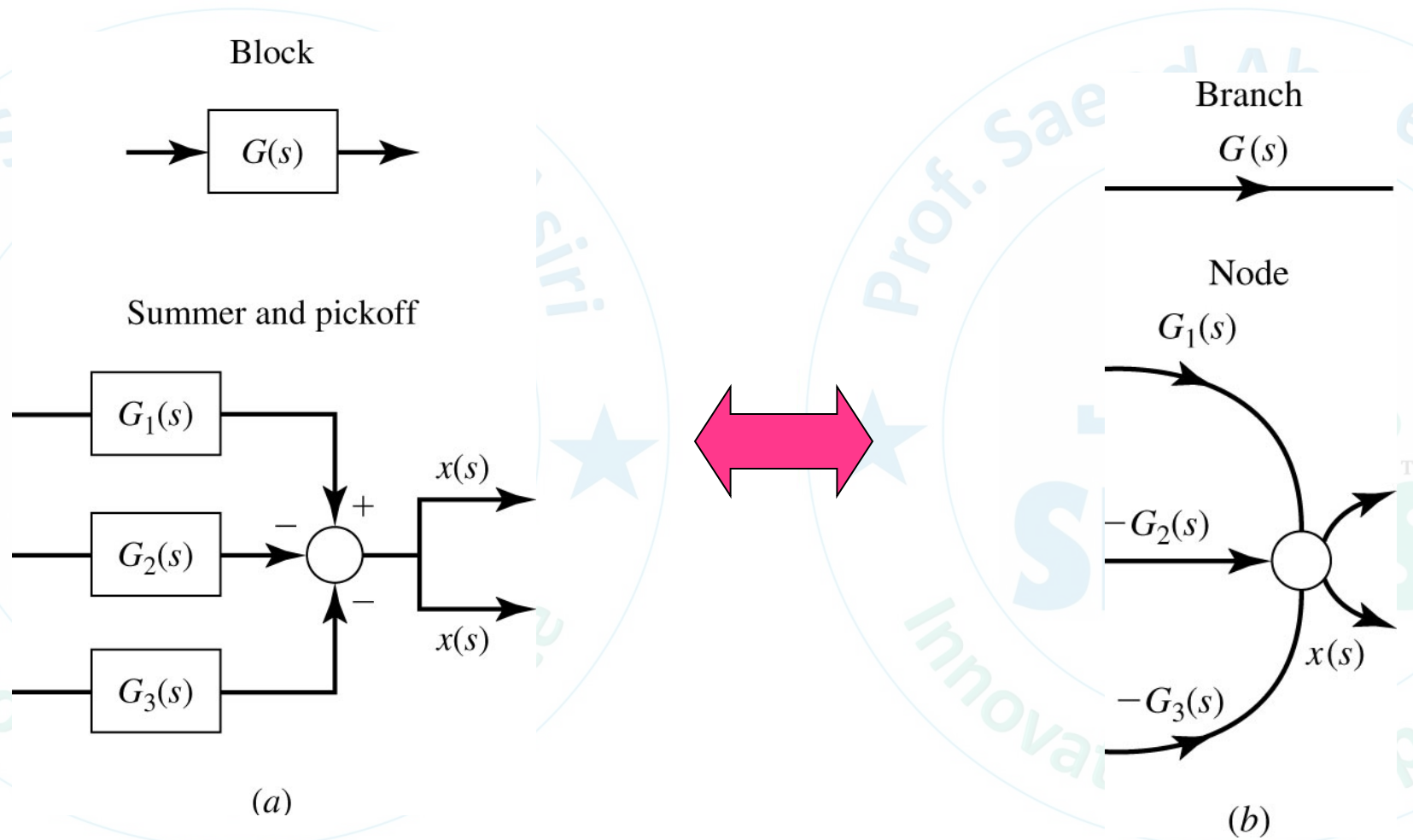


MENG366

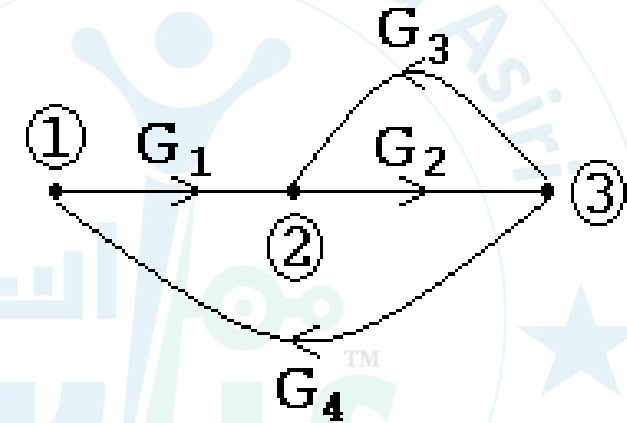
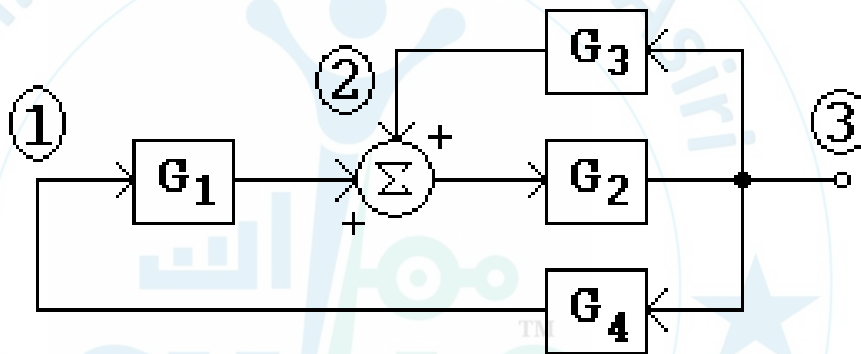
Signal Flow Graph

Prof. Saeed Asiri
saeed@asiri.net

Signal Flow Graph

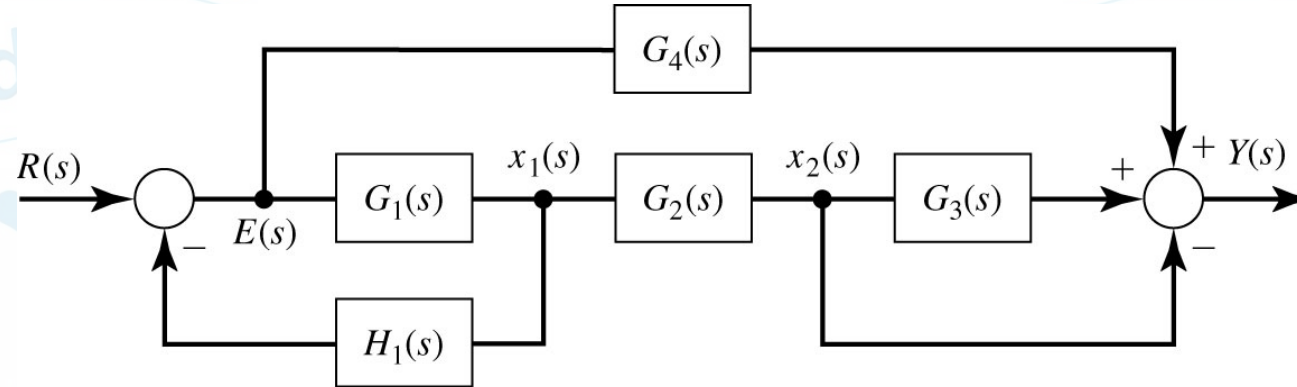


Signal Flow Graph

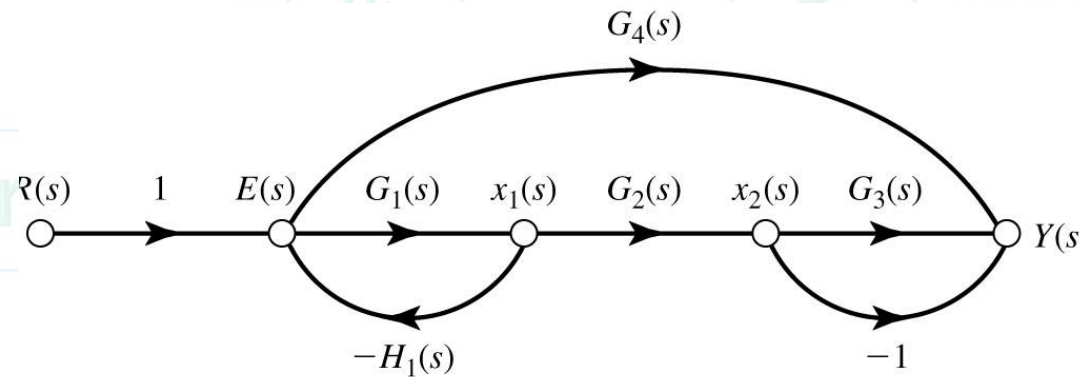
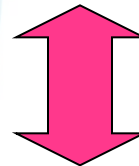


- Compact alternative ***notation to the block diagram.***
- It characterizes the system by a network of directed branches and associated transfer functions.
- The two ways of depicting signal are equivalent.

Signal Flow Graph



(a)



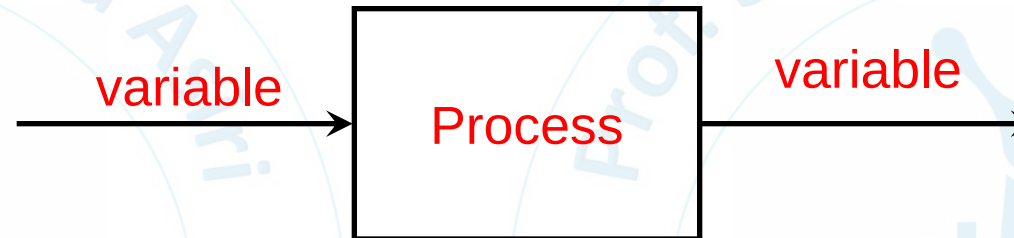
(b)

Block Diagram Models

- Block diagrams are used as schematic representations of mathematical models
- The various pieces correspond to mathematical entities
- Can be rearranged to help simplify the equations used to model the system
- We will focus on one type of schematic diagram – feedback control systems

Block Diagram Models

- Processes are represented by the blocks in block diagrams:

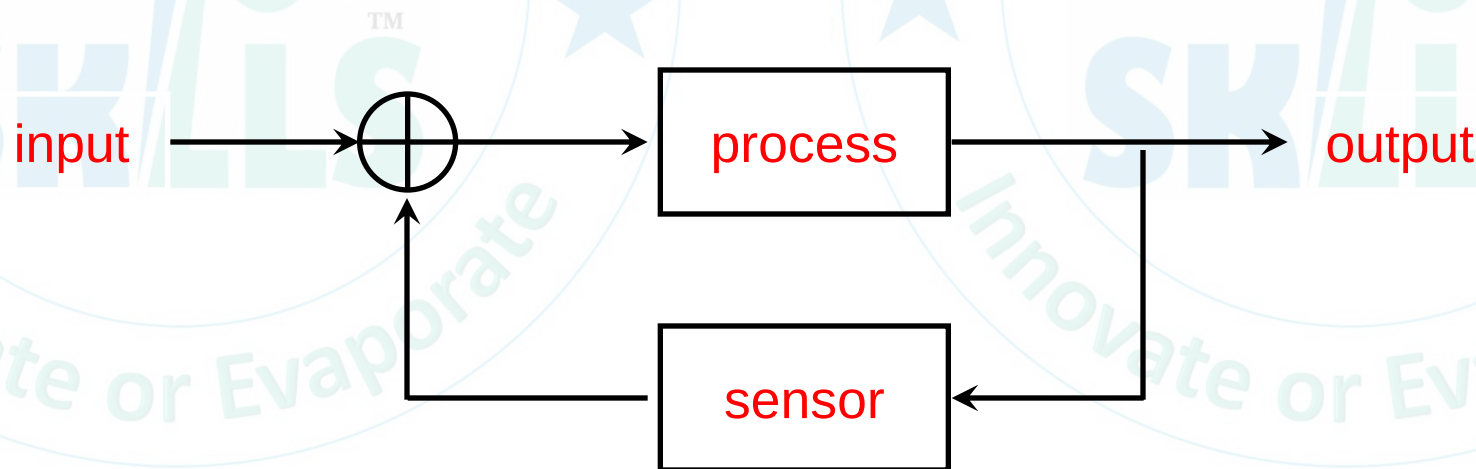


- Processes must have at least one input variable and at least one output variable
- Reclassify processes without input or output:

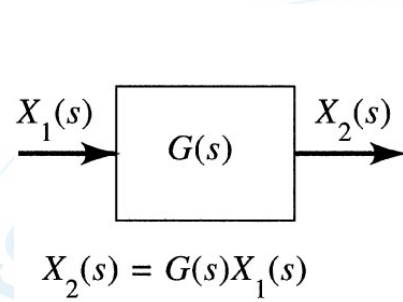
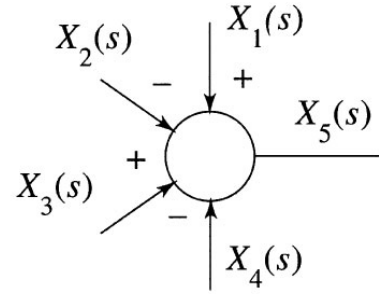
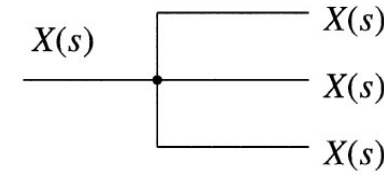
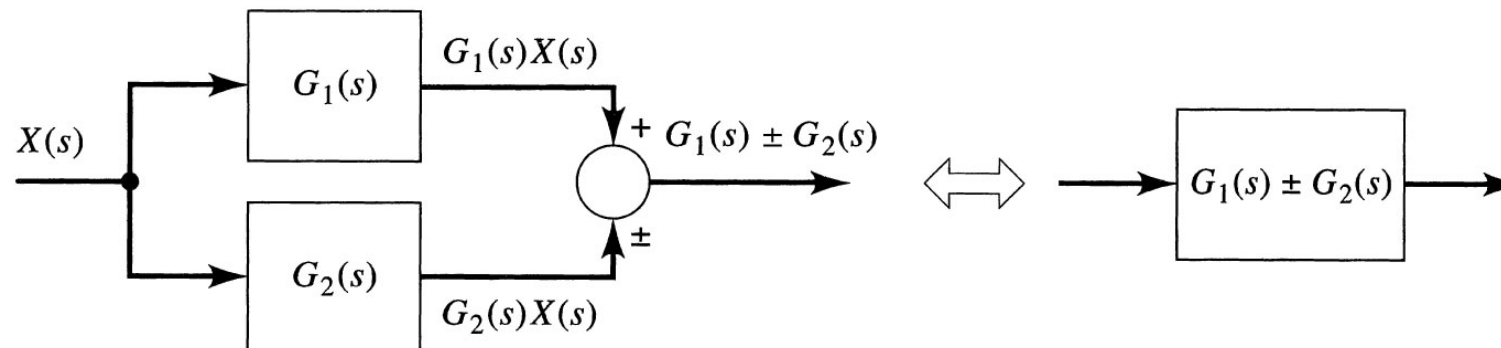
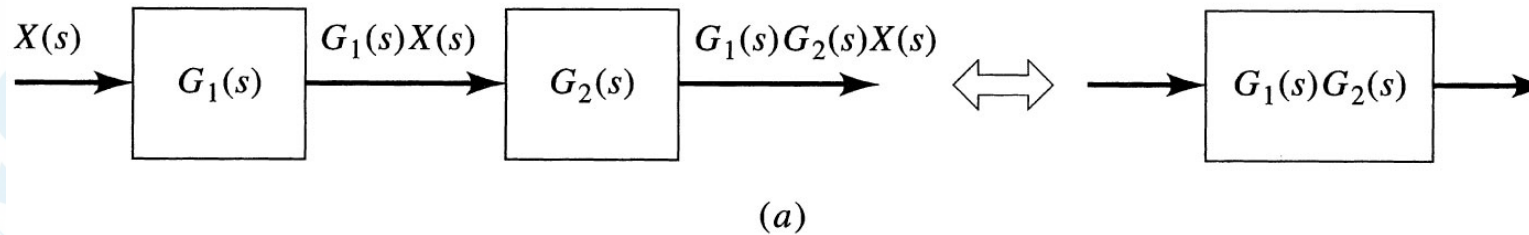


