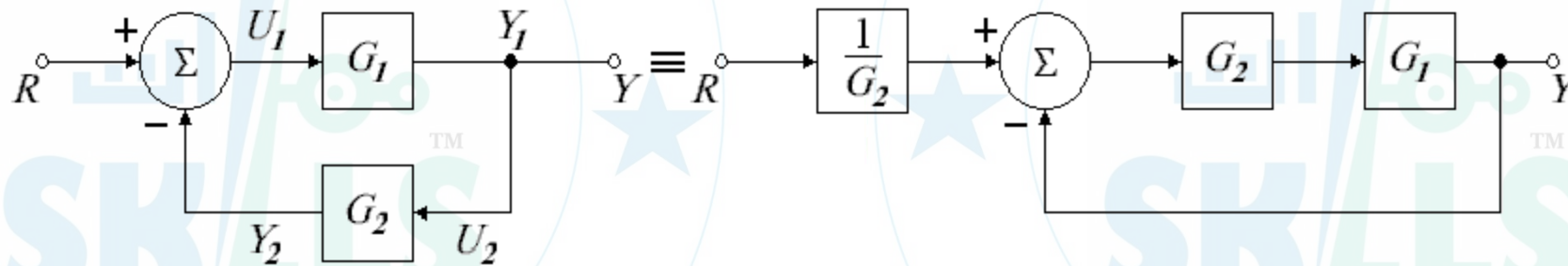
1st Elementary Principle of Block Diagram Algebra



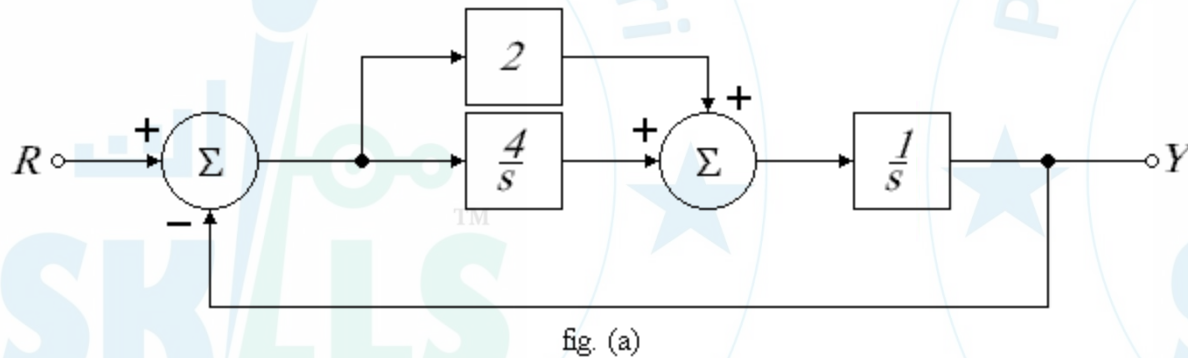
2nd Elementary Principle of Block Diagram Algebra



3rd Elementary Principle of Block Diagram Algebra



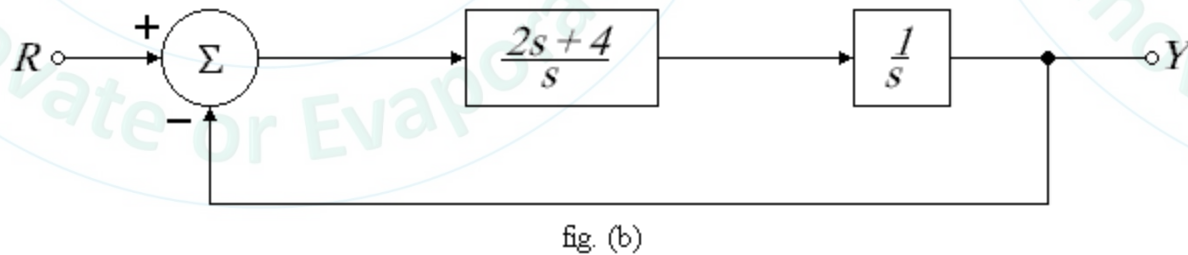
Example 1: Transfer function from a Simple Block Diagram