Block Diagram Models

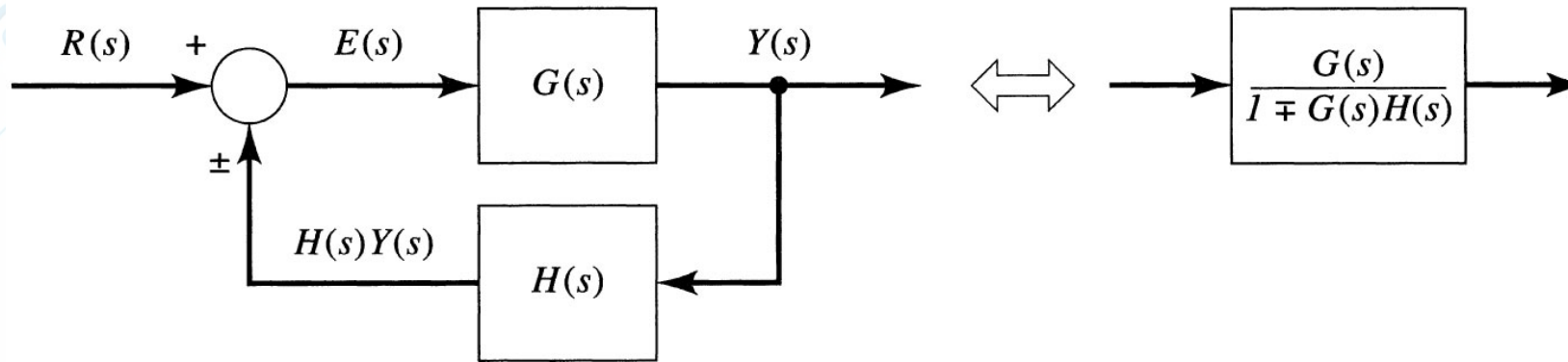
- Many systems measure their output and use this measurement to control system behavior
- This is known as feedback control – the output is “fed back” into the system
- The summing junction is a special process that compares the input and the feedback
- Inputs to summing junction must have same units!



Block Diagram Models

(a) **block**(b) **summer**(c) **pickoff point**

Block Diagram Models



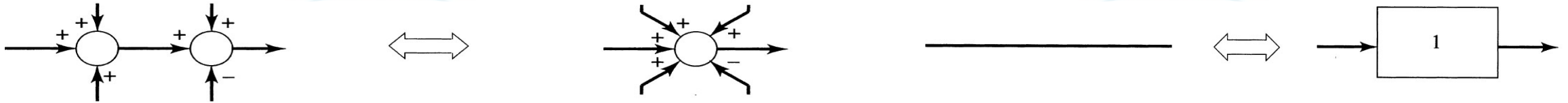
$$Y(s) = G(s)E(s)$$

$$E(s) = R(s) \pm H(s)Y(s)$$

$$Y(s) = G(s)[R(s) \pm H(s)Y(s)] = G(s)R(s) \pm G(s)H(s)Y(s)$$

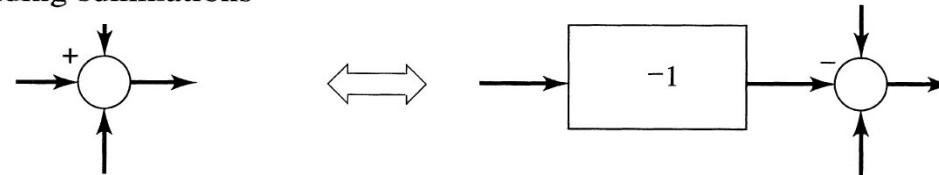
$$T(s) = \frac{Y(s)}{R(s)} = \frac{G(s)}{1 \mp G(s)H(s)}$$

Block Diagram Models

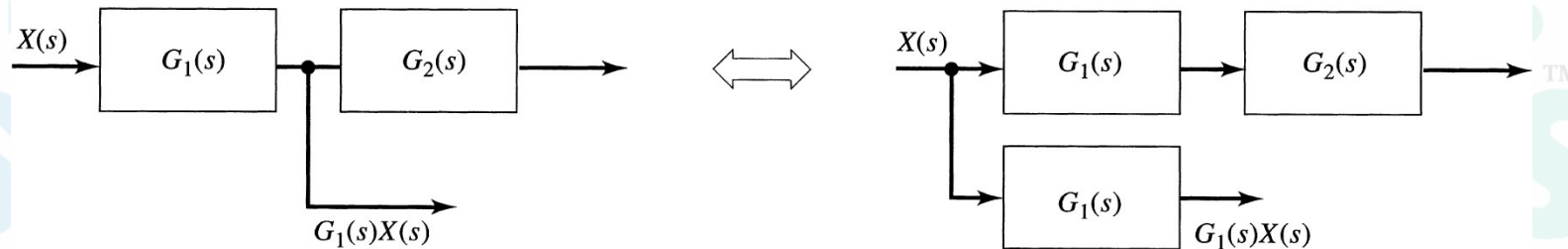


(a) Insertion or removal of unity gain

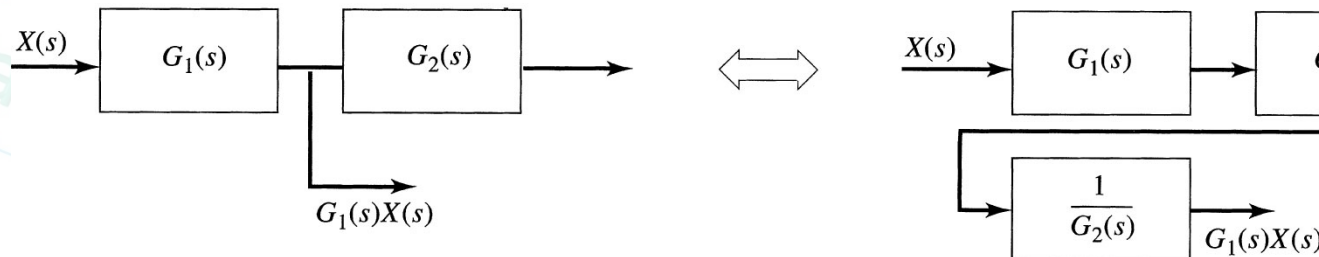
(e) Combining or expanding summations



(b) Changing a summer sign



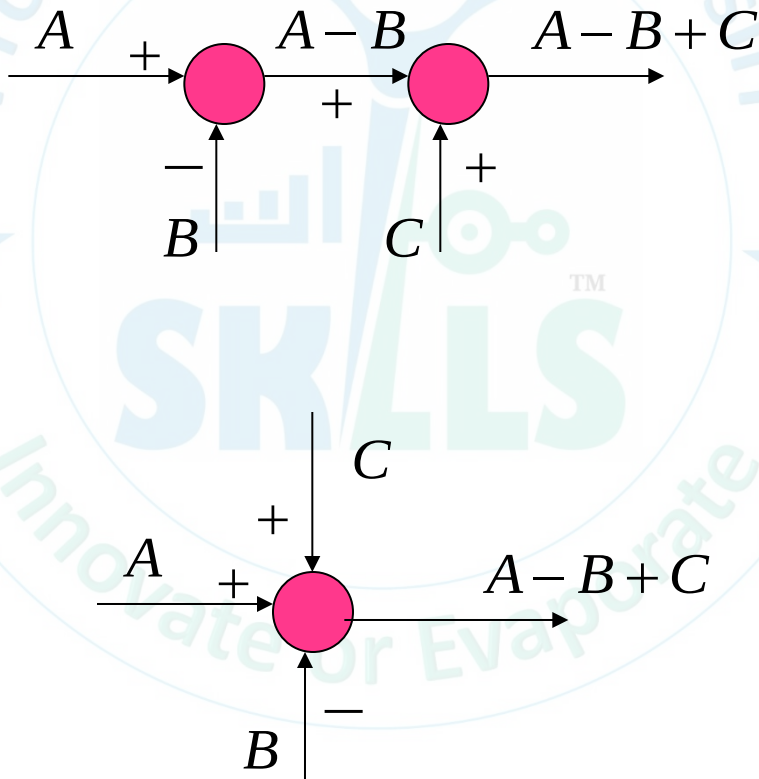
(c) Moving a pickoff point back



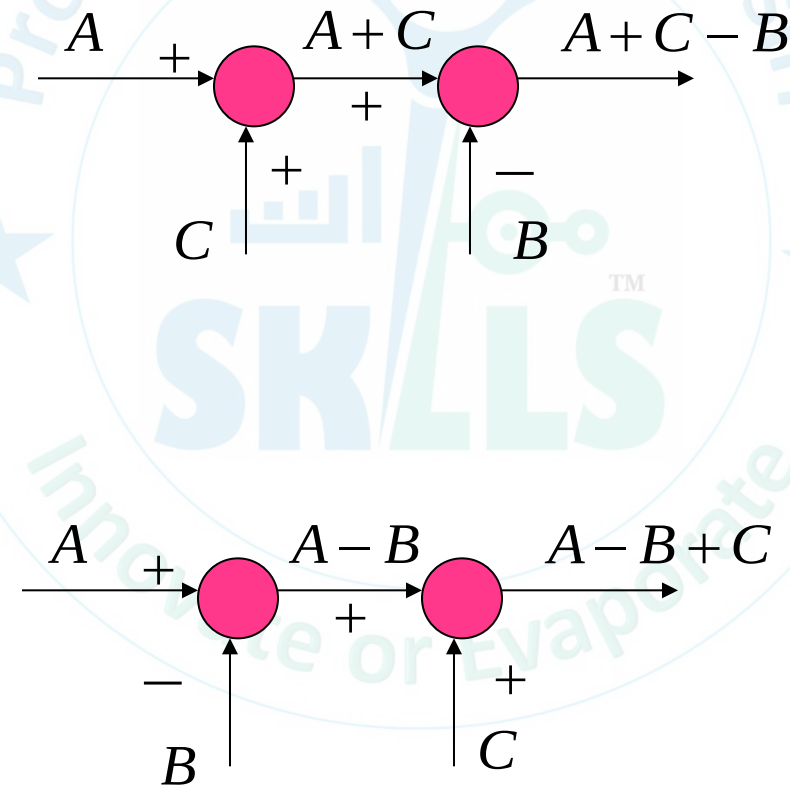
(d) Moving a pickoff point forward

Block Diagram Models

original

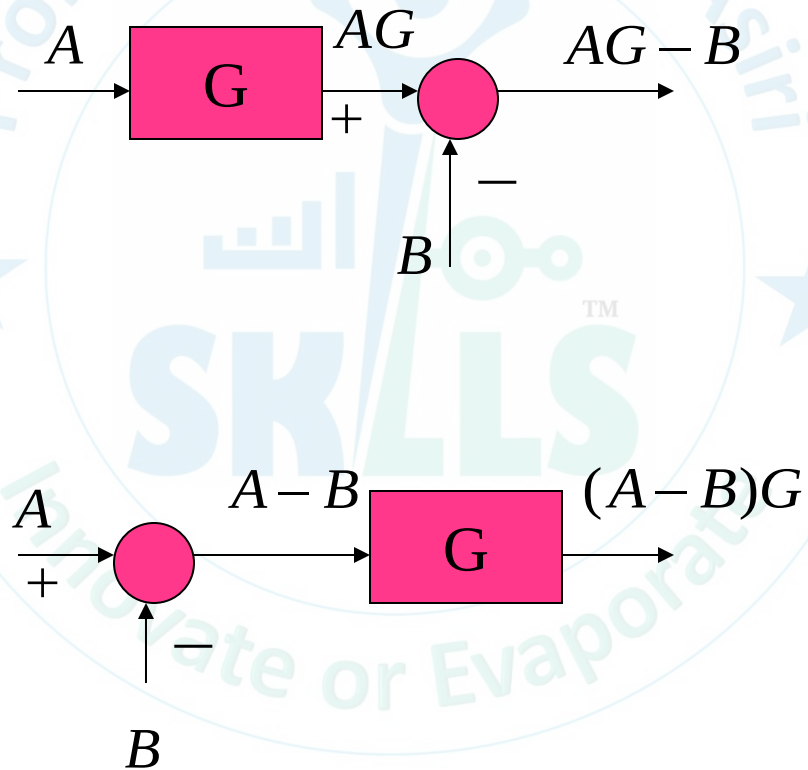


equivalent

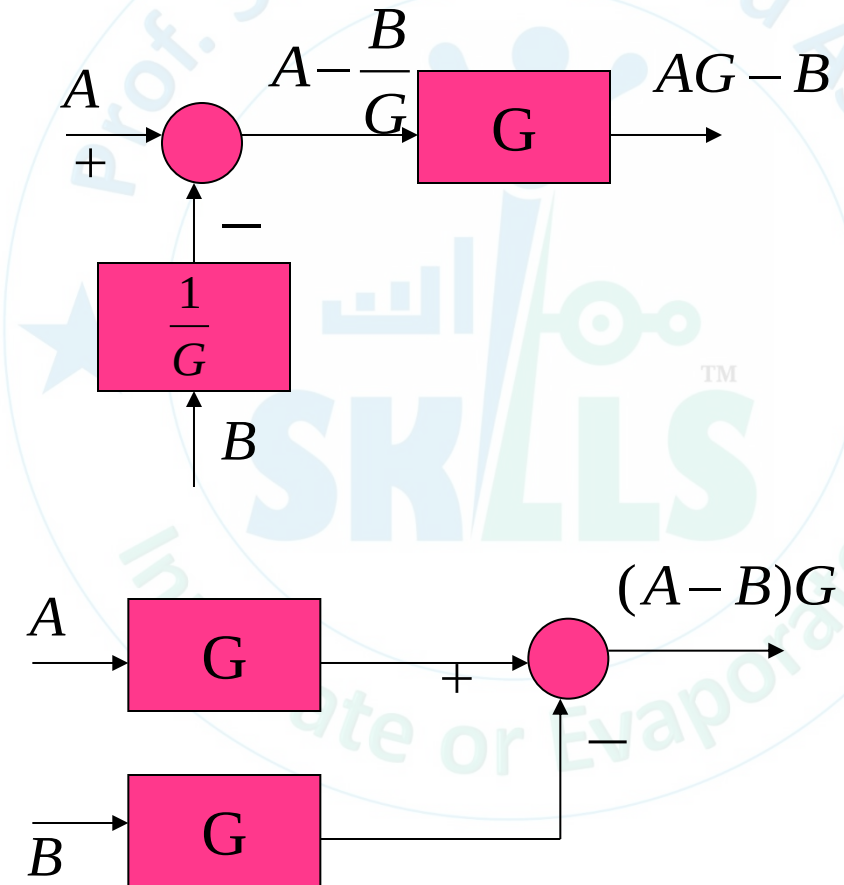


Block Diagram Models

original

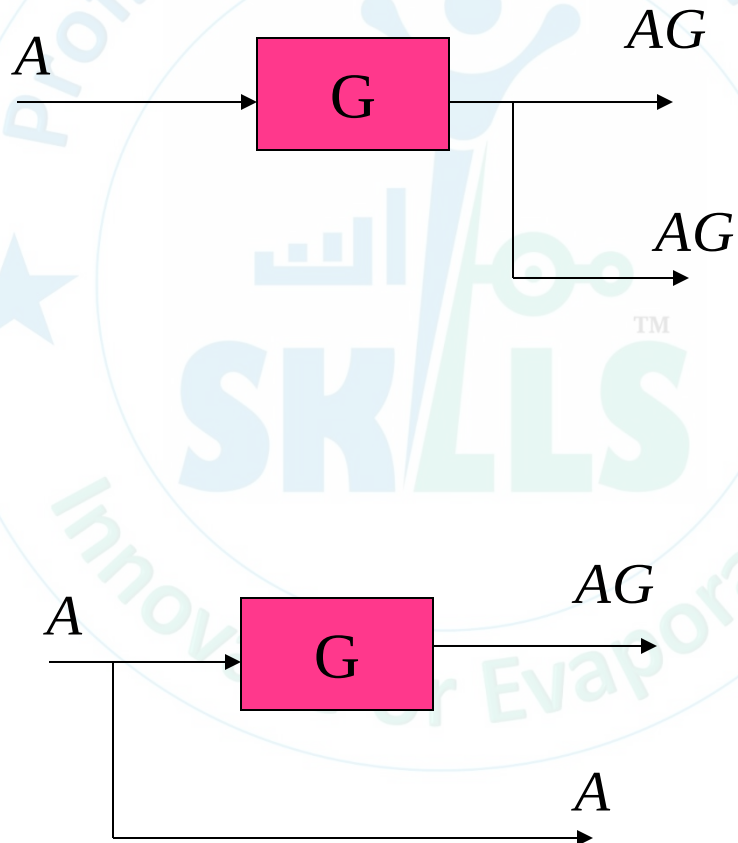


equivalent

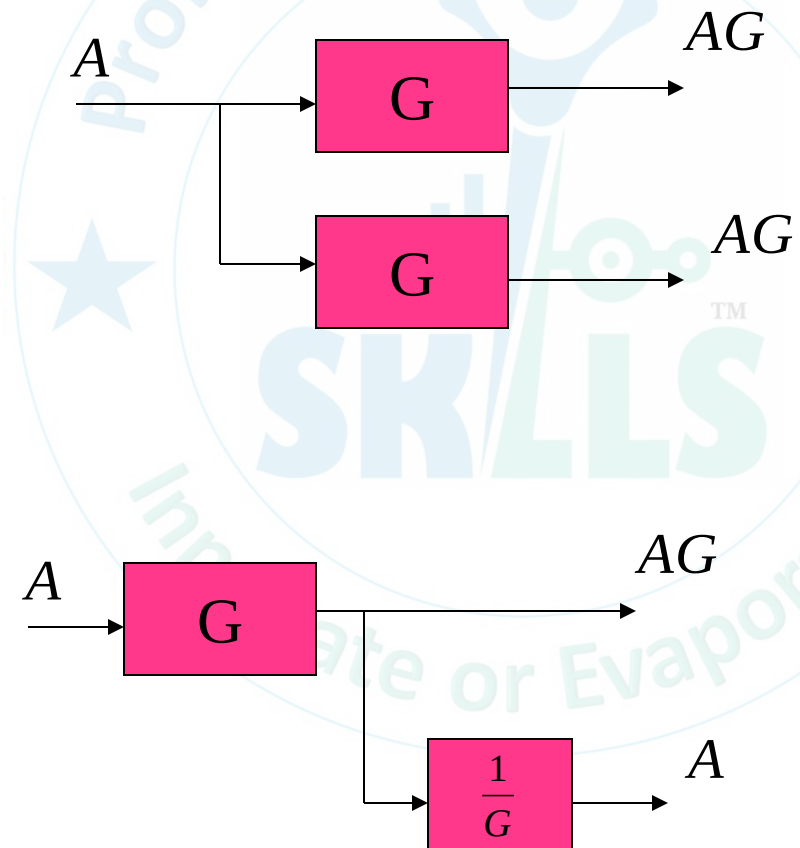


Block Diagram Models

original

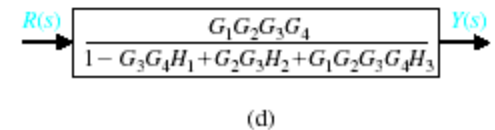
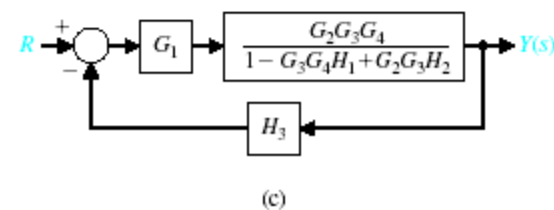
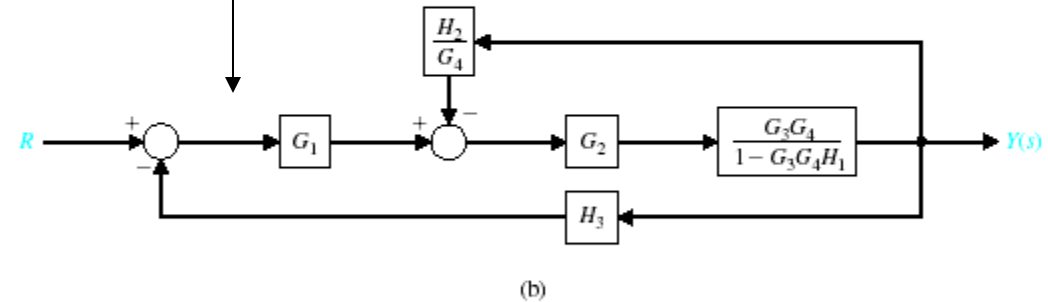
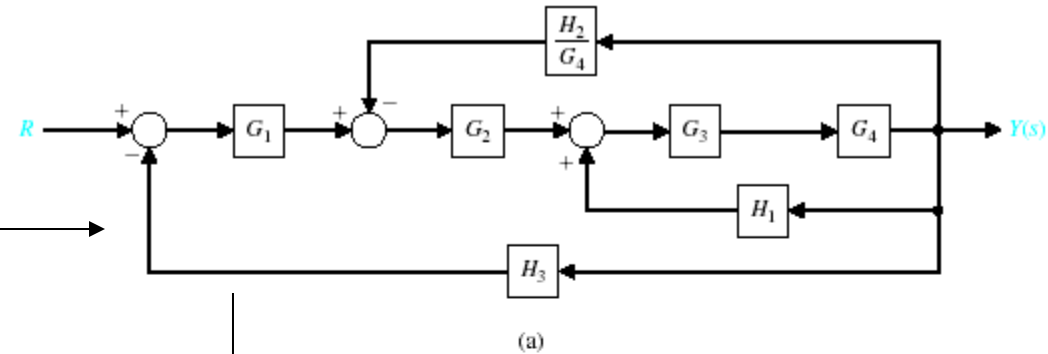
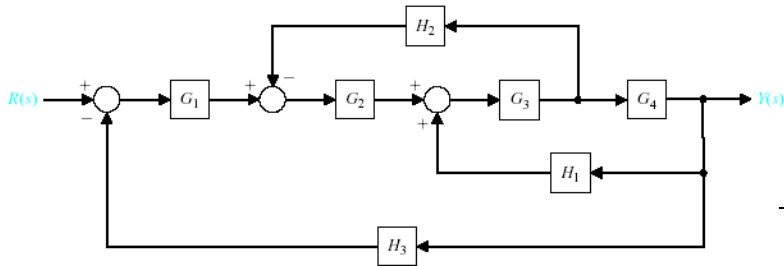


equivalent



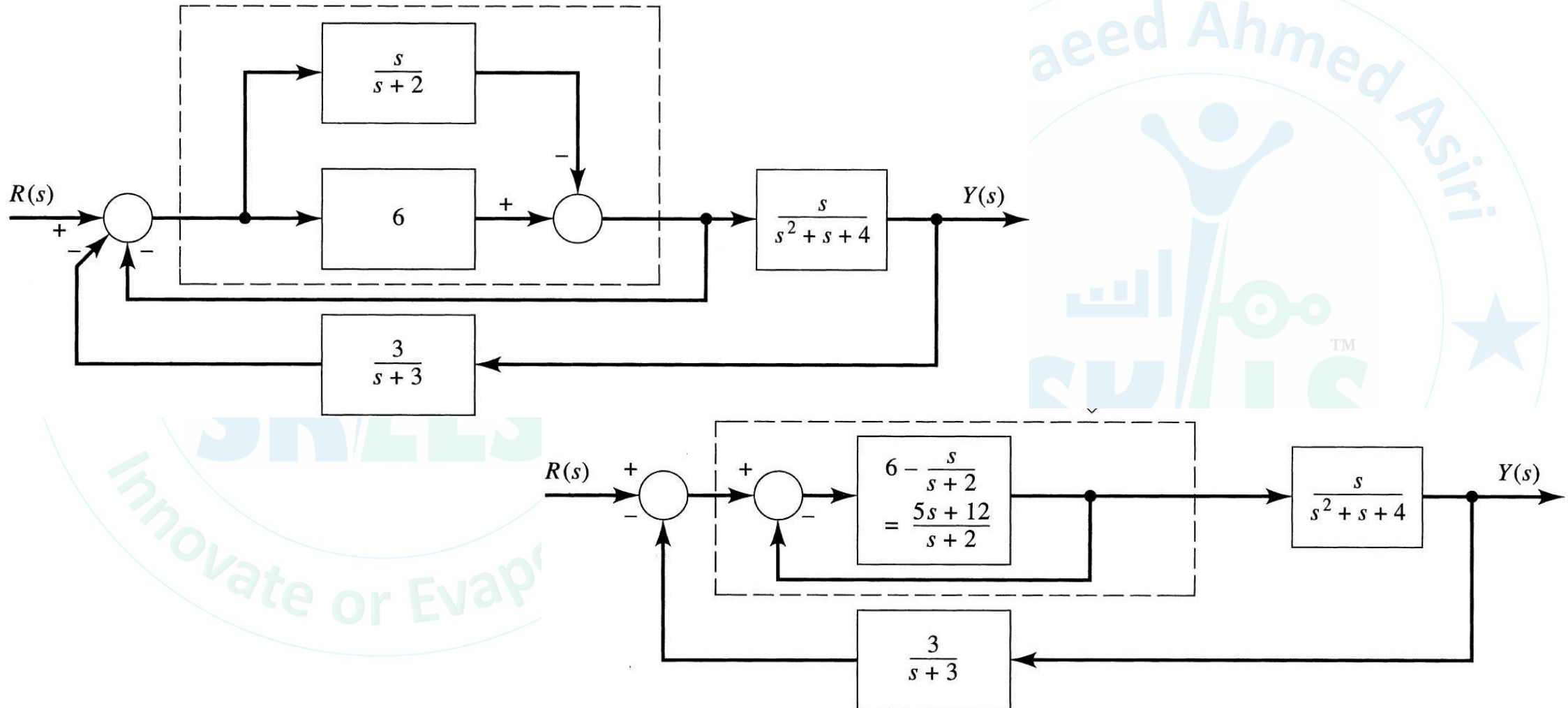
Block Diagram Models

Example 1



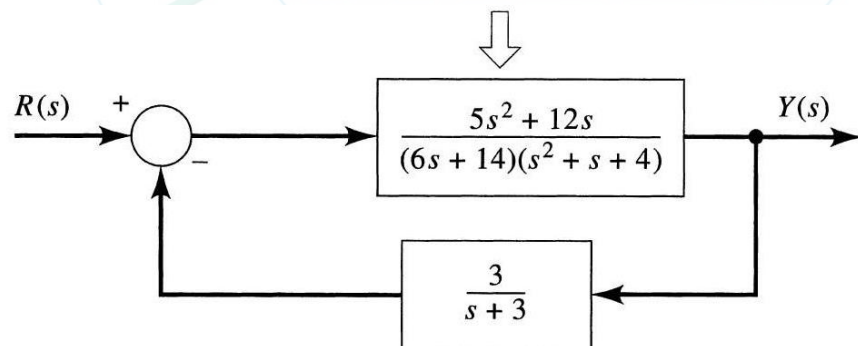
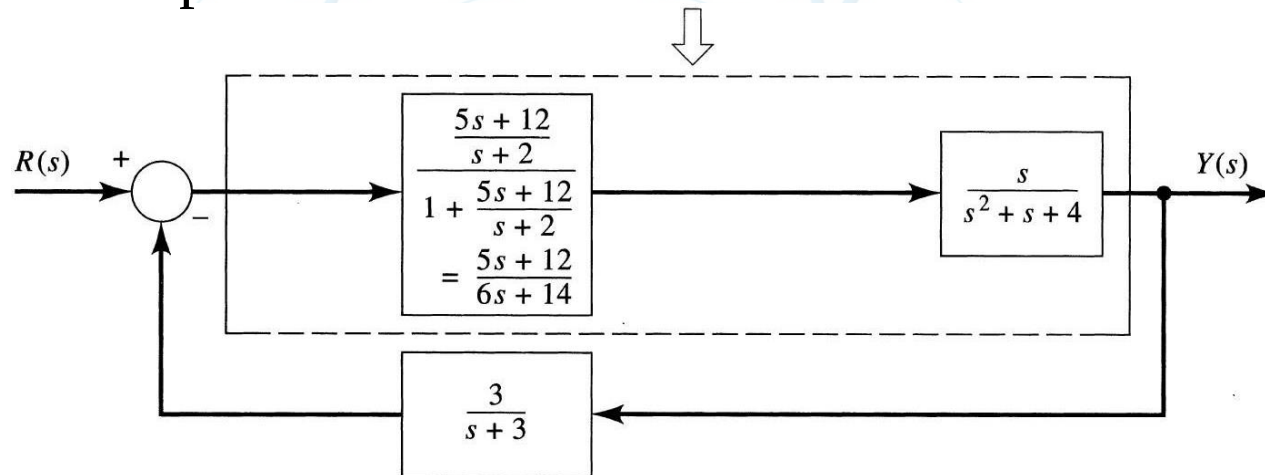
Block Diagram Models