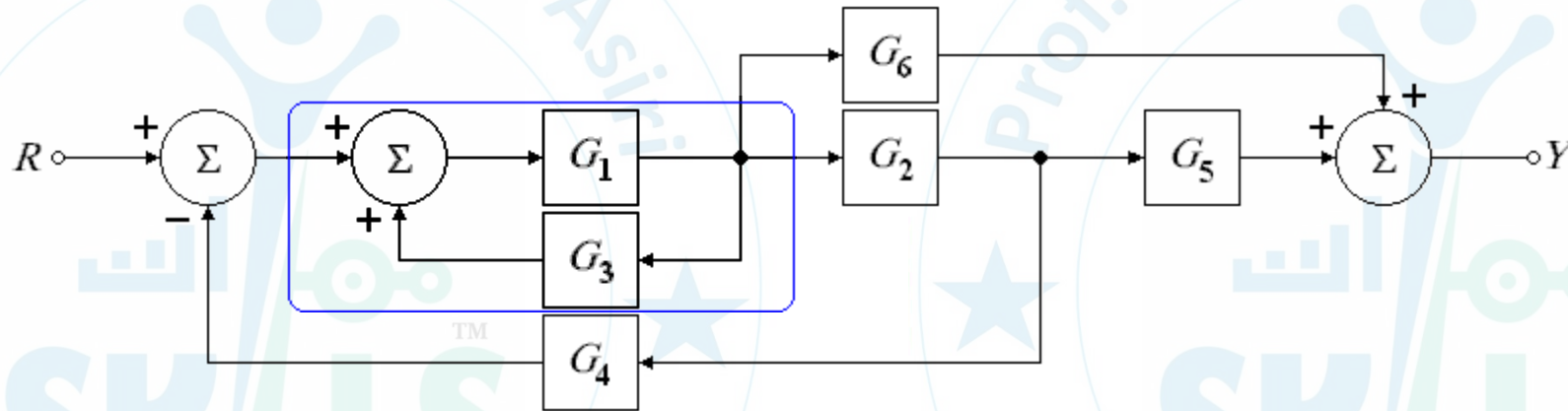
$$T(s) = \frac{Y(s)}{R(s)}$$

$$T(s) = \frac{2s + 4}{1 + \frac{2s + 4}{s^2}}$$

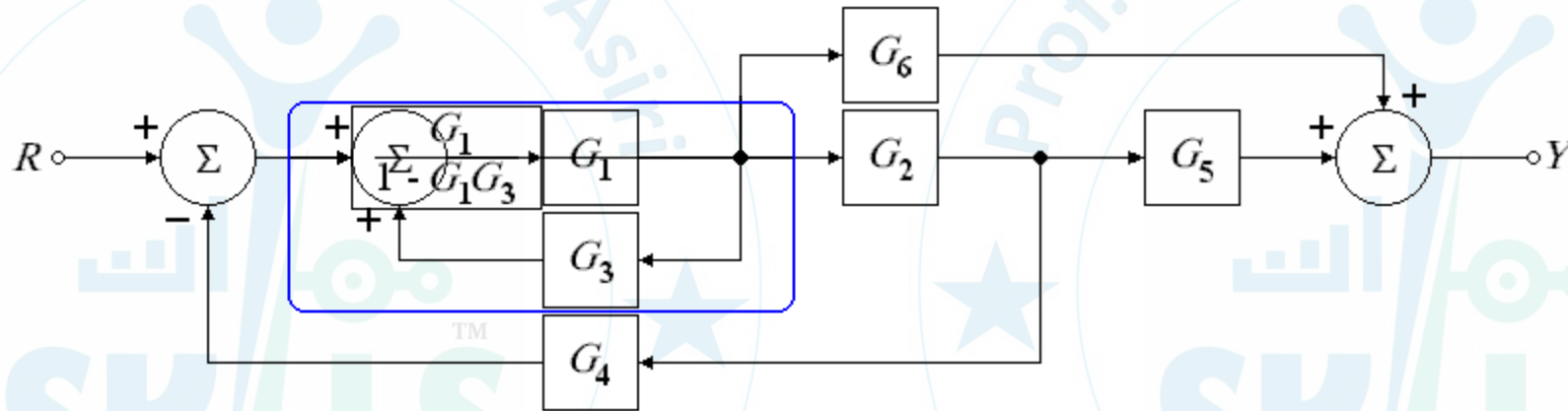
$$T(s) = \frac{2s + 4}{s^2 + 2s + 4}$$



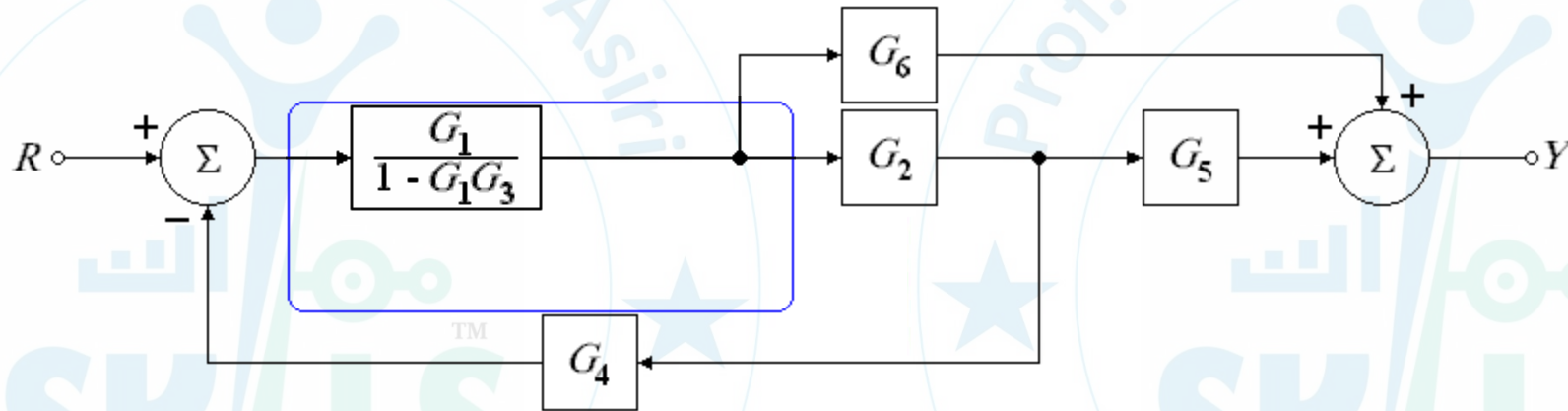
Example 2: TF from the Block Diagram



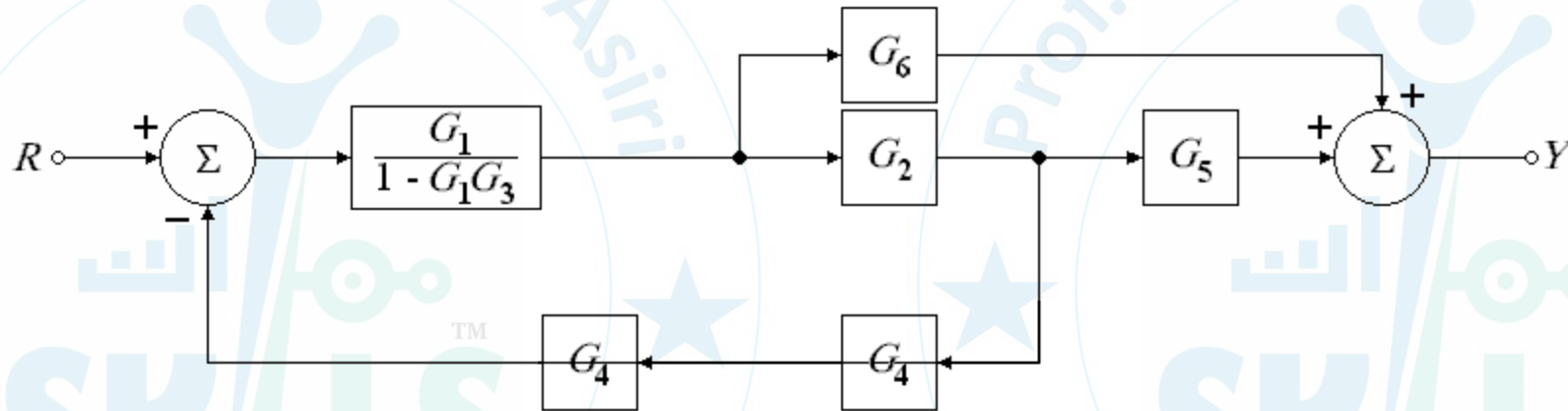
Example 2: TF from the Block Diagram



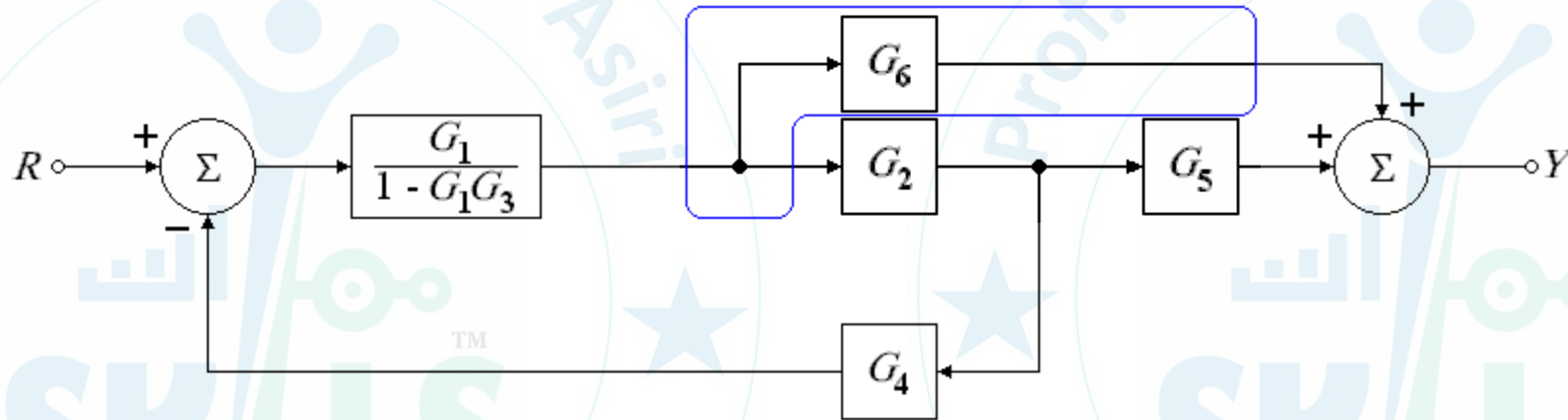
Example 2: TF from the Block Diagram



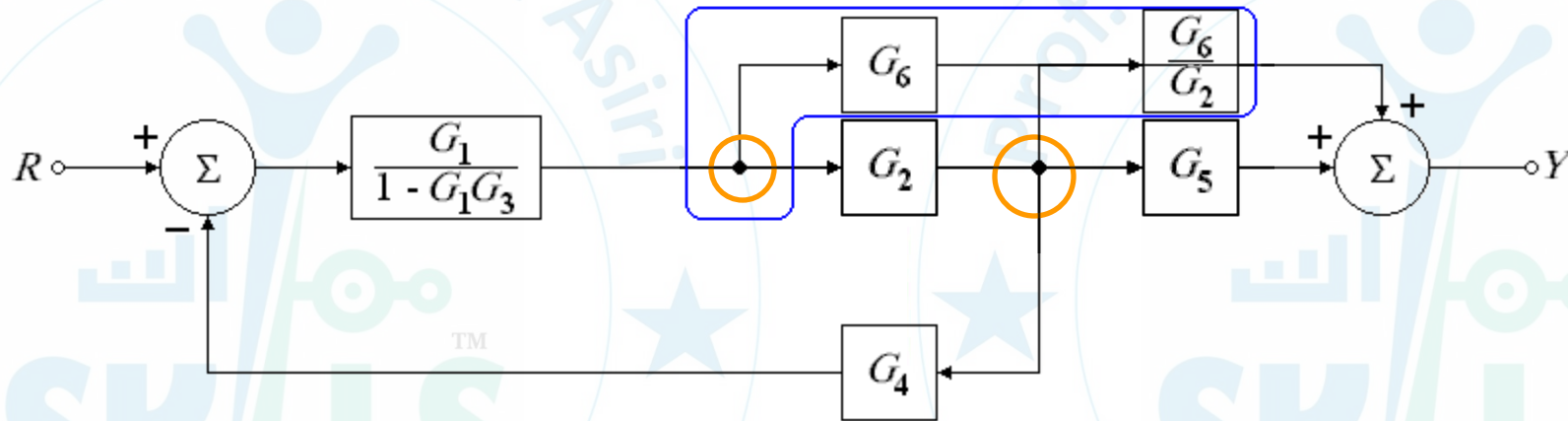
Example 2: TF from the Block Diagram



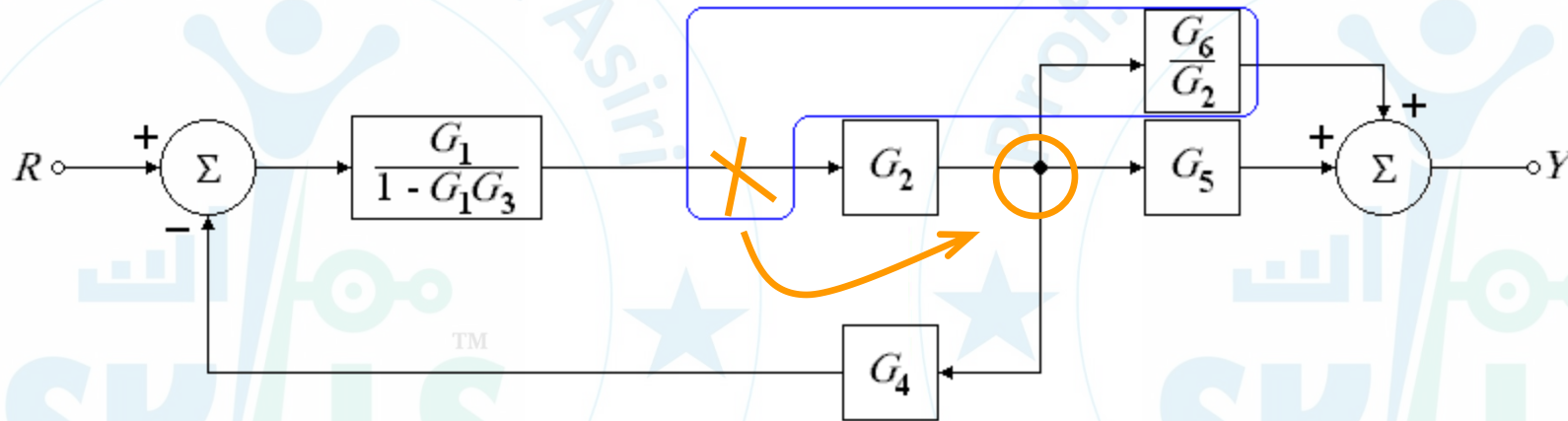
Example 2: TF from the Block Diagram



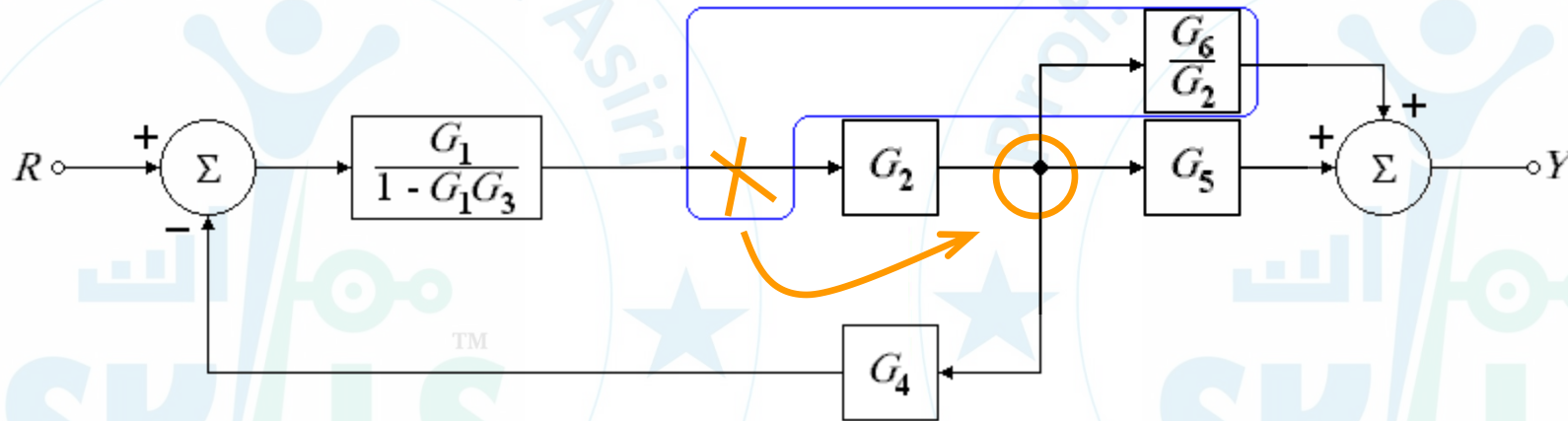
Example 2: TF from the Block Diagram



Example 2: TF from the Block Diagram



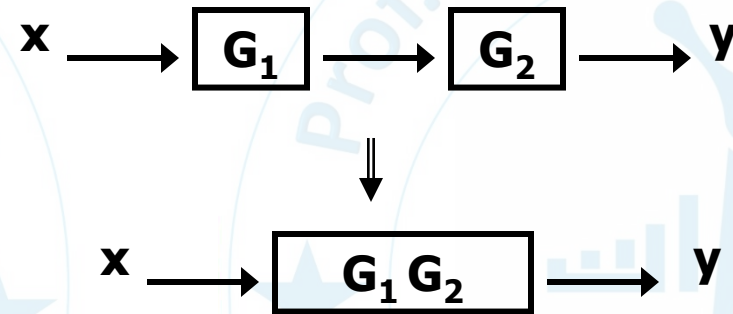
Example 2: TF from the Block Diagram



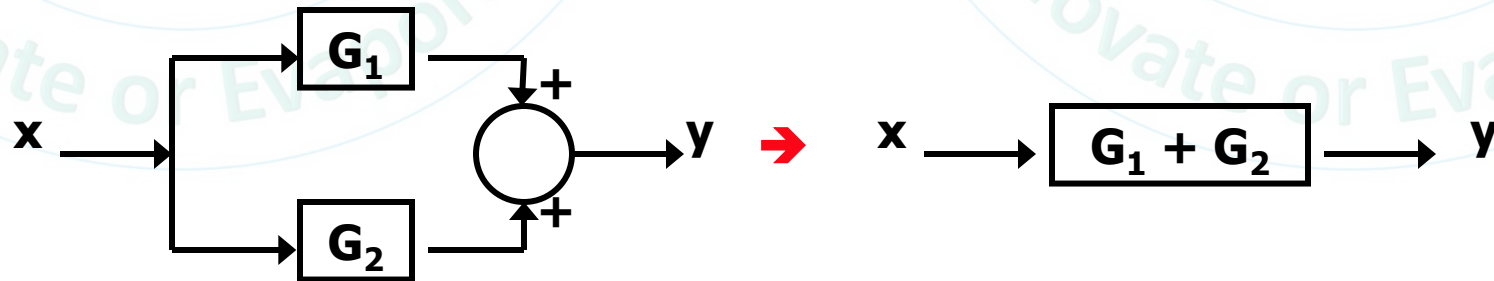
$$T(s) = \frac{G_1 G_2 G_5 + G_1 G_6}{1 - G_1 G_3 + G_1 G_2 G_4}$$

Block Diagram Reduction

- Series:

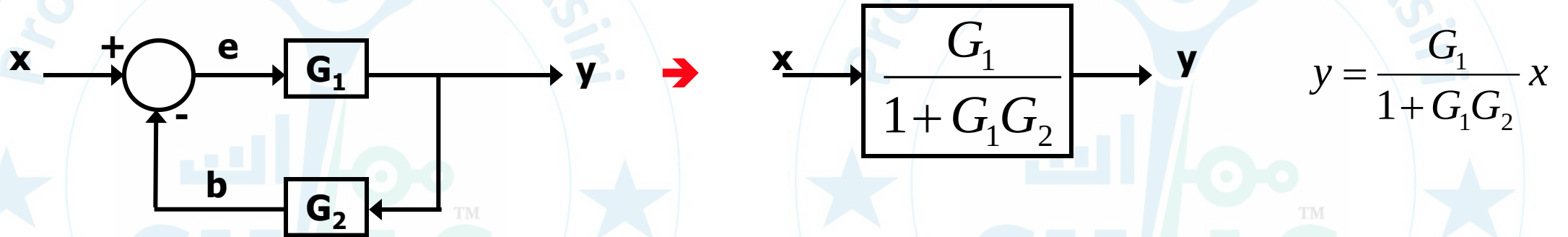


- Parallel:



Block Diagram Reduction

- Feedback:

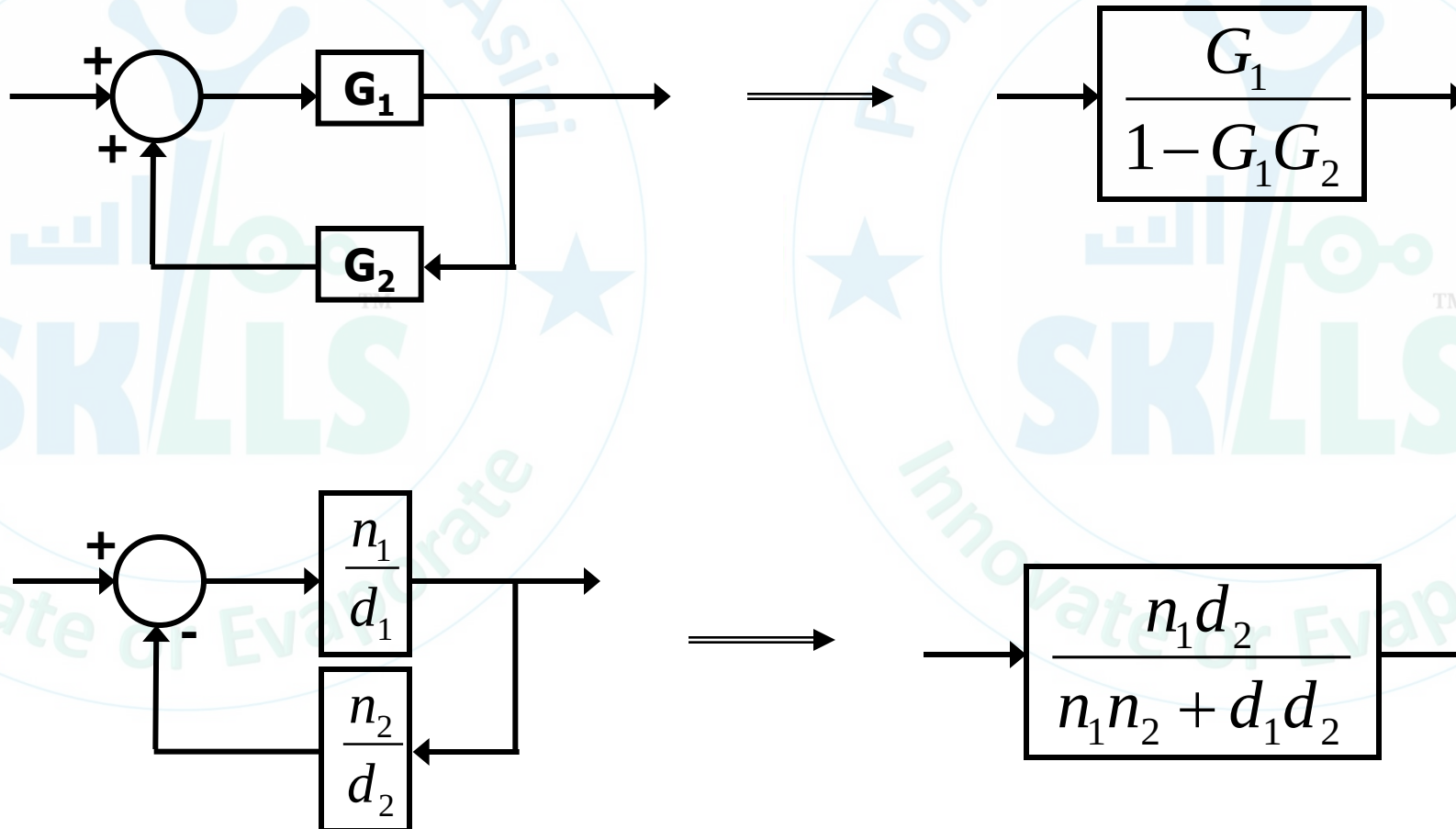


Proof: $e = x - b$, $b = G_2y$, $y = G_1e \Rightarrow y = \frac{G_1}{1 + G_1G_2} x$

$$e = x - G_2G_1e$$

$$(1 + G_1G_2)e = x \Rightarrow e = \frac{1}{1 + G_1G_2} x$$

Block Diagram Reduction



Block Diagram Reduction

>> s=tf('s')

Transfer function:

s

>> G1=(s+1)/(s+2)

Transfer function:

s + 1

s + 2

>> G2=5/(s+5)

Transfer function:

5

s + 5

>> G=G1*G2

Transfer function:

5 s + 5

s² + 7 s + 10

>> H=G1+G2

Transfer function:

s² + 11 s + 15

s² + 7 s + 10

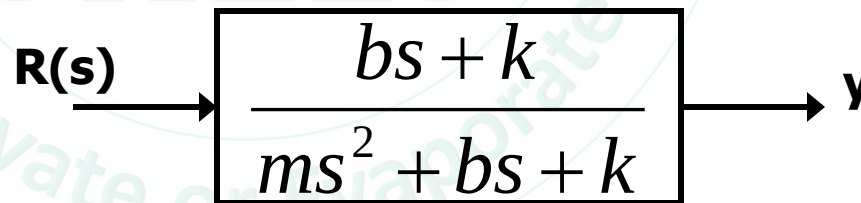
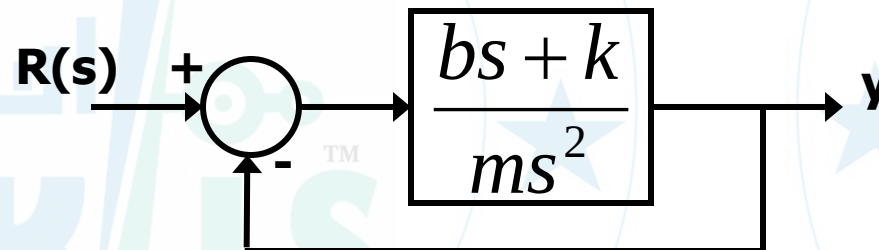
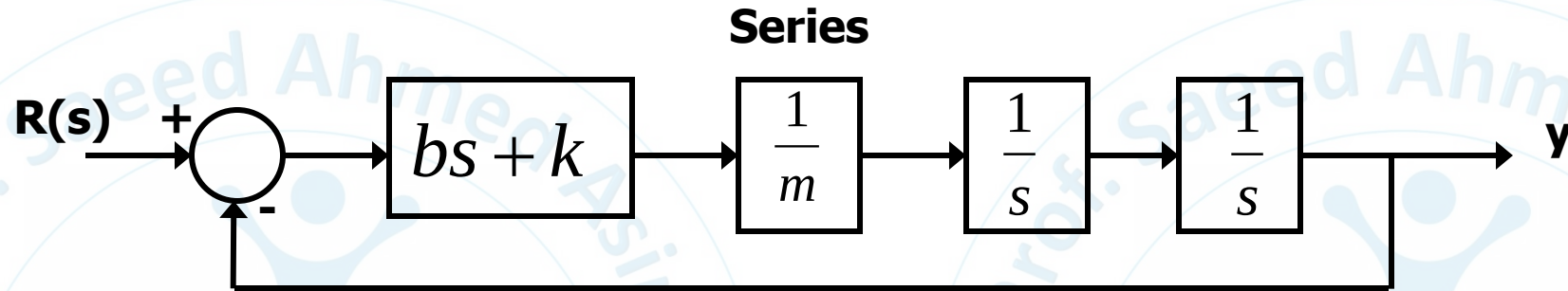
>> HF=feedback(G1, G2)

Transfer function:

s² + 6 s + 5

s² + 12 s + 15

Quarter car suspension



$$TF = H(s) = \frac{bs + k}{ms^2 + bs + k}$$

Block Diagram Reduction

```
>> b=sym('b');  
>> m=sym('m');  
>> k=sym('k');  
>> s=sym('s');  
>> G1=b*s+k  
G1 =  
b*s+k
```

```
>> G2=1/m*1/s*1/s  
G2 =  
1/m/s^2
```

```
>> G=G1*G2  
G =  
(b*s+k)/m/s^2
```

```
>> Gcl=G/(1+G)
```

Gcl =

$(b*s+k)/m/s^2/(1+(b*s+k)/m/s^2)$

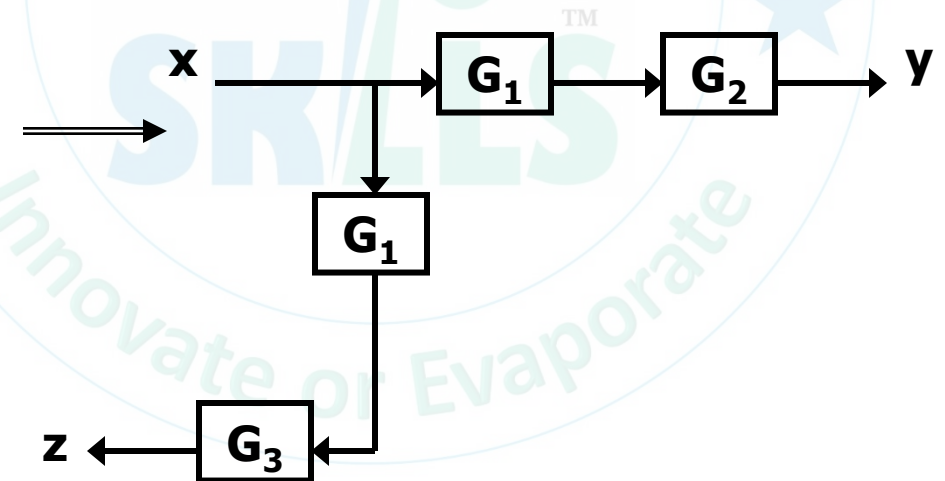
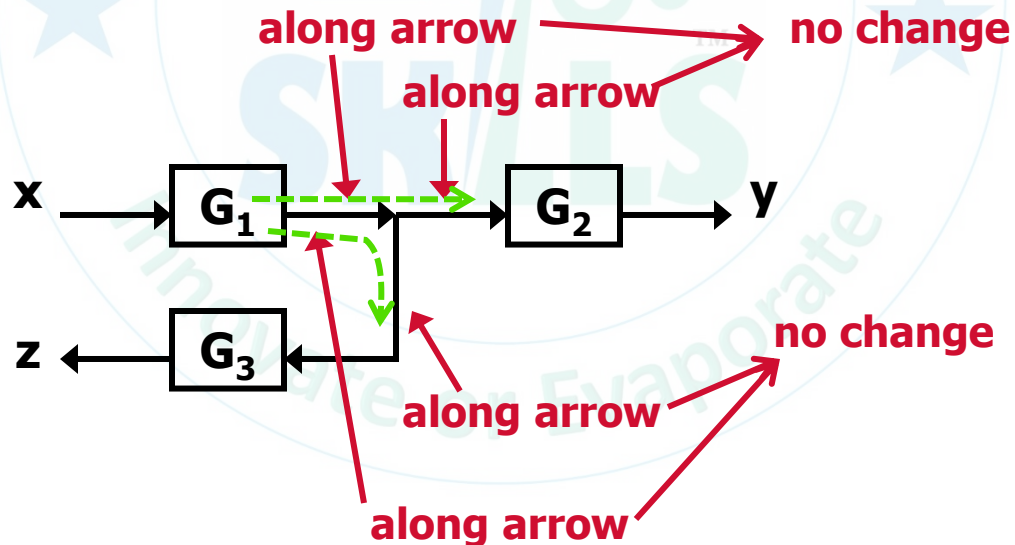
```
>> simplify(Gcl)
```