Example 2



Block Diagram Models

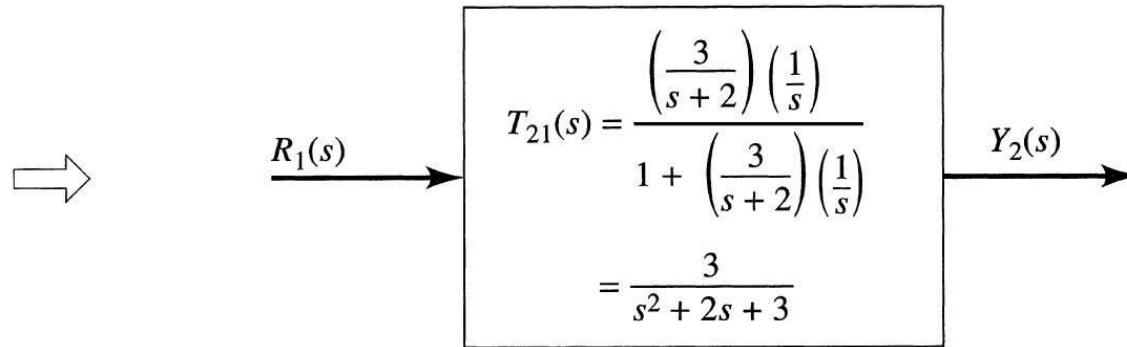
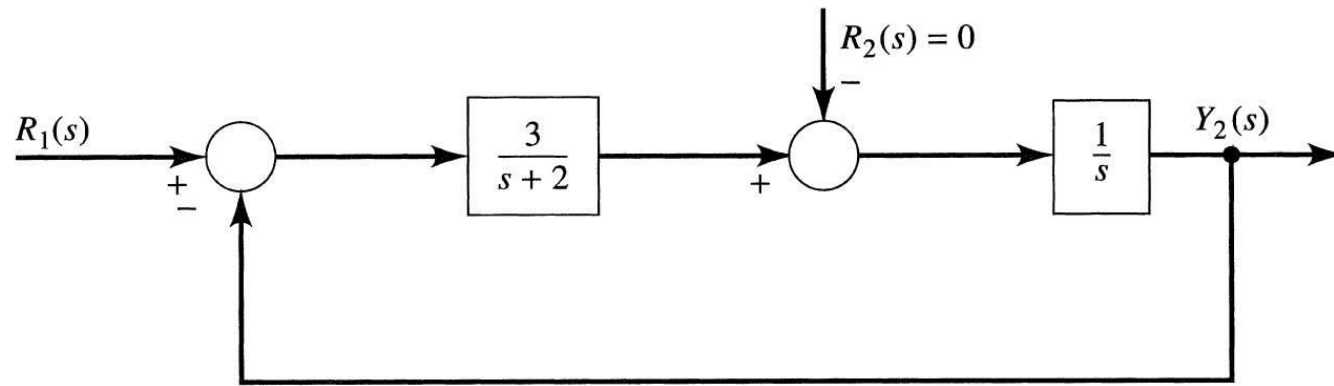
Example 2 cont.



$$\begin{aligned}
 & \downarrow \\
 & \begin{aligned}
 & R(s) \rightarrow \frac{\frac{5s^2 + 12s}{(6s + 14)(s^2 + s + 4)}}{1 + \frac{(5s^2 + 12s)(3)}{(6s + 14)(s^2 + s + 4)(s + 3)}} \rightarrow Y(s) \\
 & = \frac{5s^3 + 27s^2 + 36s}{6s^4 + 38s^3 + 113s^2 + 206s + 168}
 \end{aligned}
 \end{aligned}$$

Block Diagram Models

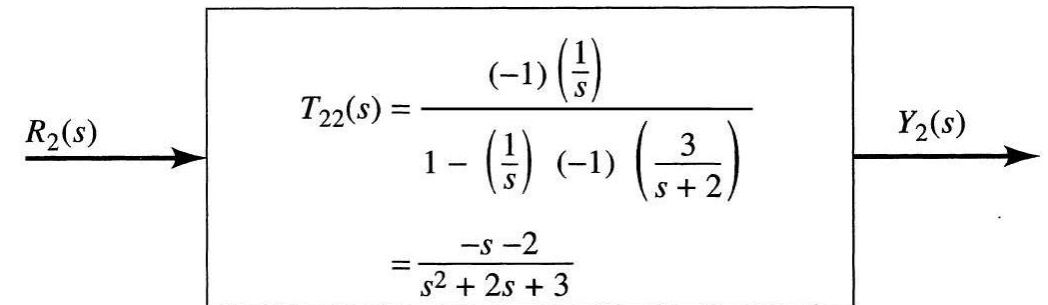
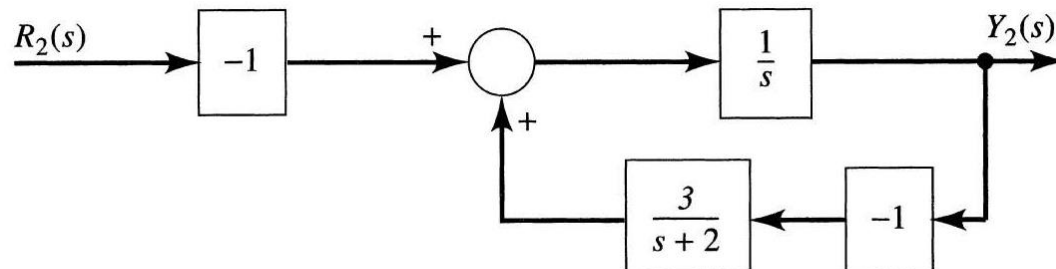
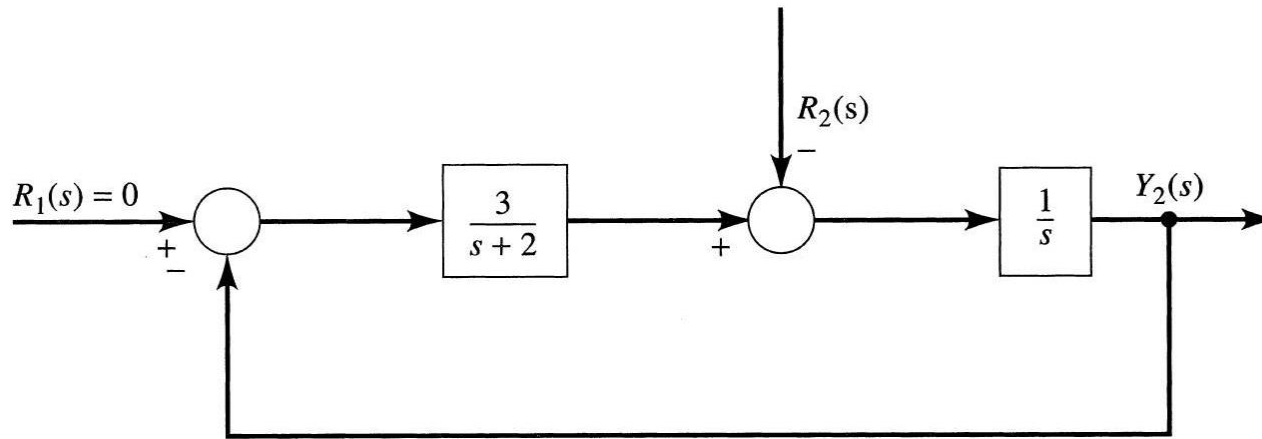
Example 3



(a)

Block Diagram Models

Example 3 cont.



(b)

Introduction to Signal Flow

- Alternative method to block diagram representation, developed by **Samuel Jefferson Mason**.
- Advantage: the availability of a flow graph gain formula, also called Mason's gain formula.
- A signal-flow graph consists of a network in which nodes are connected by directed branches.
- It depicts the flow of signals from one point of a system to another and gives the relationships among the signals.

Fundamentals of Signal Flow Graphs

- Consider a simple equation below and draw its signal flow graph:

$$y = ax$$

- The signal flow graph of the equation is shown below;

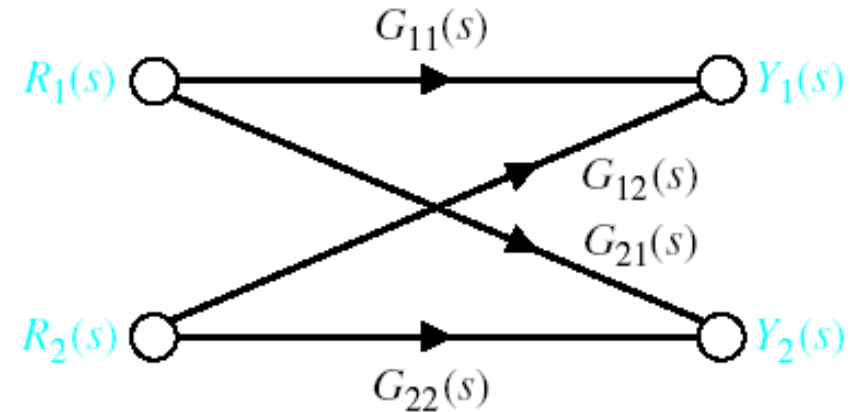


- Every variable in a signal flow graph is designed by a **Node**.
- Every transmission function in a signal flow graph is designed by a **Branch**.
- Branches are always **unidirectional**.
- The arrow in the branch denotes the **direction** of the signal flow.

Signal-Flow Graph Models

$$Y_1(s) = G_{11}(s) \cdot R_1(s) + G_{12}(s) \cdot R_2(s)$$

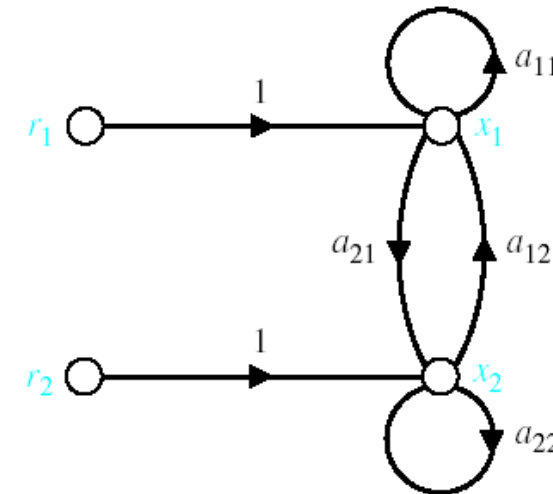
$$Y_2(s) = G_{21}(s) \cdot R_1(s) + G_{22}(s) \cdot R_2(s)$$



r_1 and r_2 are inputs and x_1 and x_2 are outputs

$$a_{11} \cdot x_1 + a_{12} \cdot x_2 + r_1 = x_1$$

$$a_{21} \cdot x_1 + a_{22} \cdot x_2 + r_2 = x_2$$



Signal-Flow Graph Models

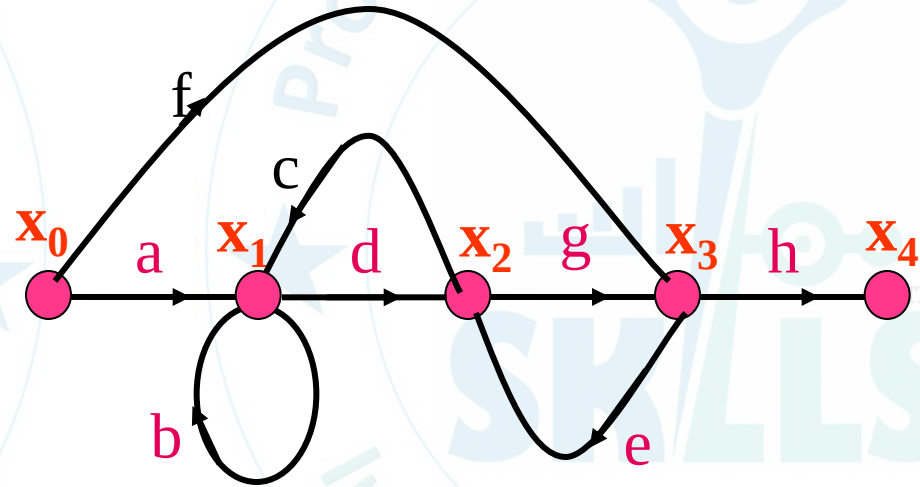
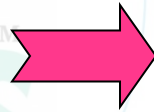
x_0 is input and x_4 is output

$$x_1 = ax_0 + bx_1 + cx_2$$

$$x_2 = dx_1 + ex_3$$

$$x_3 = fx_0 + gx_2$$

$$x_4 = hx_3$$



Signal-Flow Graph Models

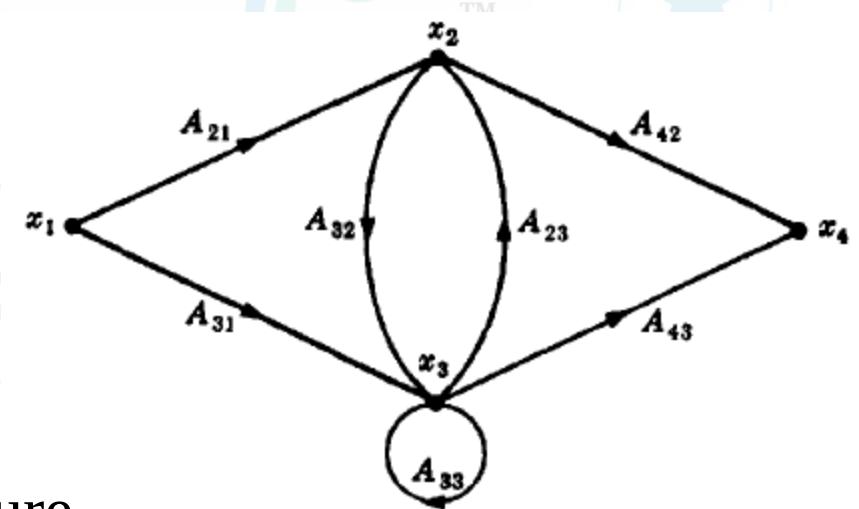
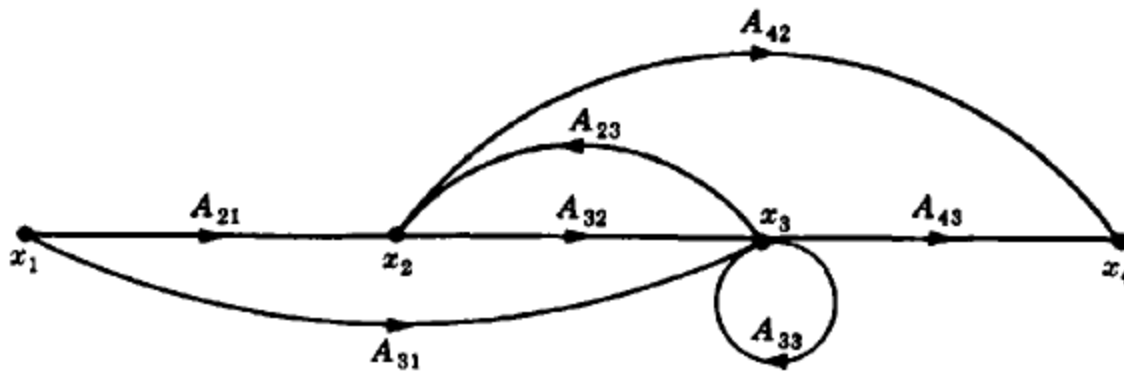
Construct the signal flow graph for the following set of simultaneous equations.

$$x_2 = A_{21}x_1 + A_{23}x_3$$

$$x_3 = A_{31}x_1 + A_{32}x_2 + A_{33}x_3$$

$$x_4 = A_{42}x_2 + A_{43}x_3$$

- There are four variables in the equations (i.e., x_1, x_2, x_3 , and x_4) therefore four nodes are required to construct the signal flow graph.
- Arrange these four nodes from left to right and connect them with the associated branches.

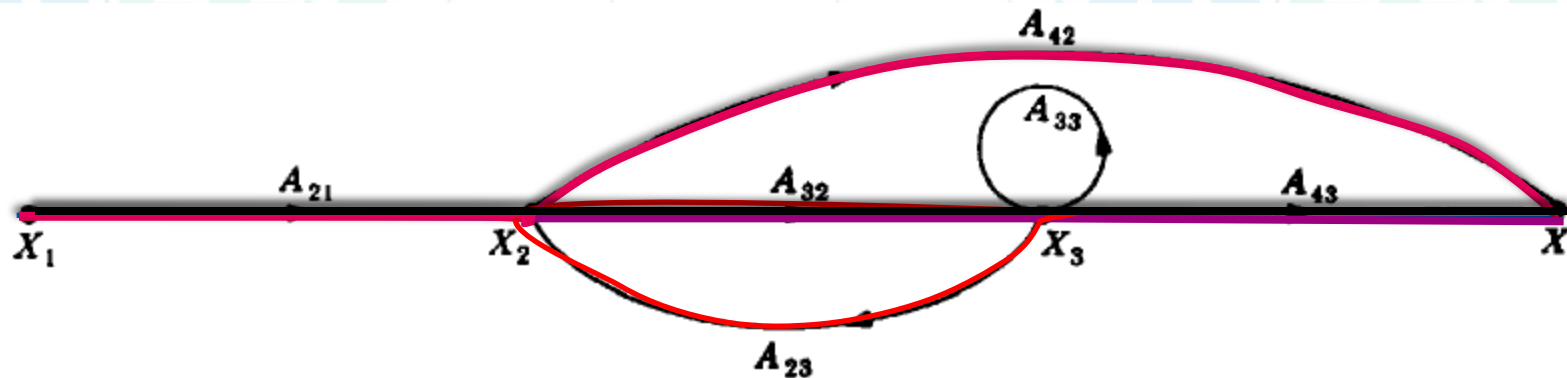


- Another way to arrange this graph is shown in the figure.

Signal-Flow Graph Models

Terminologies:

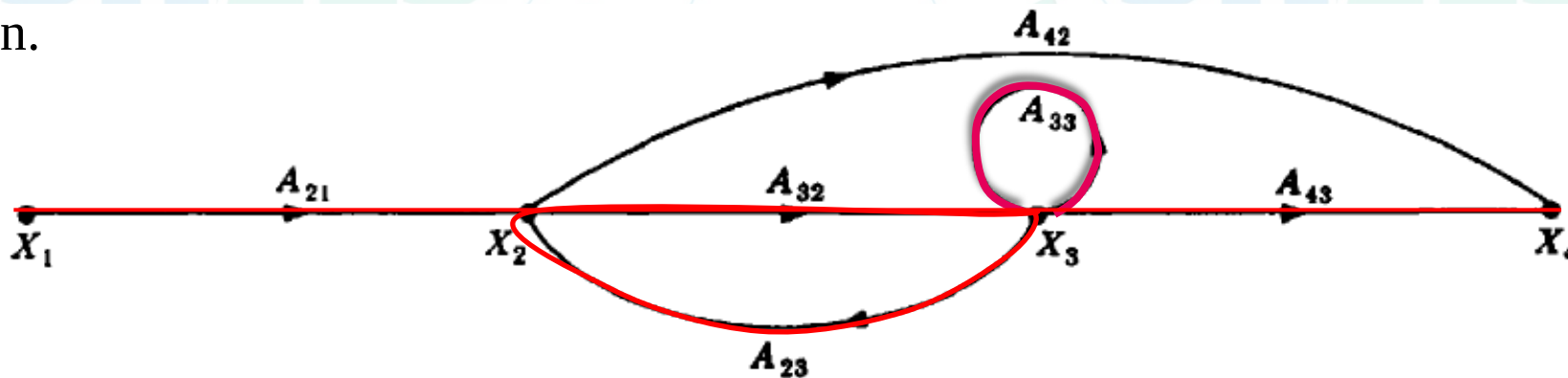
- An **input node** or source contain only the outgoing branches. i.e., X_1
- An **output node** or sink contain only the incoming branches. i.e., X_4
- A **path** is a continuous, unidirectional succession of branches along which no node is passed more than ones. i.e., $X_1 \text{ to } X_2 \text{ to } X_3 \text{ to } X_4$ $X_1 \text{ to } X_2 \text{ to } X_4$ $X_2 \text{ to } X_3 \text{ to } X_4$
- A **forward path** is a path from the input node to the output node. i.e., $X_1 \text{ to } X_2 \text{ to } X_3 \text{ to } X_4$, and $X_1 \text{ to } X_2 \text{ to } X_4$, are forward paths.
- A **feedback path** or feedback loop is a path which originates and terminates on the same node. i.e.; $X_2 \text{ to } X_3 \text{ and back to } X_2$ is a feedback path.



Signal-Flow Graph Models

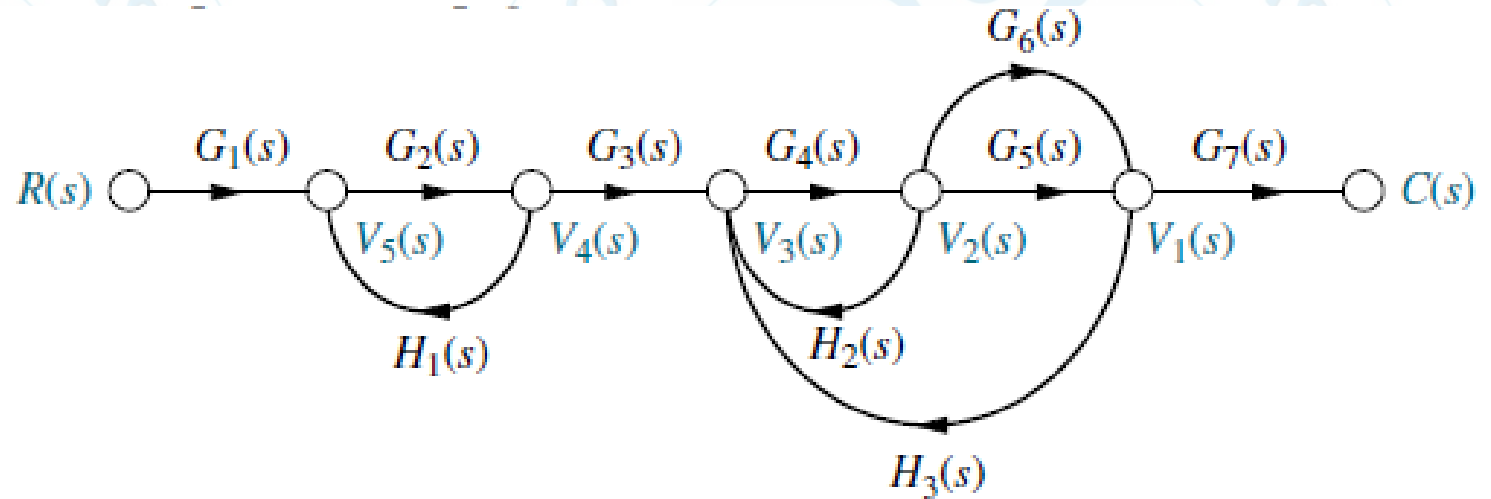
Terminologies:

- A **self-loop** is a feedback loop consisting of a single branch. i.e.; A_{33} is a self loop.
- The **gain** of a branch is the transmission function of that branch.
- The **path gain** is the product of branch gains encountered in traversing a path. i.e. the gain of forwards path X_1 to X_2 to X_3 to X_4 is $A_{21}A_{32}A_{43}$
- The **loop gain** is the product of the branch gains of the loop. i.e., the loop gain of the feedback loop from X_2 to X_3 and back to X_2 is $A_{32}A_{23}$.
- Two loops, paths, or loop and a path are said to be **non-touching** if they have no nodes in common.



Signal-Flow Graph Models

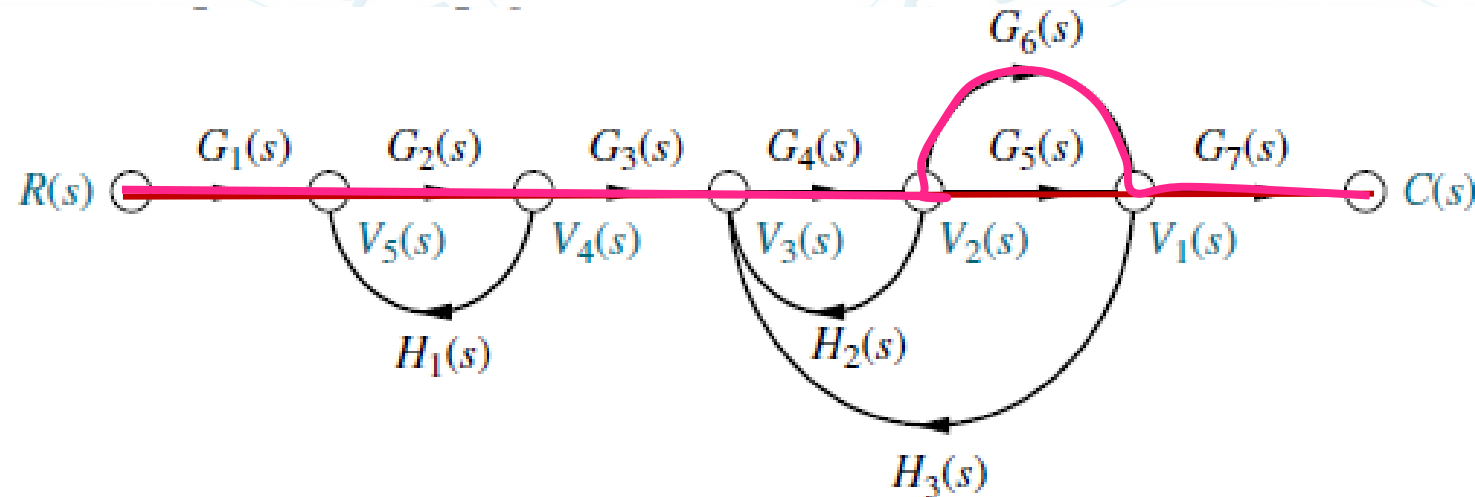
Consider the signal flow graph below and identify the following



- Input node.
- Output node.
- Forward paths.
- Feedback paths (loops).
- Determine the loop gains of the feedback loops.
- Determine the path gains of the forward paths.
- Non-touching loops

Signal-Flow Graph Models

Consider the signal flow graph below and identify the following



- There are two forward path gains;