ans =

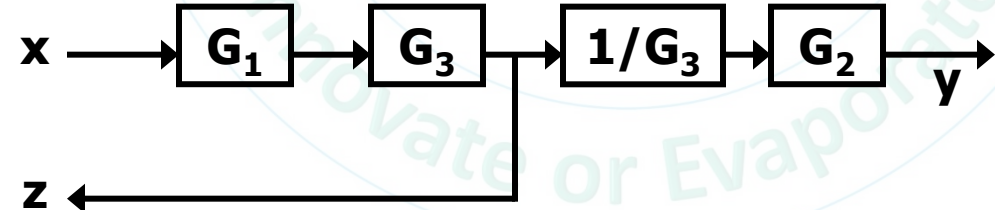
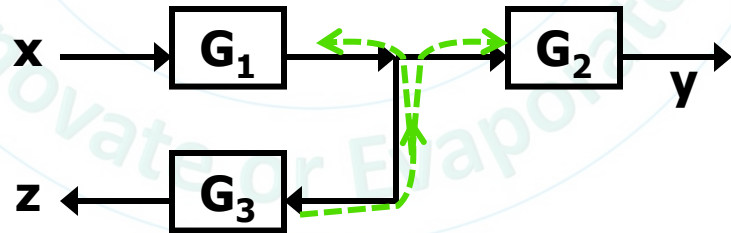
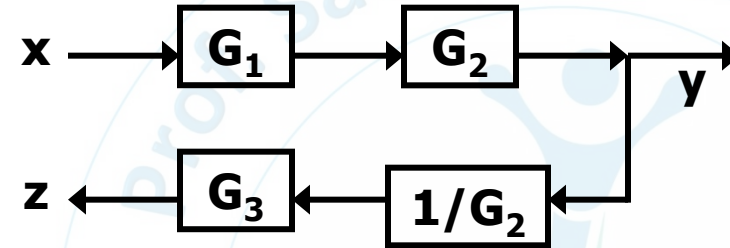
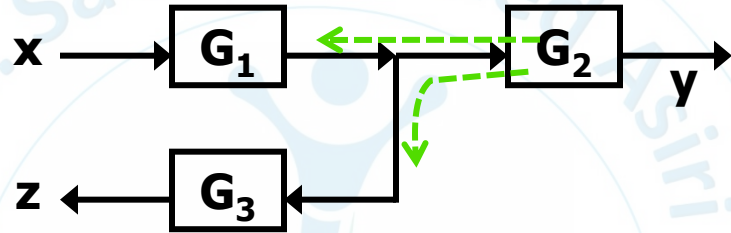
$(b*s+k)/(m*s^2+b*s+k)$

Block Diagram Reduction

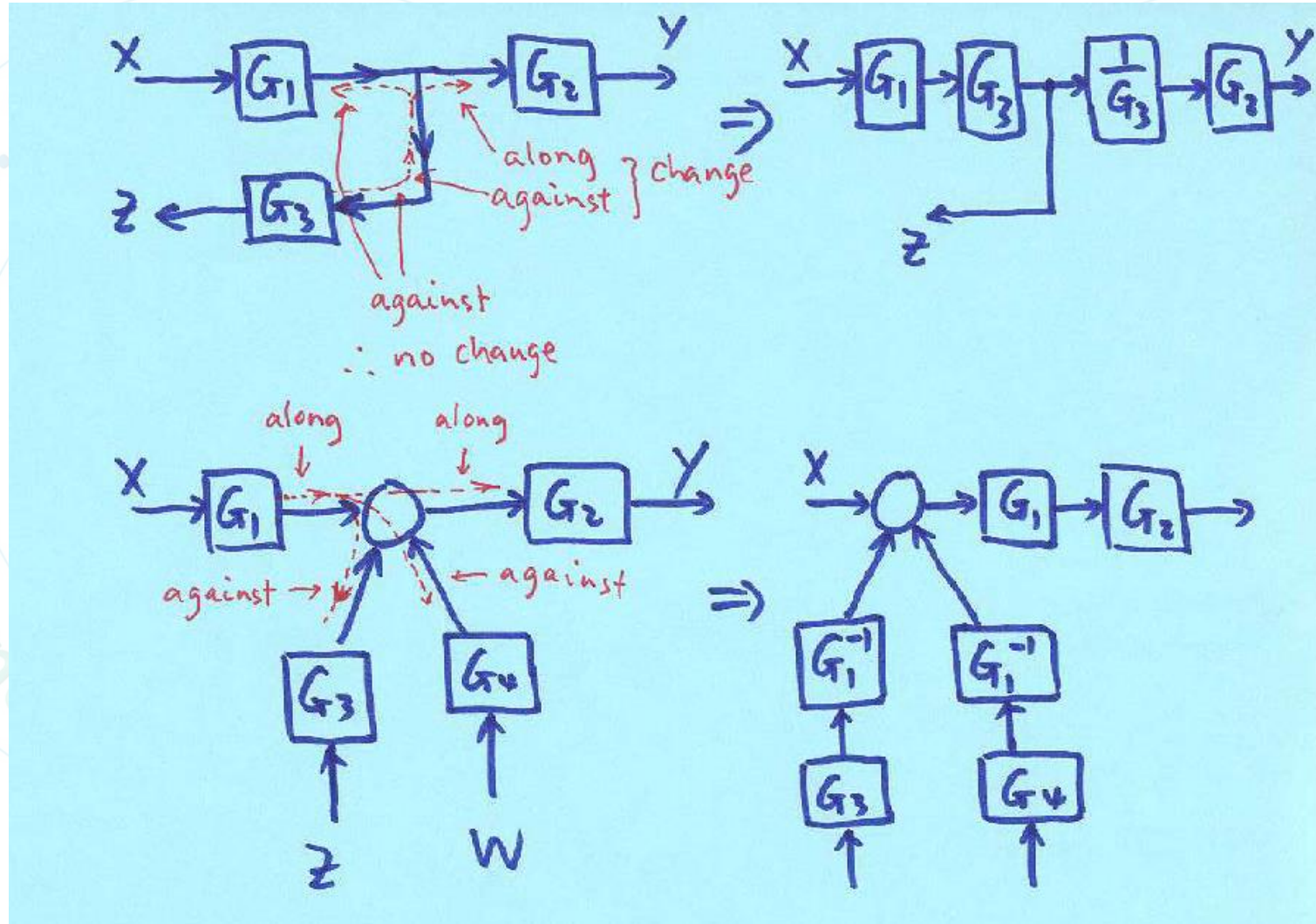
- Move a block (G_1) across a summation $\leftarrow$ pick-up point into all touching lines:



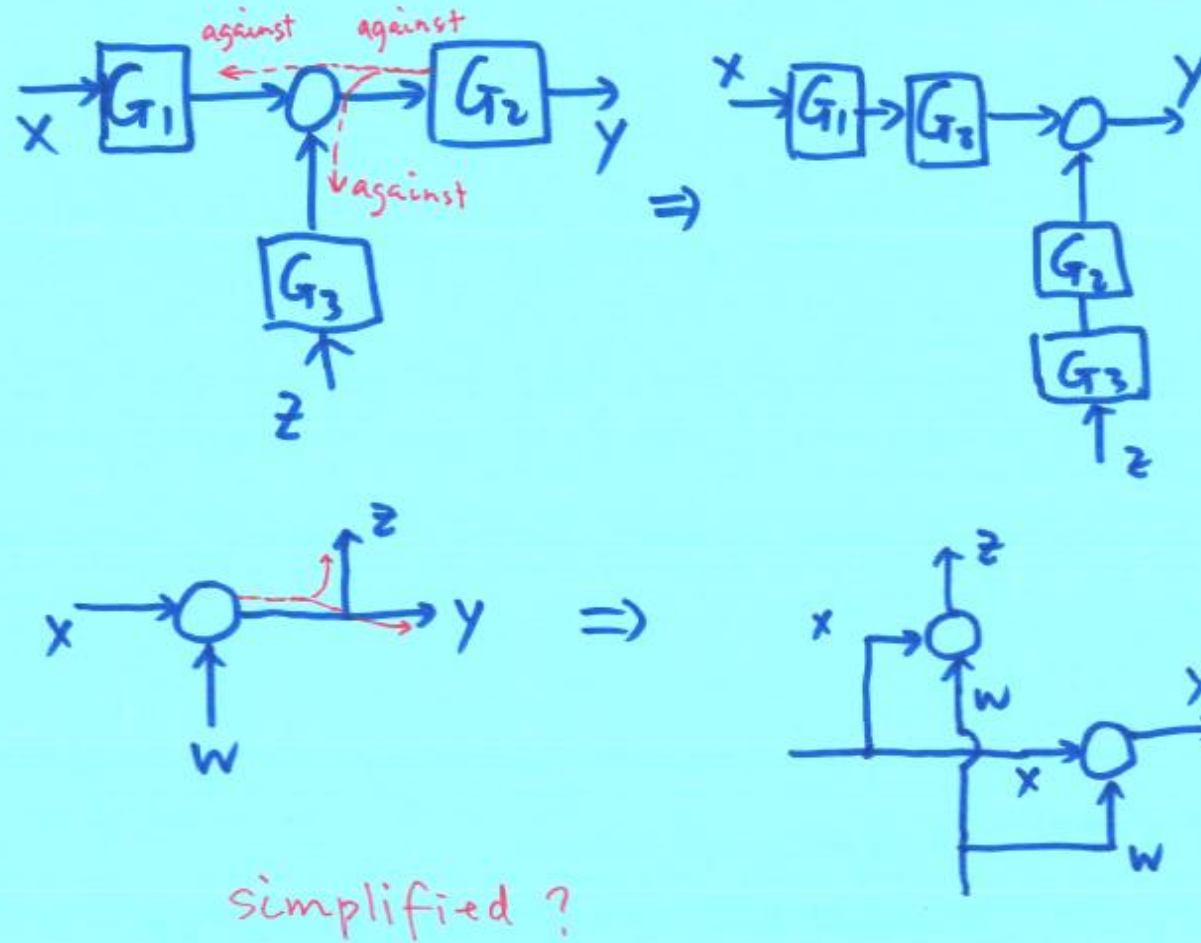
Block Diagram Reduction



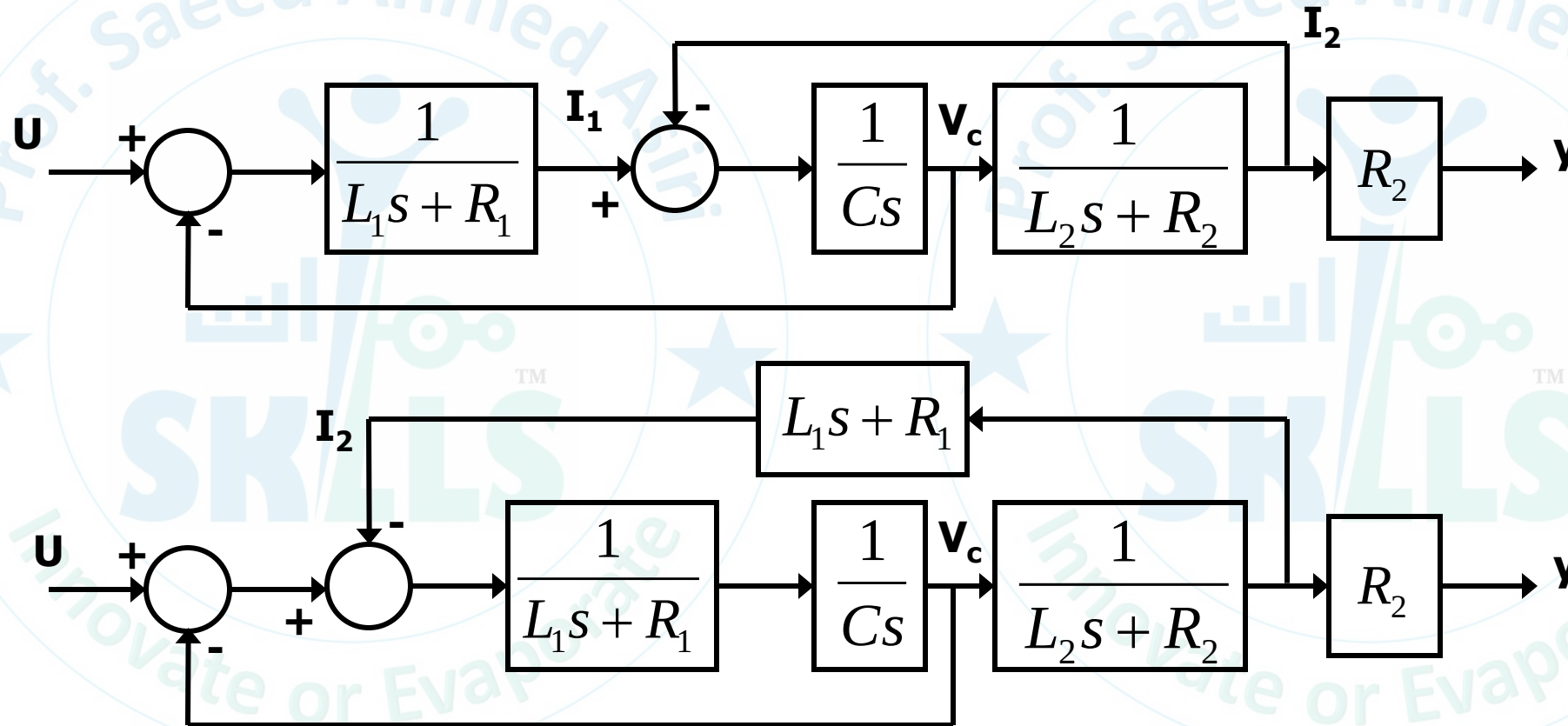
Block Diagram Reduction



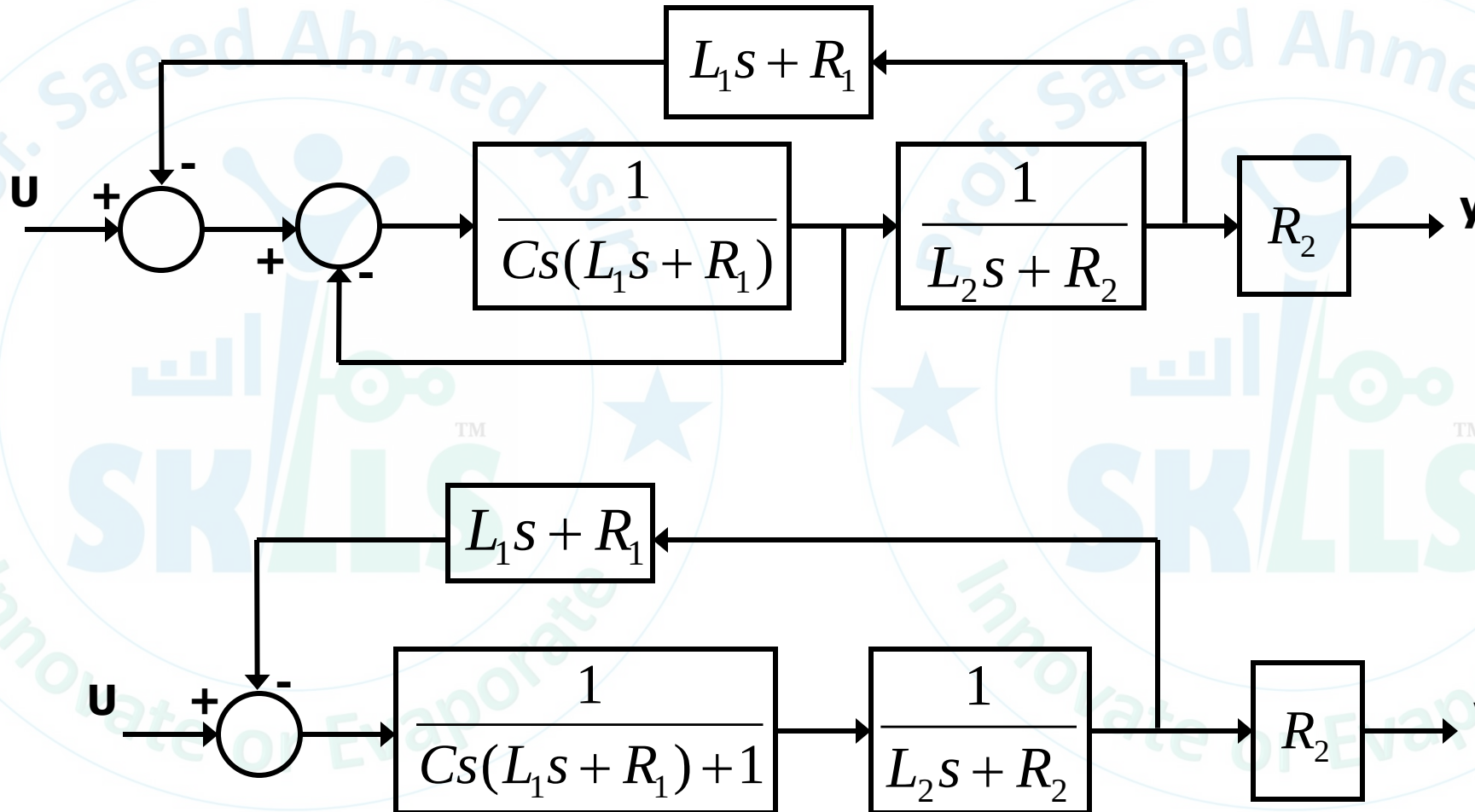
Block Diagram Reduction



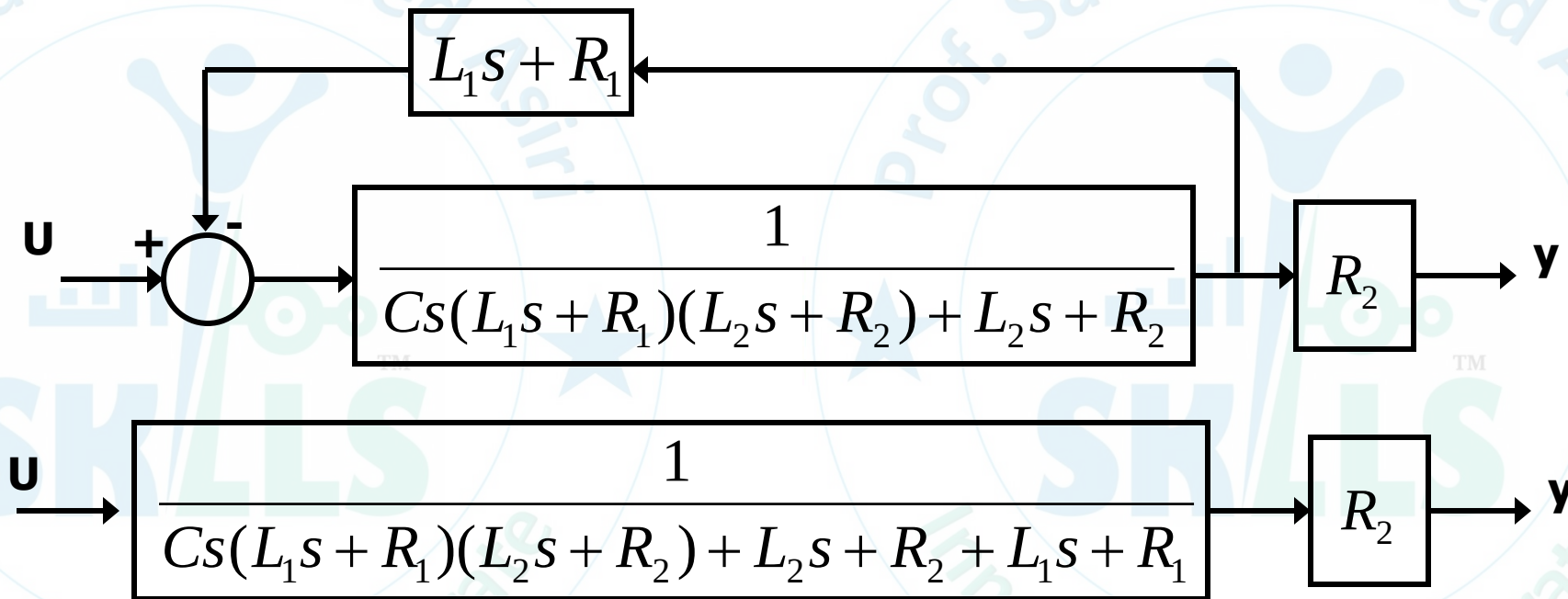
Block Diagram Reduction



Block Diagram Reduction



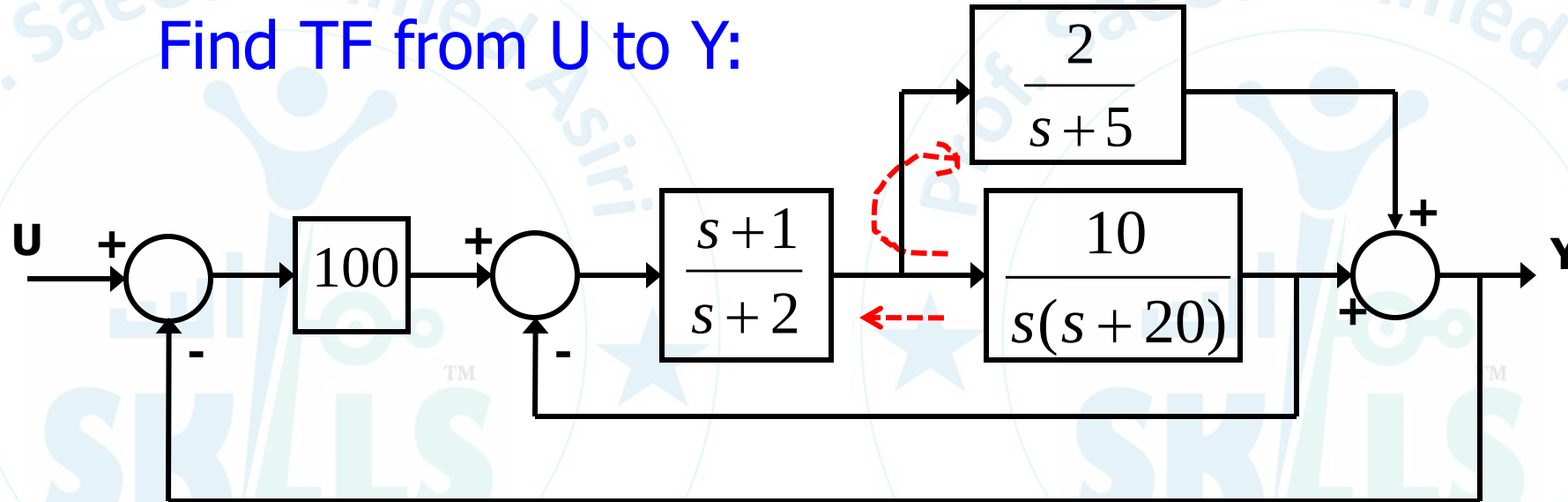
Block Diagram Reduction



$$T.F. = \frac{R_2}{Cs(L_1s + R_1)(L_2s + R_2) + L_2s + R_2 + L_1s + R_1}$$

Block Diagram Reduction

Find TF from U to Y:

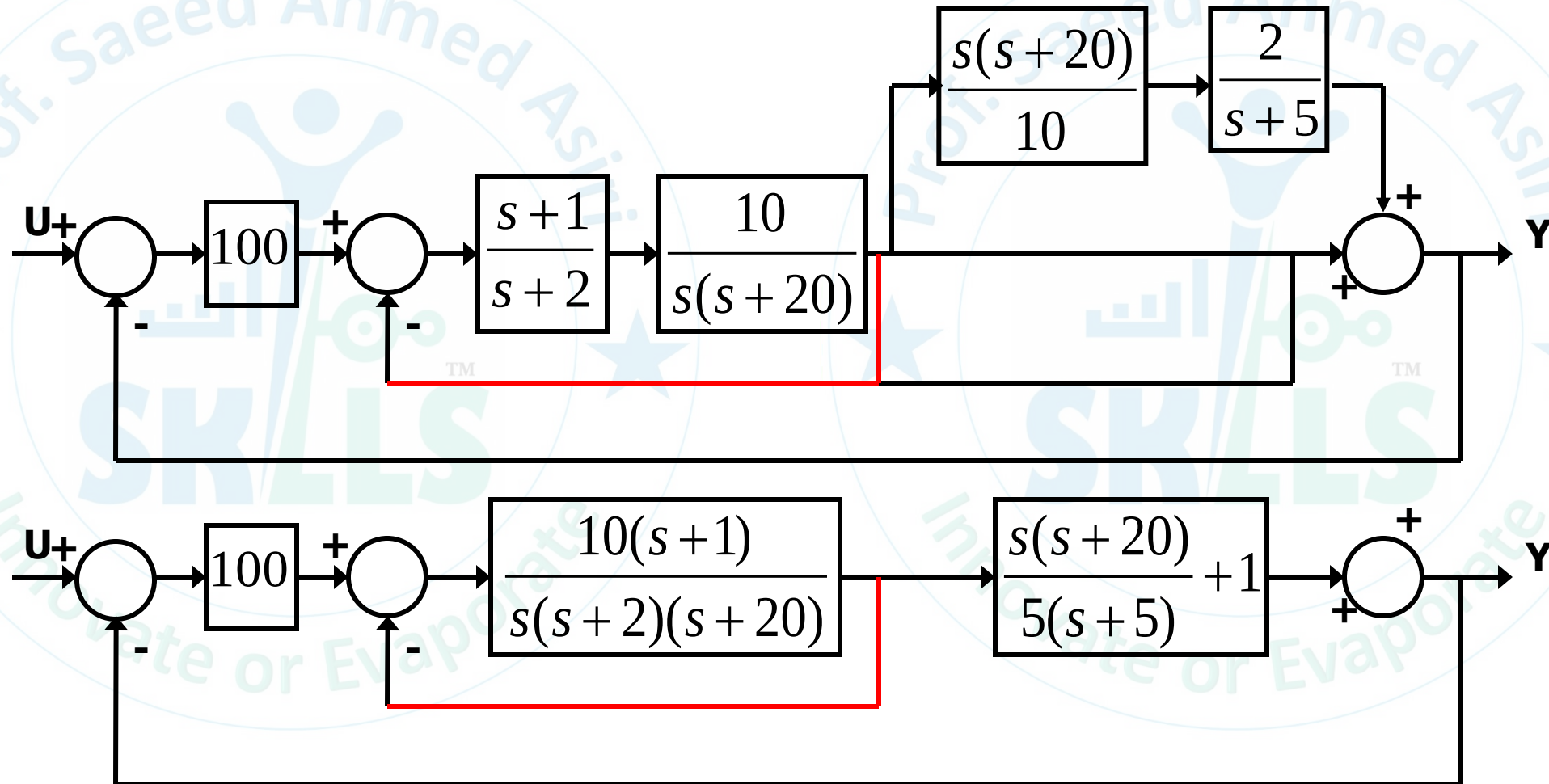


- No pure series/parallel/feedback
- Needs to move a block, but which one?

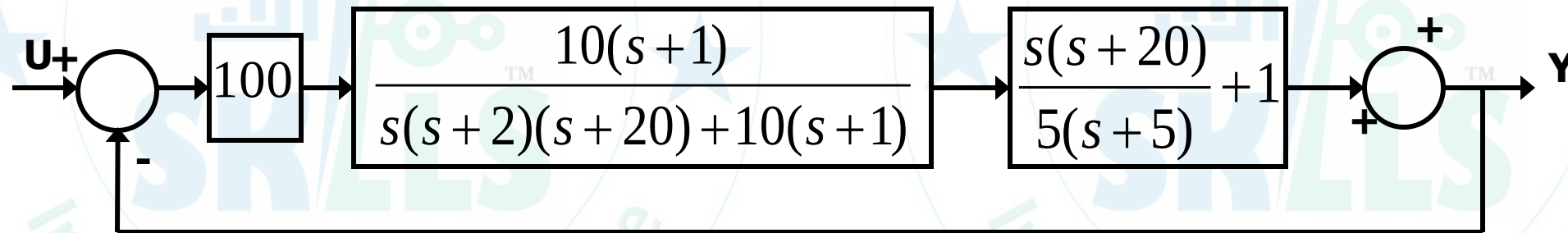
Key: move one block to create pure series or parallel or feedback!

So move $\frac{10}{s(s+20)}$ either left or right.

Block Diagram Reduction

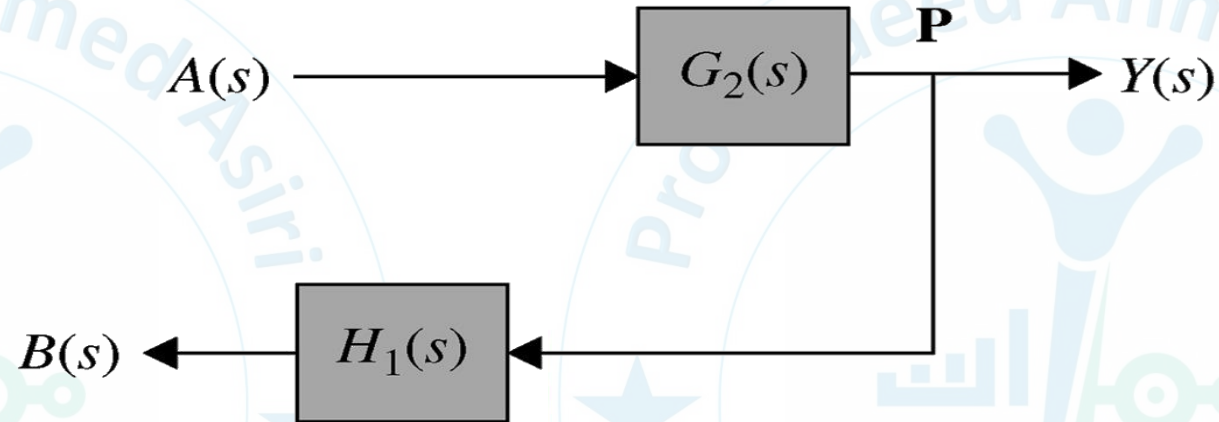


Block Diagram Reduction

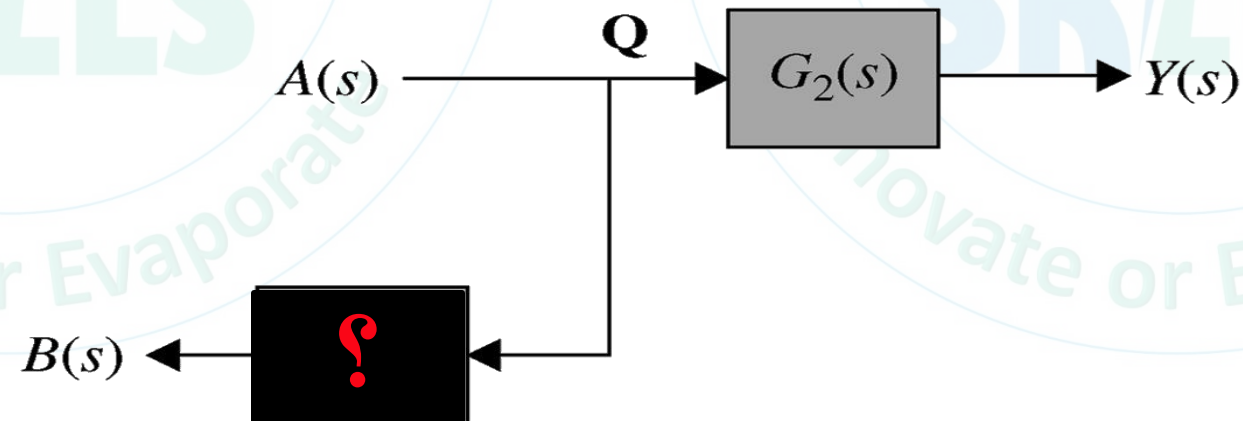


Block Diagram Reduction

(a)

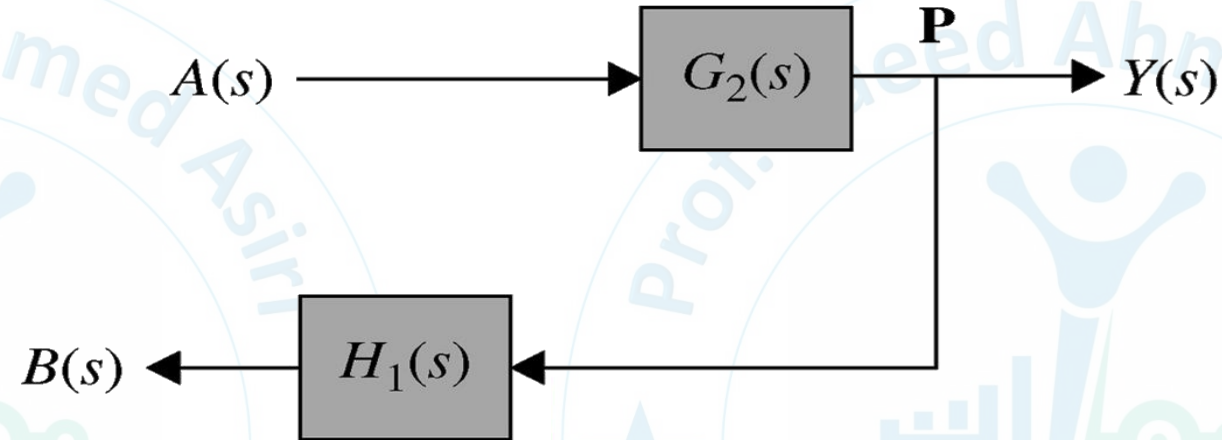


(b)