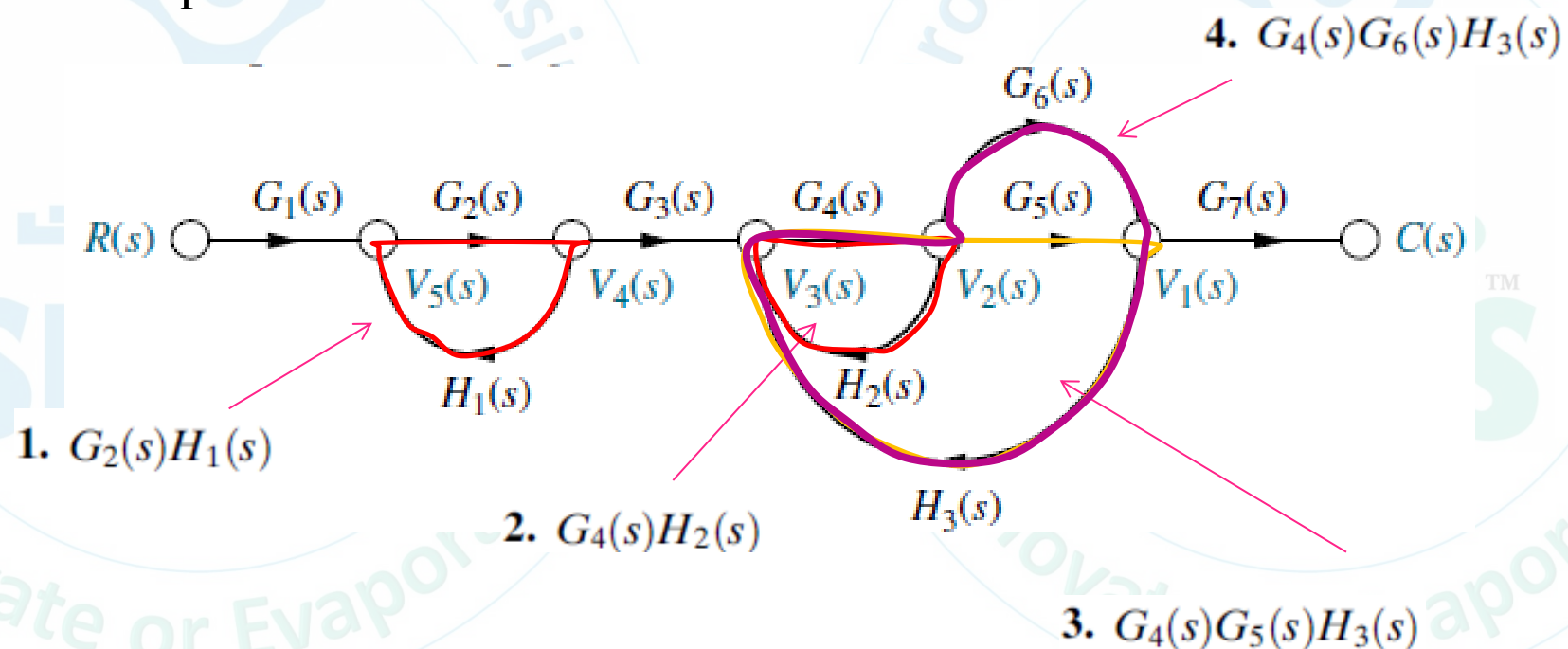
1. $G_1(s)G_2(s)G_3(s)G_4(s)G_5(s)G_7(s)$

2. $G_1(s)G_2(s)G_3(s)G_4(s)G_6(s)G_7(s)$

Signal-Flow Graph Models

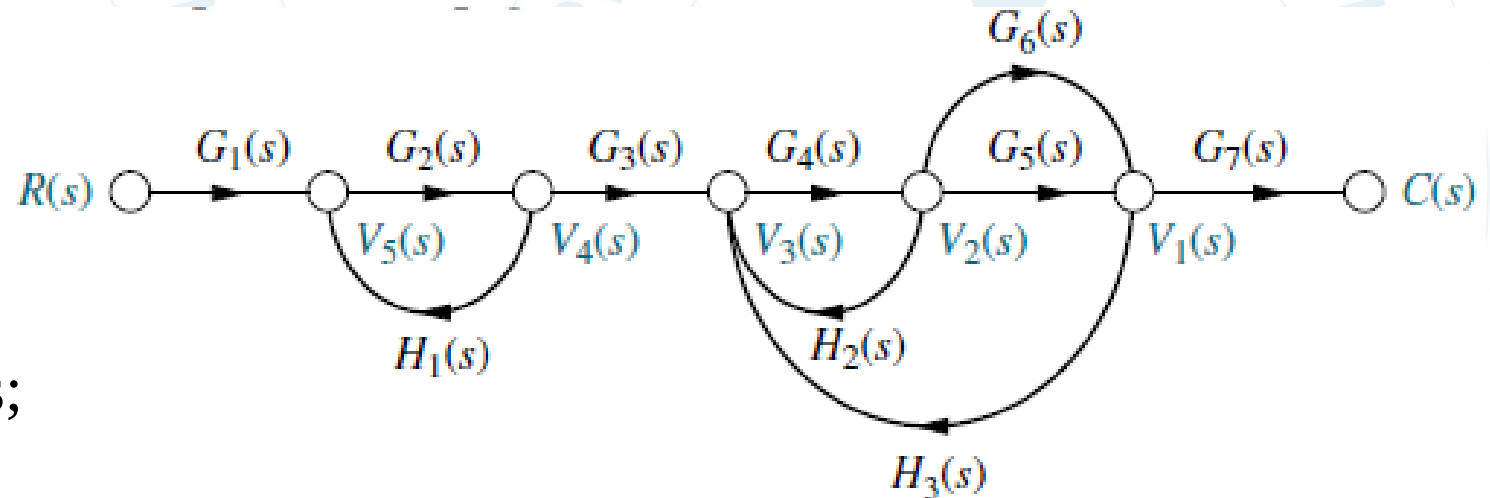
Consider the signal flow graph below and identify the following

- There are four loops



Signal-Flow Graph Models

Consider the signal flow graph below and identify the following

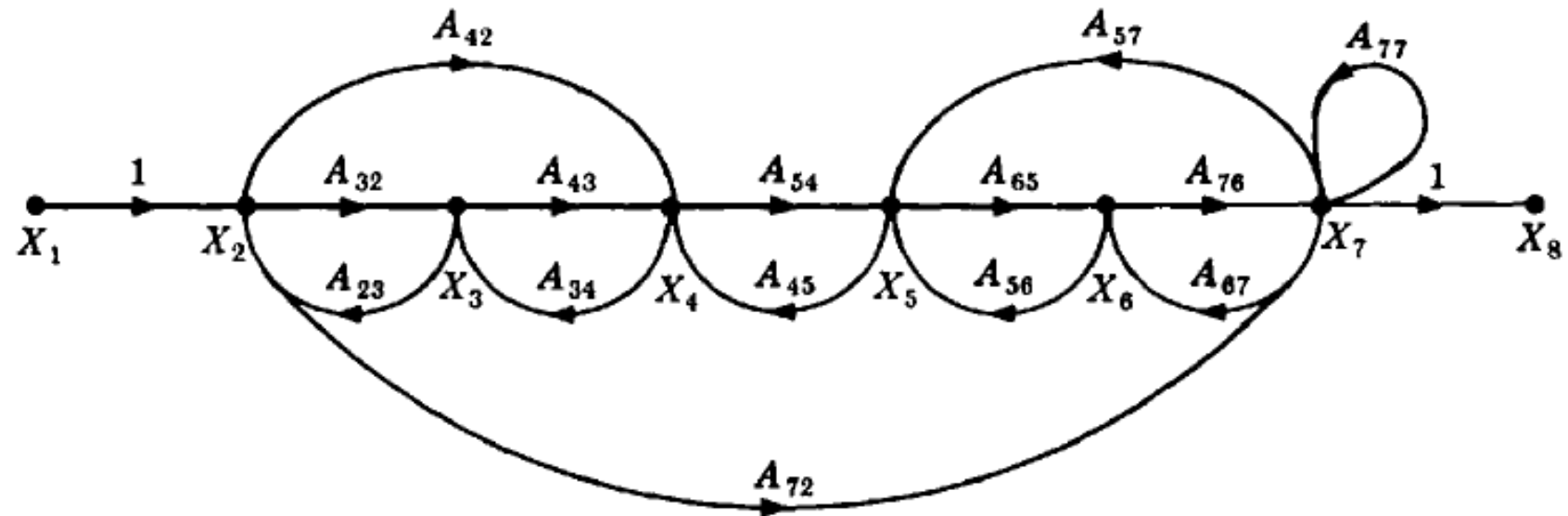


- Nontouching loop gains;

1. $[G_2(s)H_1(s)][G_4(s)H_2(s)]$
2. $[G_2(s)H_1(s)][G_4(s)G_5(s)H_3(s)]$
3. $[G_2(s)H_1(s)][G_4(s)G_6(s)H_3(s)]$

Signal-Flow Graph Models

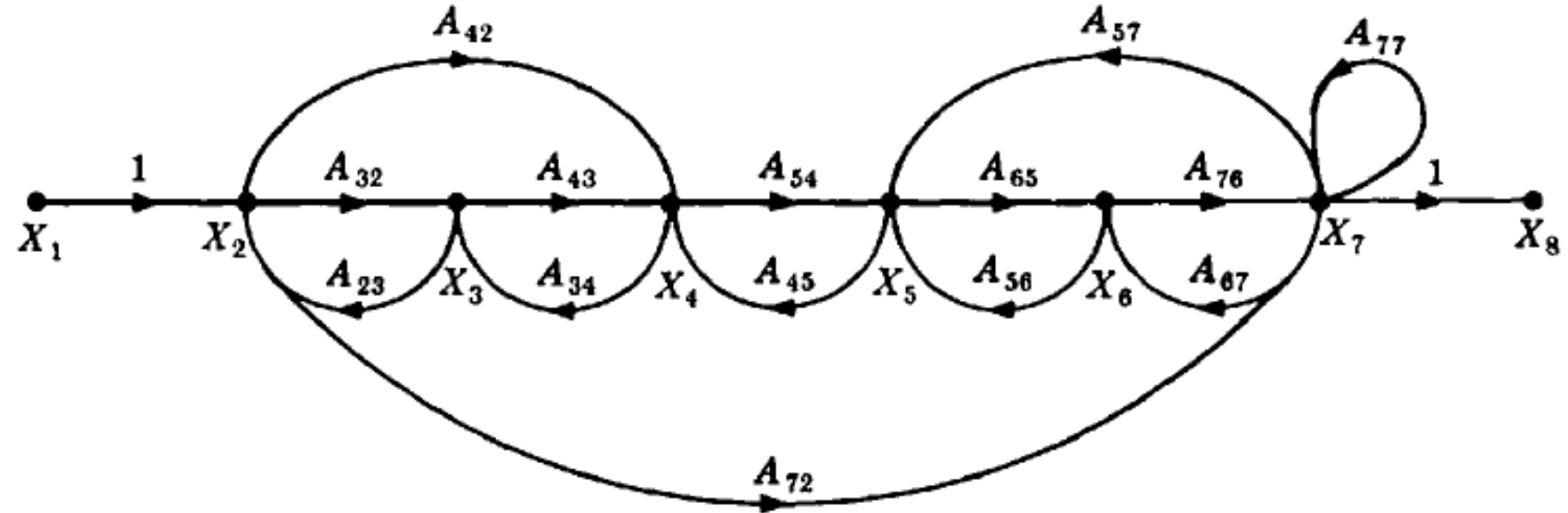
Consider the signal flow graph below and identify the following



- Input node.
- Output node.
- Forward paths.
- Feedback paths.
- Self loop.
- Determine the loop gains of the feedback loops.
- Determine the path gains of the forward paths.

Signal-Flow Graph Models

Input and output Nodes

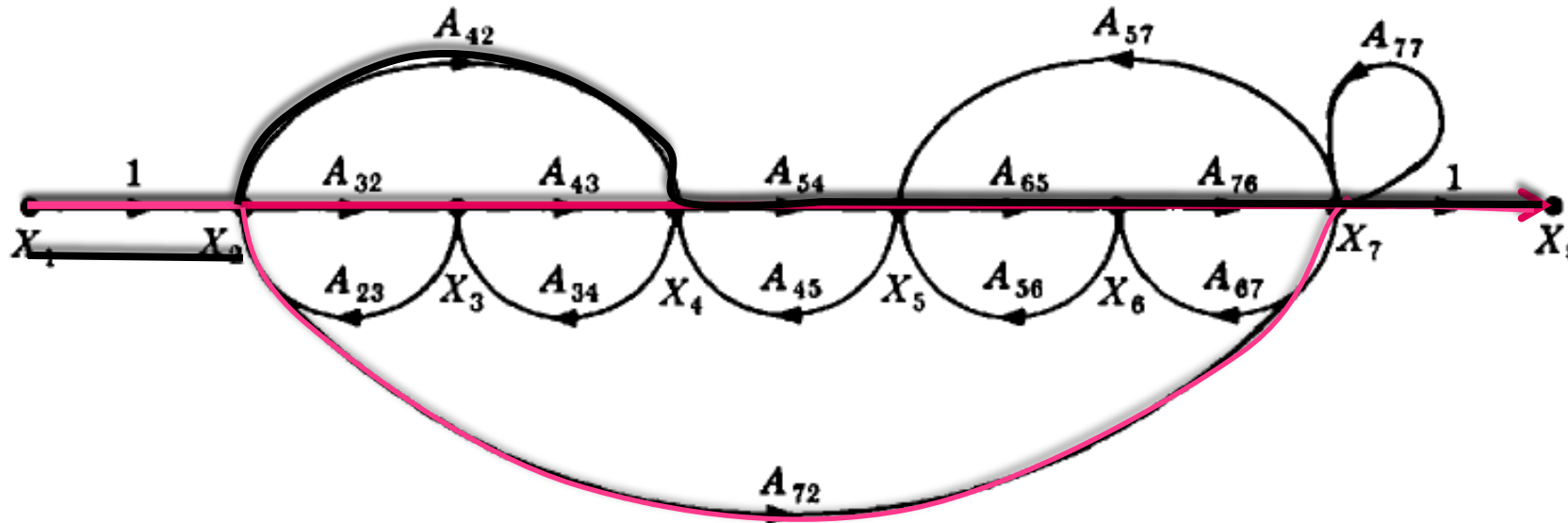


a) Input node X_1

b) Output node X_8

Signal-Flow Graph Models

(c) Forward Paths



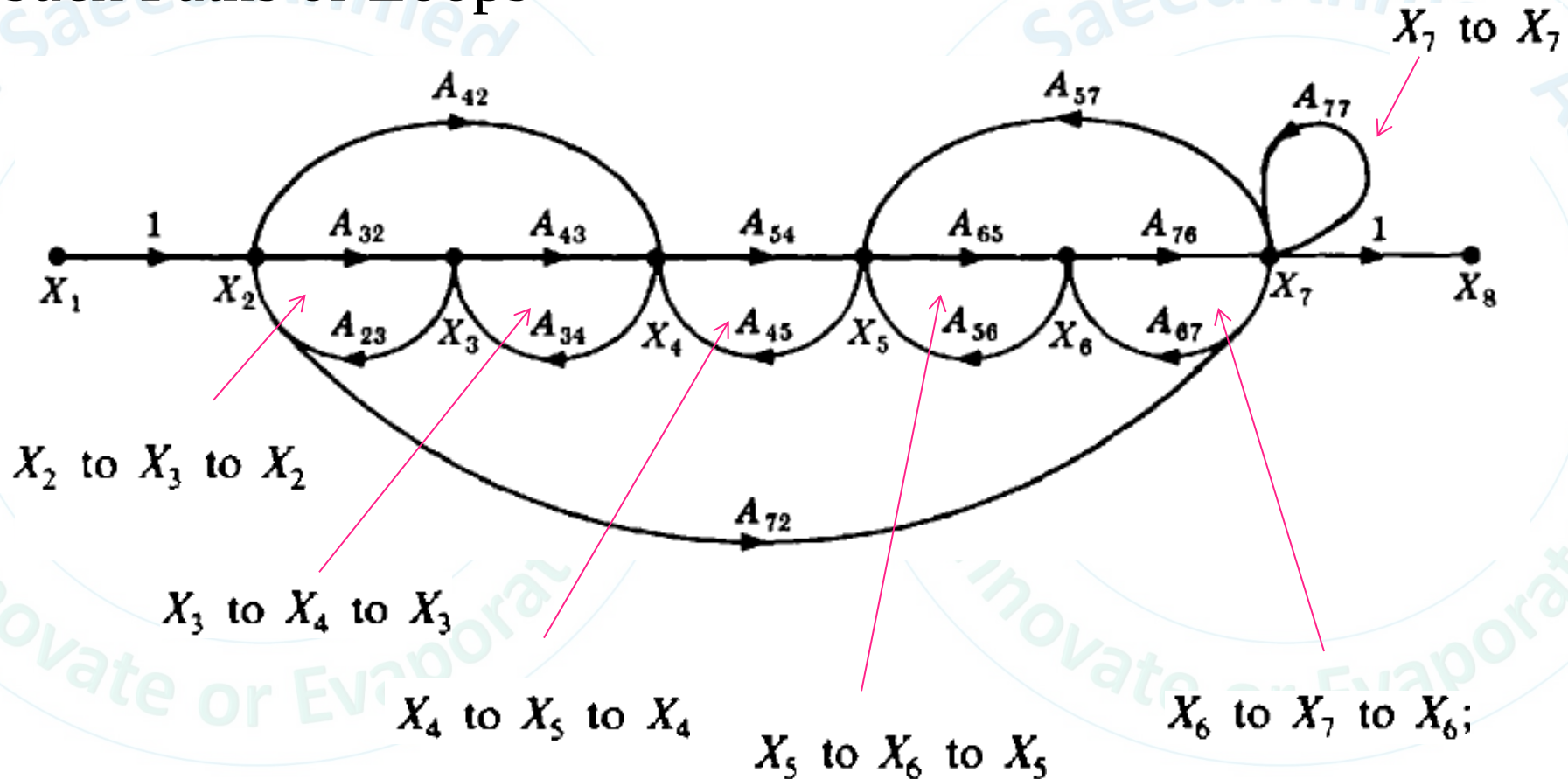
X_1 to X_2 to X_3 to X_4 to X_5 to X_6 to X_7 to X_8