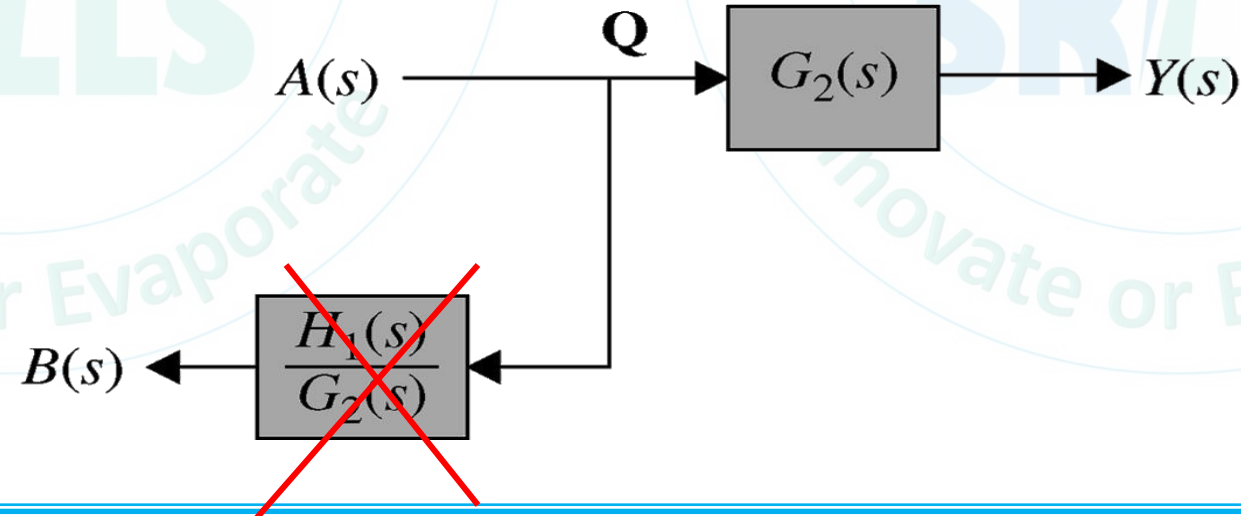


Block Diagram Reduction

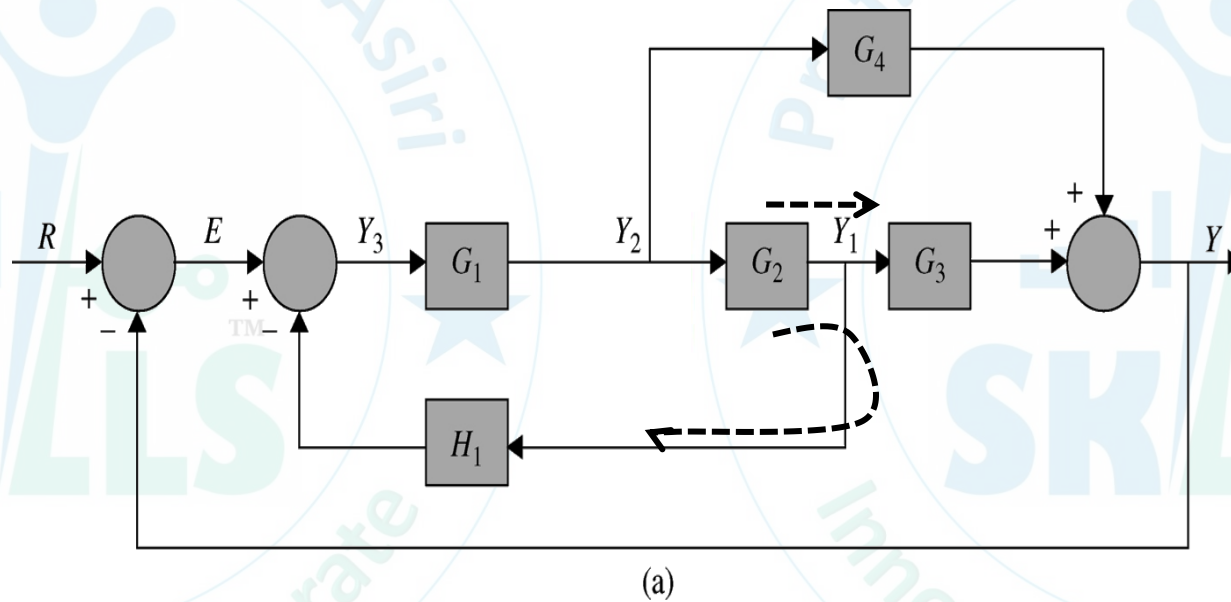
(a)



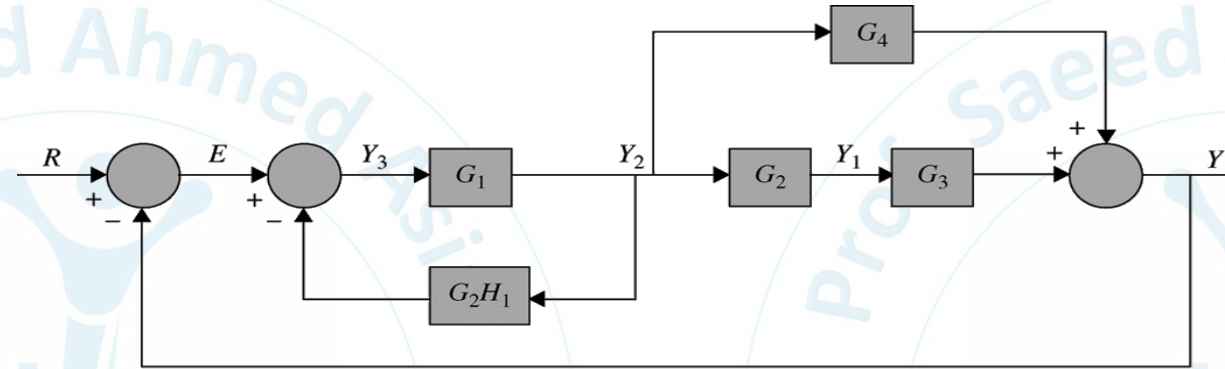
(b)

 **$H_1(s)G_2(s)$**

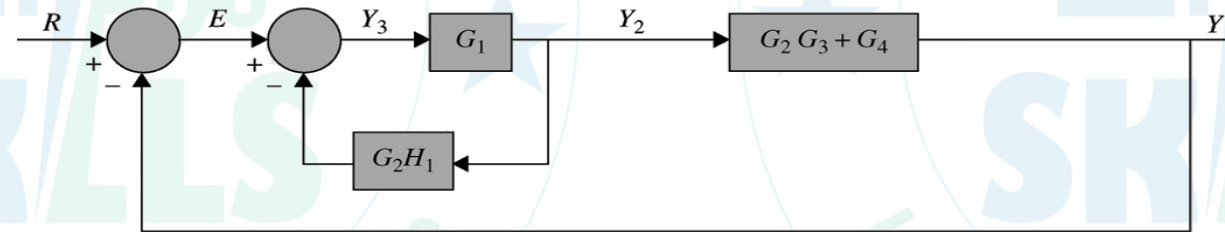
Block Diagram Reduction



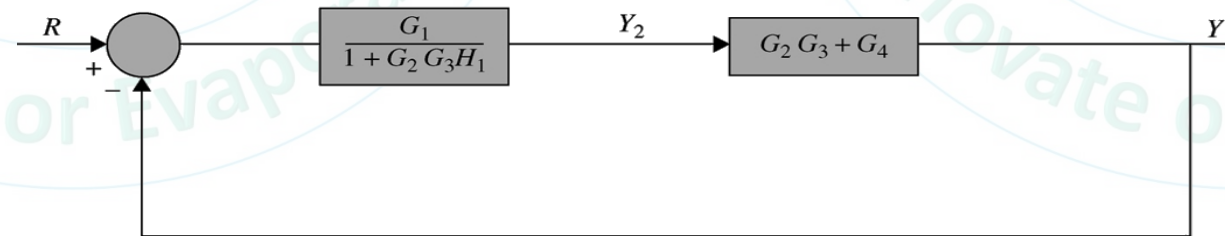
Block Diagram Reduction



(b)

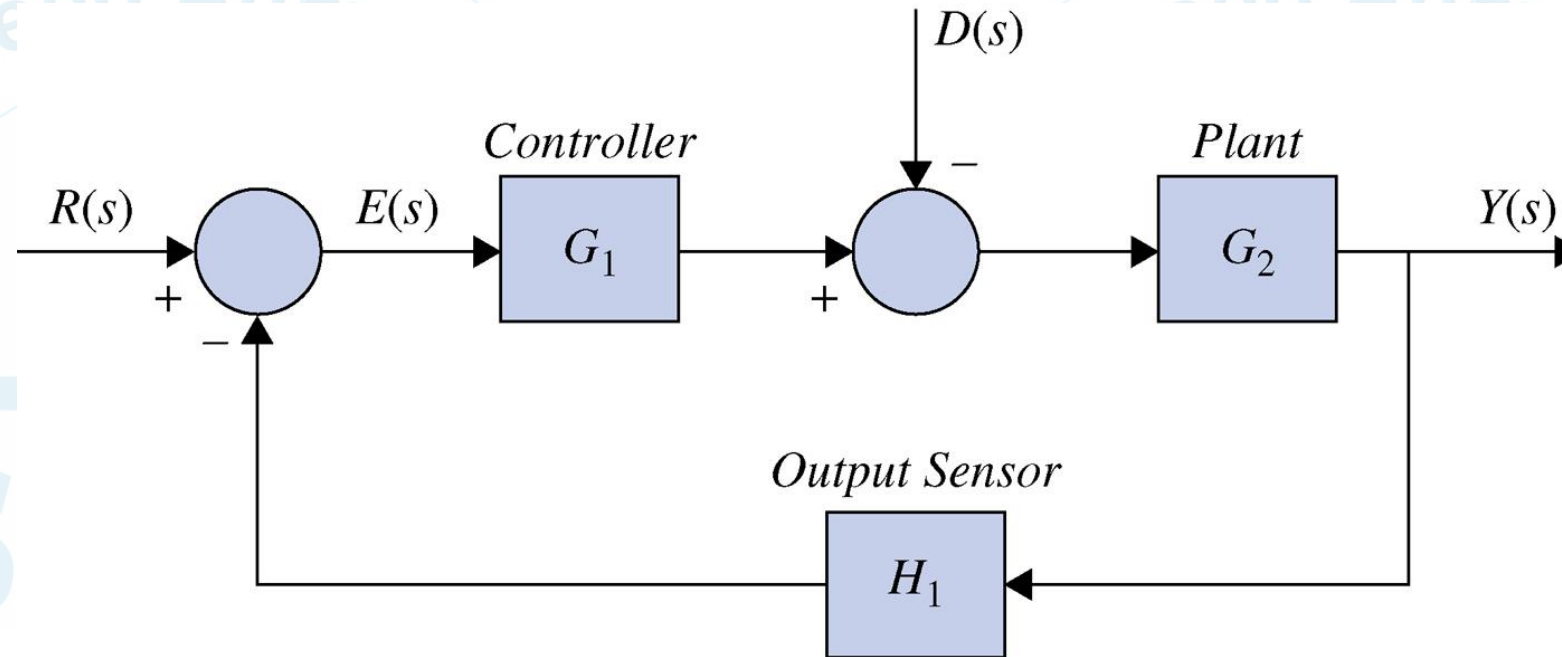


(c)



(d)

Block Diagram Reduction



Can use superposition:

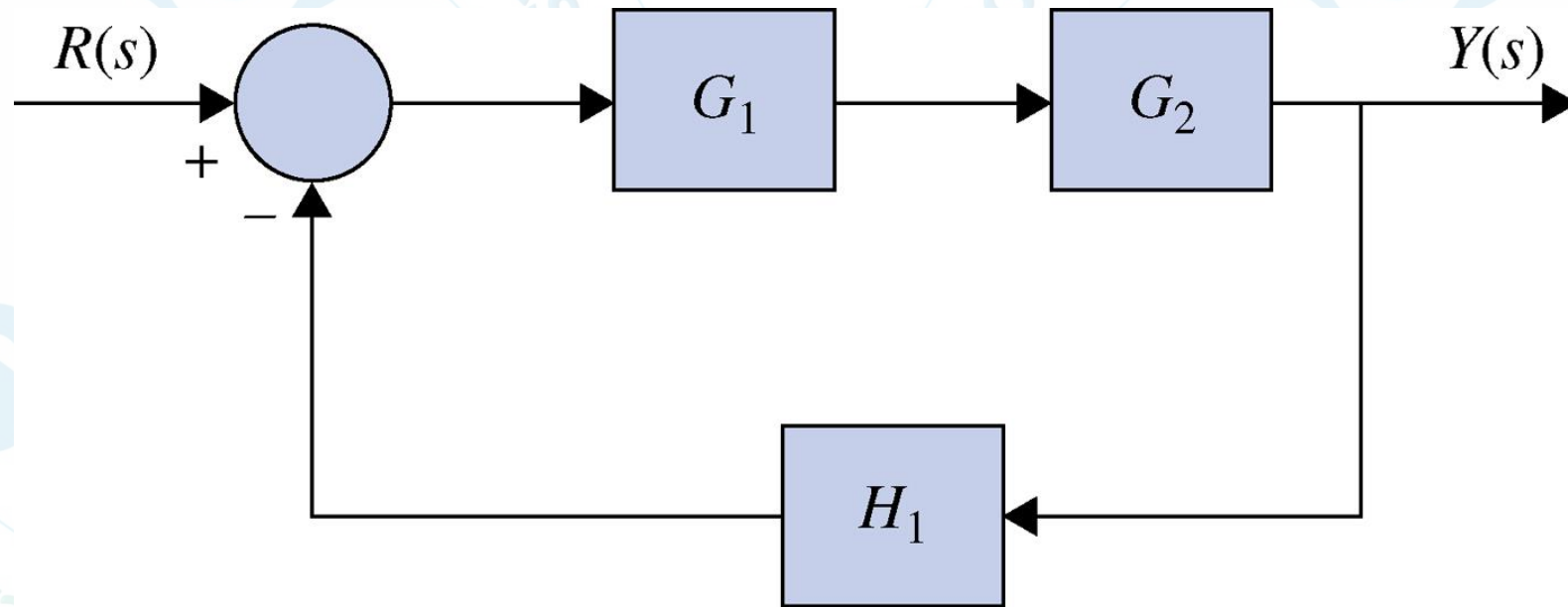
First set $D=0$, find Y due to R

Then set $R=0$, find Y due to D

Finally, add the two component to get the overall Y

Block Diagram Reduction

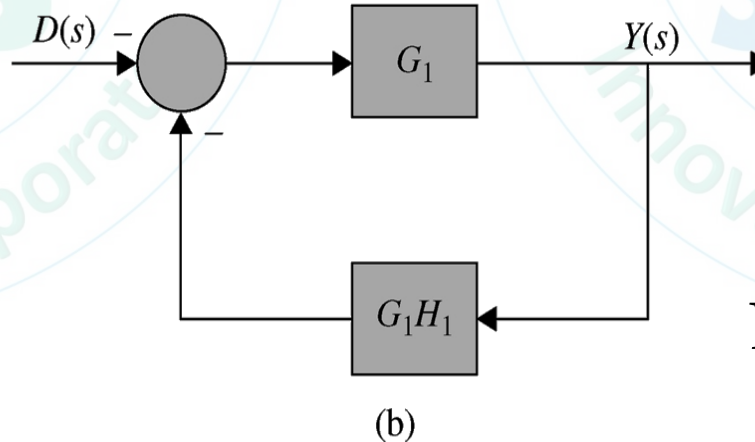
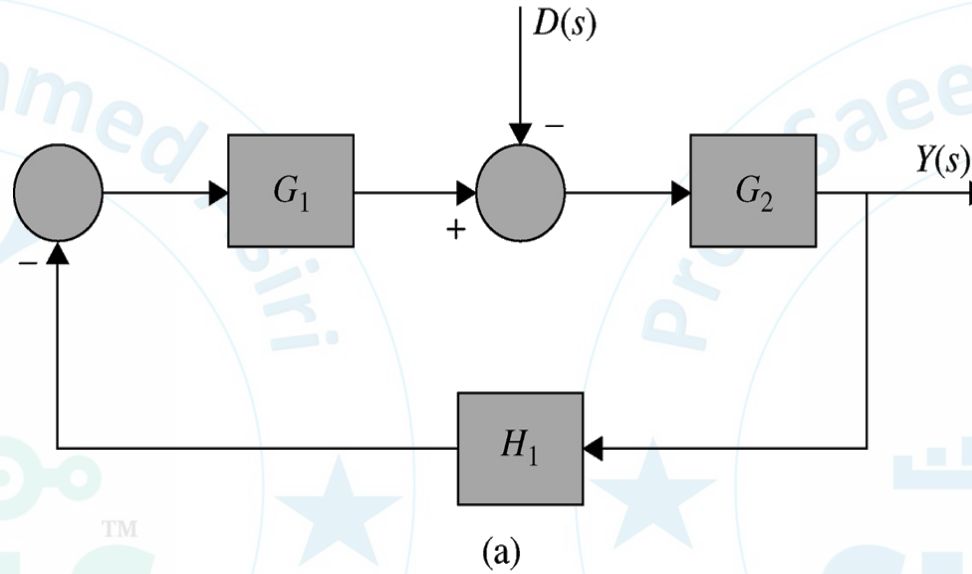
First set $D=0$, find Y due to R



$$Y_1(s) = \frac{G_1 G_2}{1 + G_1 G_2 H_1} R(s)$$

Block Diagram Reduction

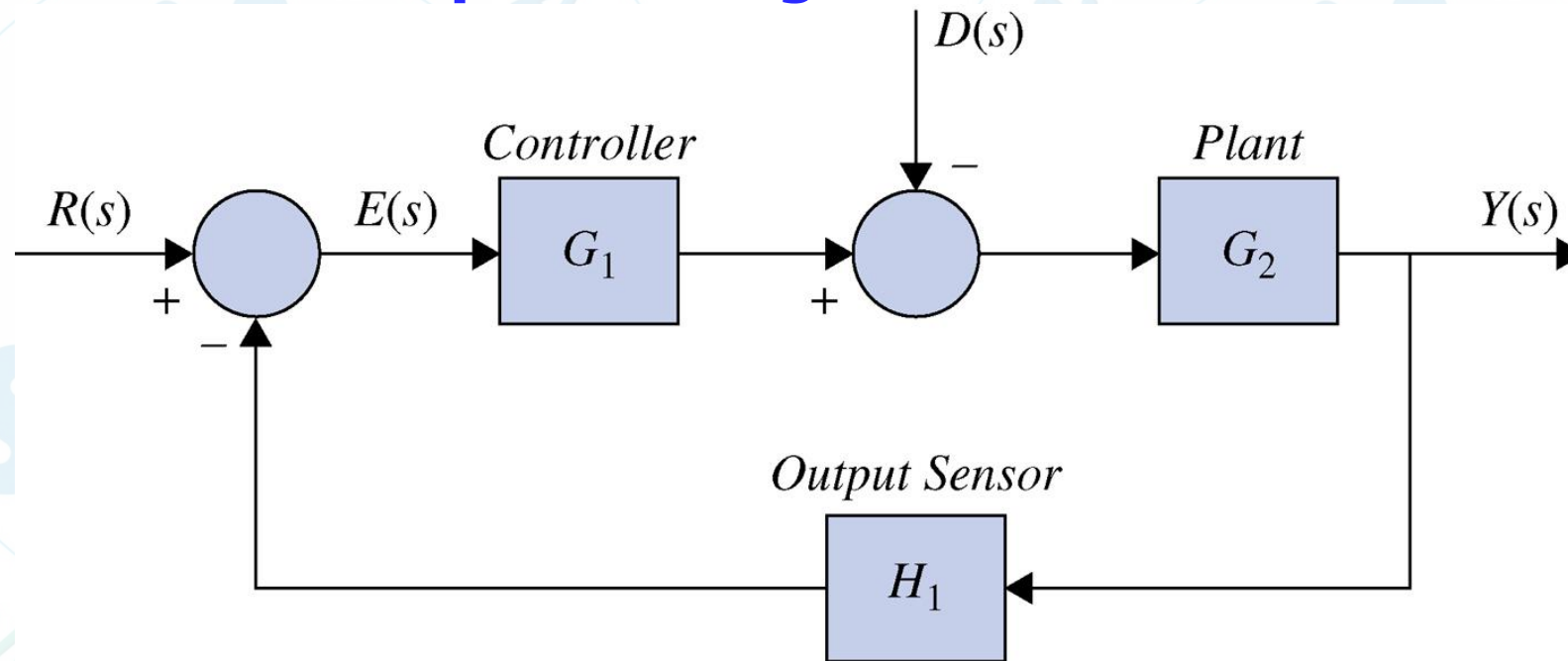
Then set $R=0$,
find Y due to
 D



$$Y_2(s) = \frac{G_2}{1 + G_1 G_2 H_1} (-D(s))$$

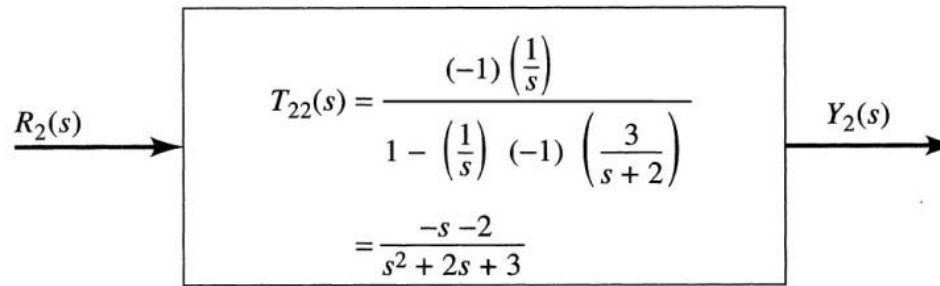
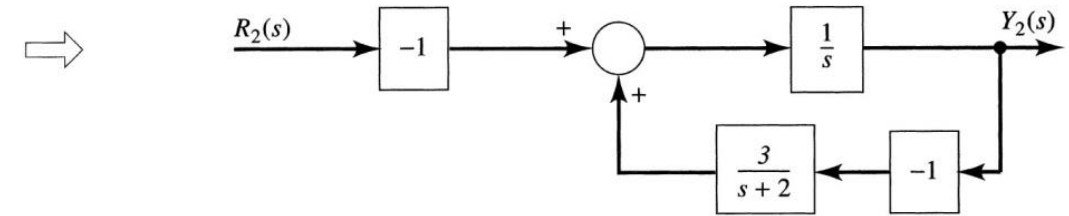
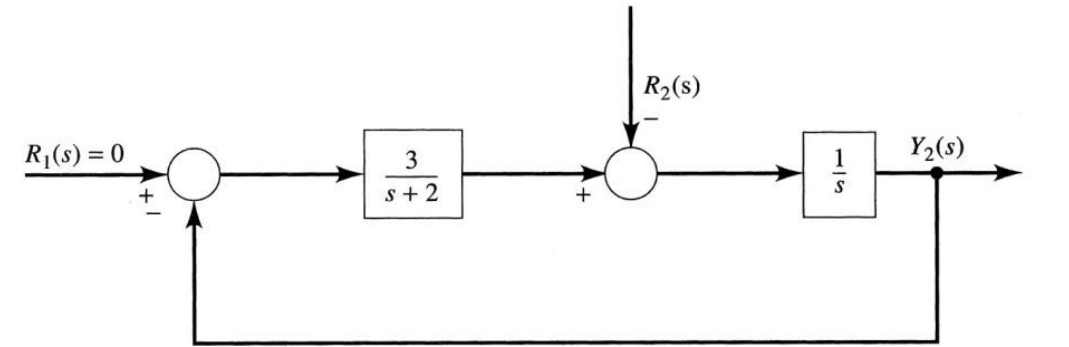
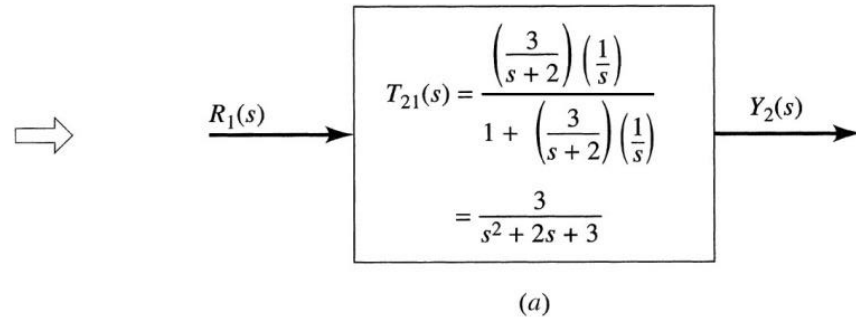
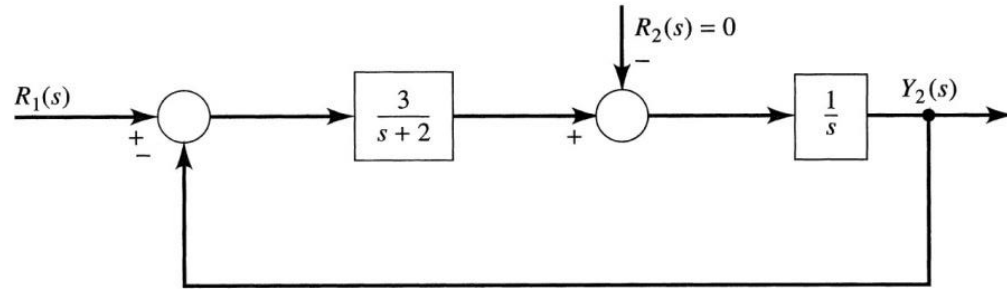
Block Diagram Reduction

Finally, add the two component to get the overall Y



$$Y(s) = \frac{G_1 G_2}{1 + G_1 G_2 H_1} R(s) - \frac{G_2}{1 + G_1 G_2 H_1} D(s)$$

Block Diagram Reduction



(b)

Block Diagram Reduction