X_1 to X_2 to X_7 to X_8

X_1 to X_2 to X_4 to X_5 to X_6 to X_7 to X_8

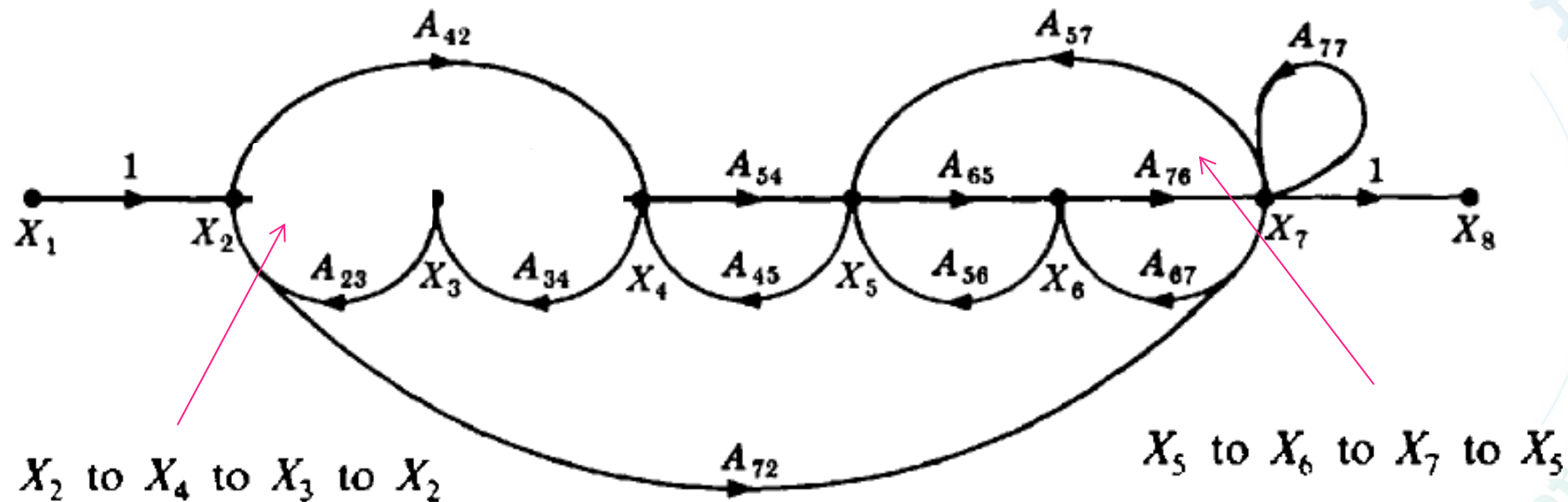
Signal-Flow Graph Models

(d) Feedback Paths or Loops



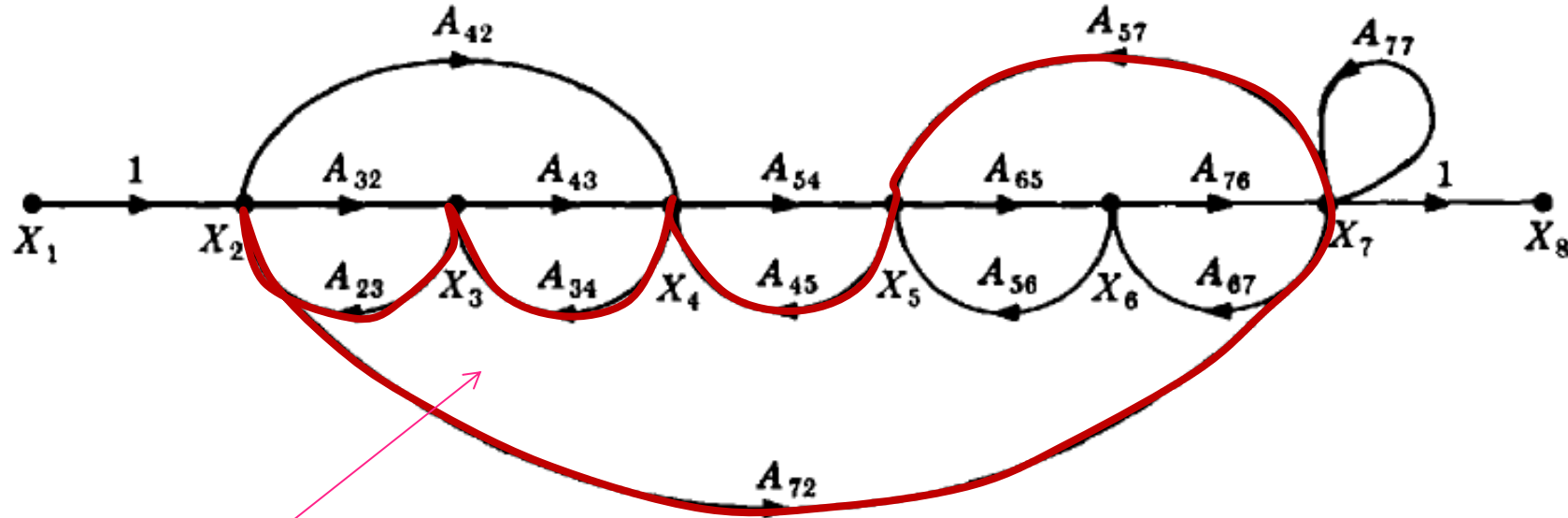
Signal-Flow Graph Models

(d) Feedback Paths or Loops



Signal-Flow Graph Models

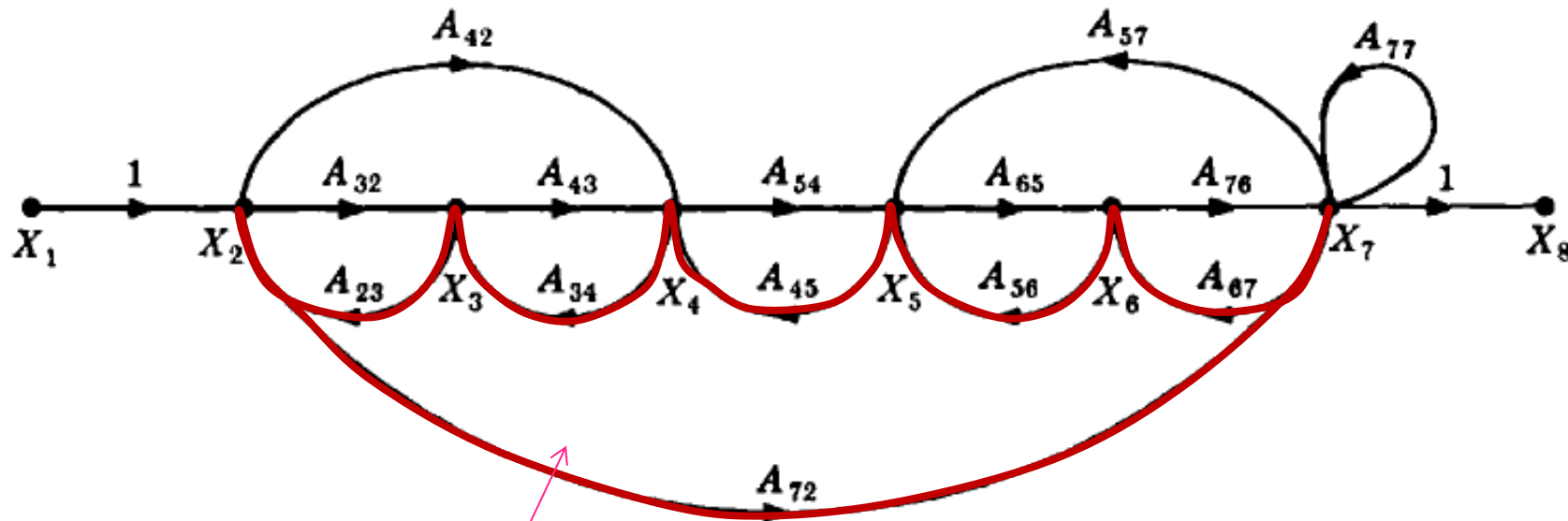
(d) Feedback Paths or Loops



X_2 to X_7 to X_5 to X_4 to X_3 to X_2

Signal-Flow Graph Models

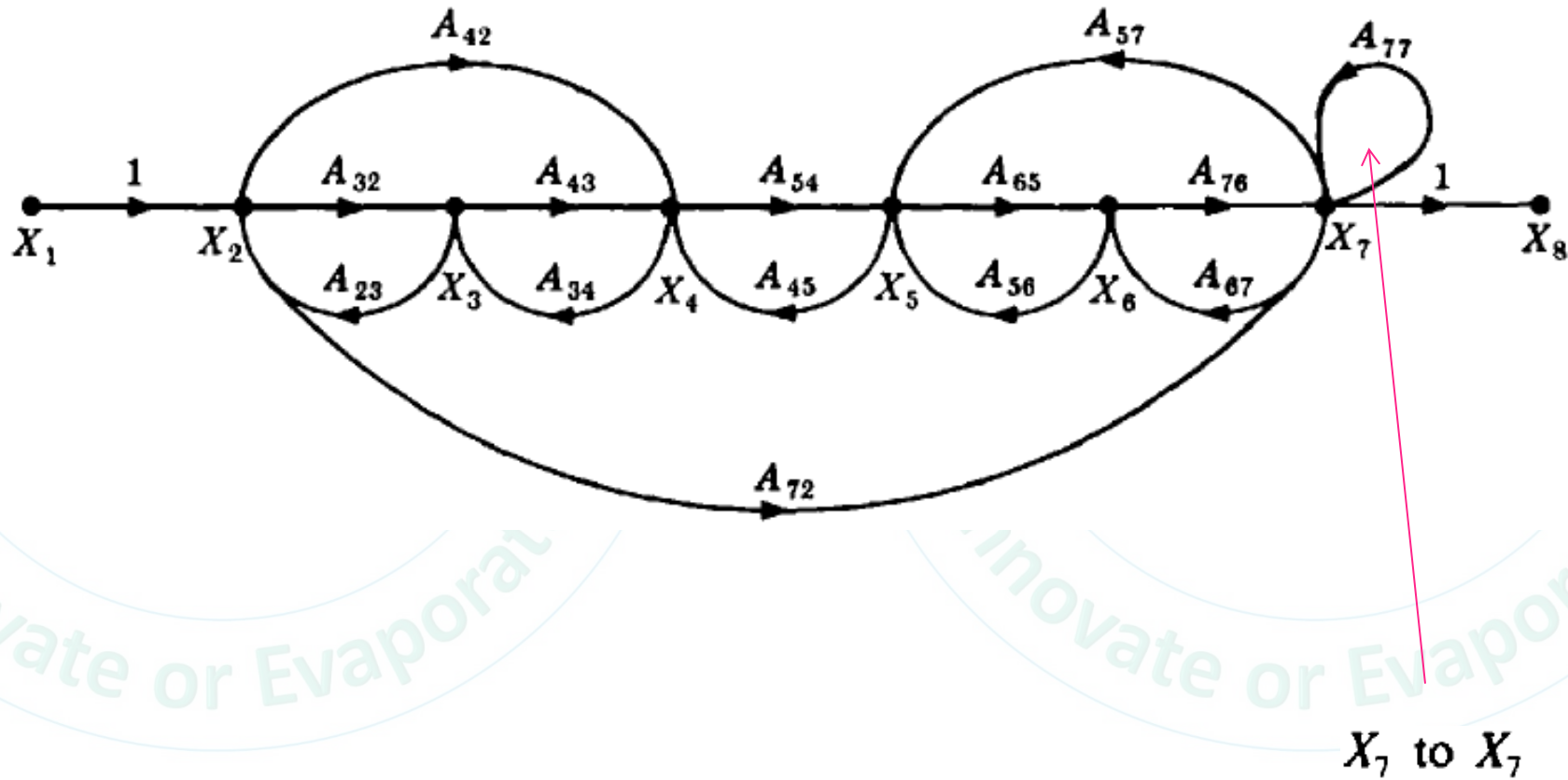
(d) Feedback Paths or Loops



X_2 to X_7 to X_6 to X_5 to X_4 to X_3 to X_2

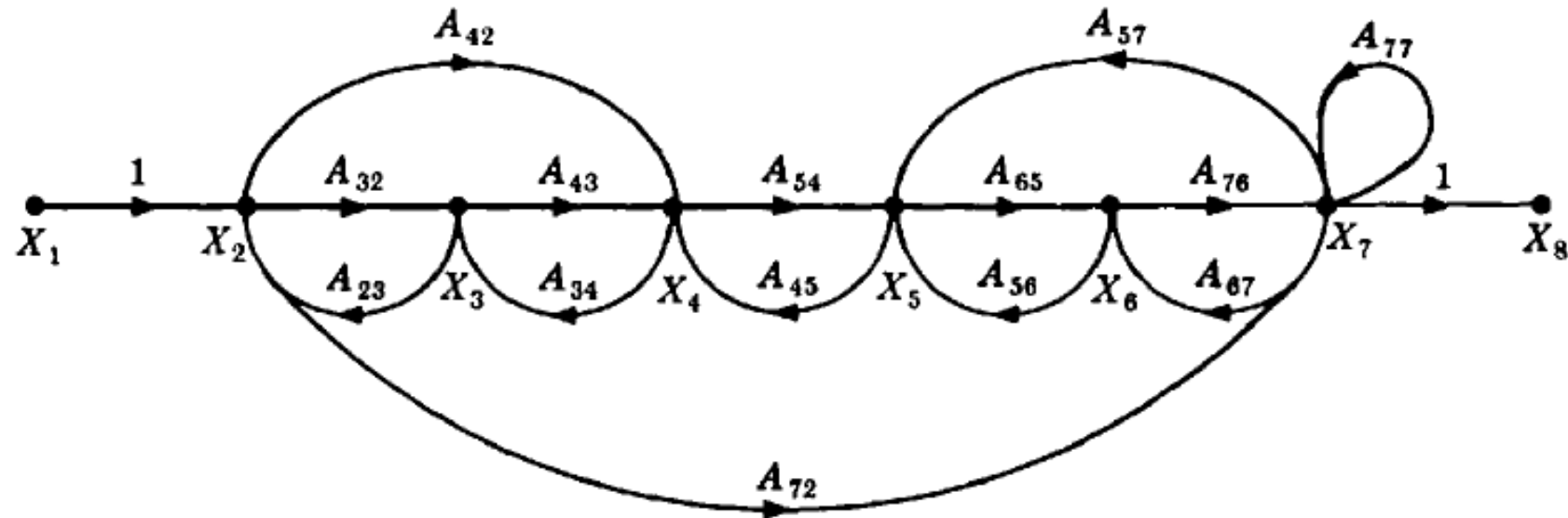
Signal-Flow Graph Models

(e) Self Loop(s)



Signal-Flow Graph Models

(f) Loop Gains of the Feedback Loops



$$A_{32} A_{23}$$

$$A_{76} A_{67}$$

$$A_{72} A_{57} A_{45} A_{34} A_{23}$$

$$A_{43} A_{34}$$

$$A_{65} A_{76} A_{57}$$

$$A_{72} A_{67} A_{56} A_{45} A_{34} A_{23}$$

$$A_{54} A_{45}$$

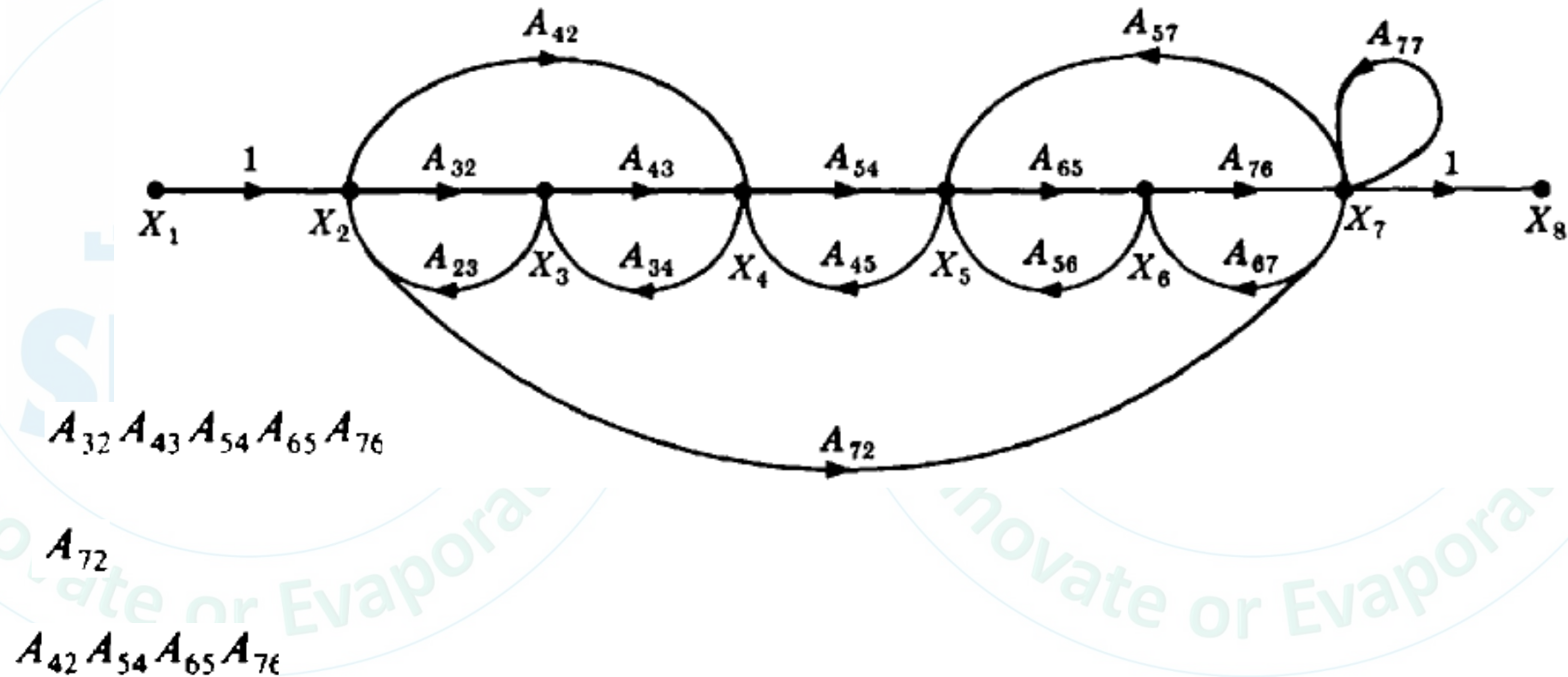
$$A_{77}$$

$$A_{65} A_{56}$$

$$A_{42} A_{34} A_{23}$$

Signal-Flow Graph Models

(g) Path Gains of the Forward Paths



Mason's Rule (Mason, 1953)

- The block diagram reduction technique requires successive application of fundamental relationships in order to arrive at the system transfer function.
- On the other hand, Mason's rule for reducing a signal-flow graph to a single transfer function requires the application of one formula.
- The formula was derived by S. J. Mason when he related the signal-flow graph to the simultaneous equations that can be written from the graph.

Mason's Rule

- The transfer function, $C(s)/R(s)$, of a system represented by a signal-flow graph is;

$$\frac{C(s)}{R(s)} = \frac{\sum_{i=1}^n P_i \Delta_i}{\Delta}$$

Where

n = number of forward paths.

P_i = the i^{th} forward-path gain.

Δ = Determinant of the system

Δ_i = Determinant of the i^{th} forward path

- Δ is called the signal flow graph determinant or characteristic function. Since $\Delta=0$ is the system characteristic equation.

Mason's Rule

$$\frac{C(s)}{R(s)} = \frac{\sum_{i=1}^n P_i \Delta_i}{\Delta}$$

$\Delta = 1$ - (sum of all individual loop gains) + (sum of the products of the gains of all possible two loops that do not touch each other) – (sum of the products of the gains of all possible three loops that do not touch each other) + ... and so forth with sums of higher number of non-touching loop gains

Δ_i = value of Δ for the part of the block diagram that does not touch the i -th forward path ($\Delta_i = 1$ if there are no non-touching loops to the i -th path.)

Signal-Flow Graph Models

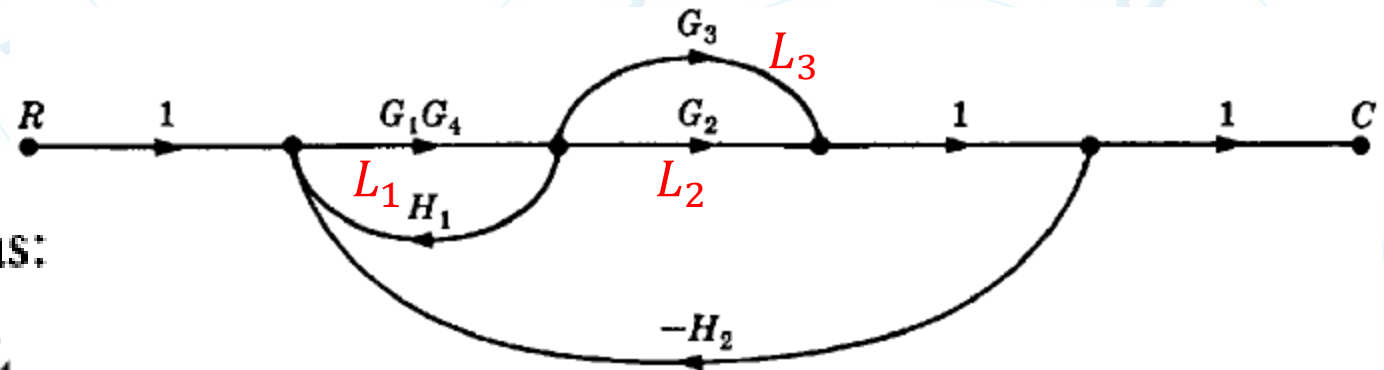
Systematic approach

1. Calculate forward path gain P_i for each forward path i .
2. Calculate all loop transfer functions
3. Consider non-touching loops 2 at a time
4. Consider non-touching loops 3 at a time
5. etc
6. Calculate Δ from steps 2,3,4 and 5
7. Calculate Δ_i as portion of Δ not touching forward path i

$$\frac{C(s)}{R(s)} = \frac{\sum_{i=1}^n P_i \Delta_i}{\Delta}$$

Signal-Flow Graph Models

Example.1



There are two forward paths:

$$P_1 = G_1 G_2 G_4 \quad P_2 = G_1 G_3 G_4$$

Therefore,

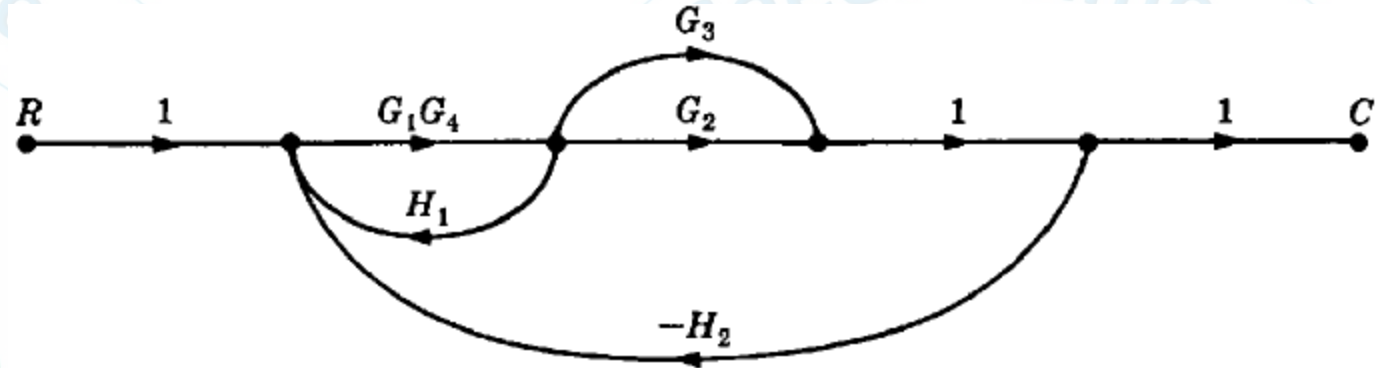
$$\frac{C}{R} = \frac{P_1 \Delta_1 + P_2 \Delta_2}{\Delta}$$

There are three feedback loops

$$L_1 = G_1 G_4 H_1, \quad L_2 = -G_1 G_2 G_4 H_2, \quad L_3 = -G_1 G_3 G_4 H_2$$

Signal-Flow Graph Models

Example.1



There are no non-touching loops, therefore

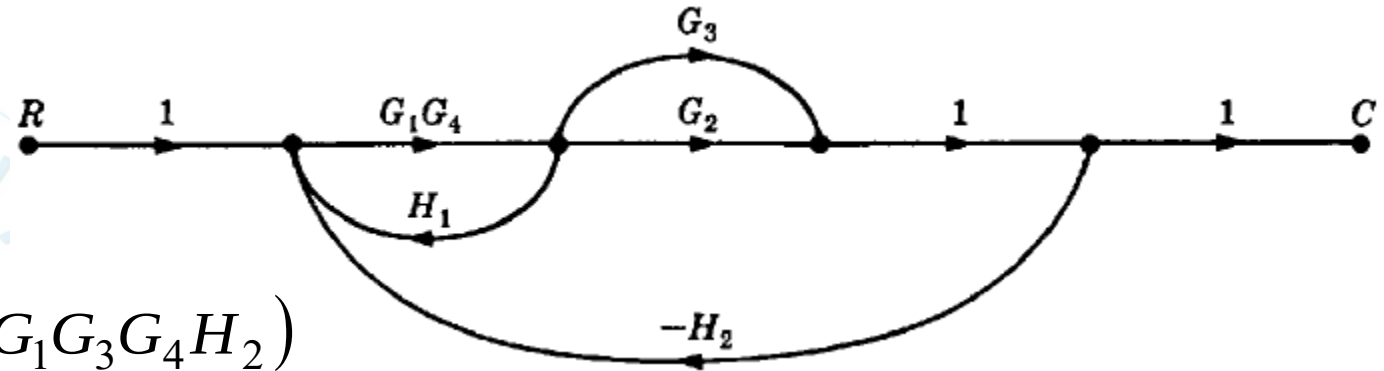
$$\Delta = 1 - (\text{sum of all individual loop gains})$$

$$\Delta = 1 - (L_1 + L_2 + L_3)$$

$$\Delta = 1 - (G_1 G_4 H_1 - G_1 G_2 G_4 H_2 - G_1 G_3 G_4 H_2)$$

Signal-Flow Graph Models

Example.1



$$\Delta = 1 - (G_1 G_4 H_1 - G_1 G_2 G_4 H_2 - G_1 G_3 G_4 H_2)$$

Eliminate forward path-1

There are two forward paths:

$$P_1 = G_1 G_2 G_4$$

$$P_2 = G_1 G_3 G_4$$

$$\Delta_1 = 1 - (\text{sum of all individual loop gains}) + \dots$$

$$\Delta_1 = 1$$

Eliminate forward path-2

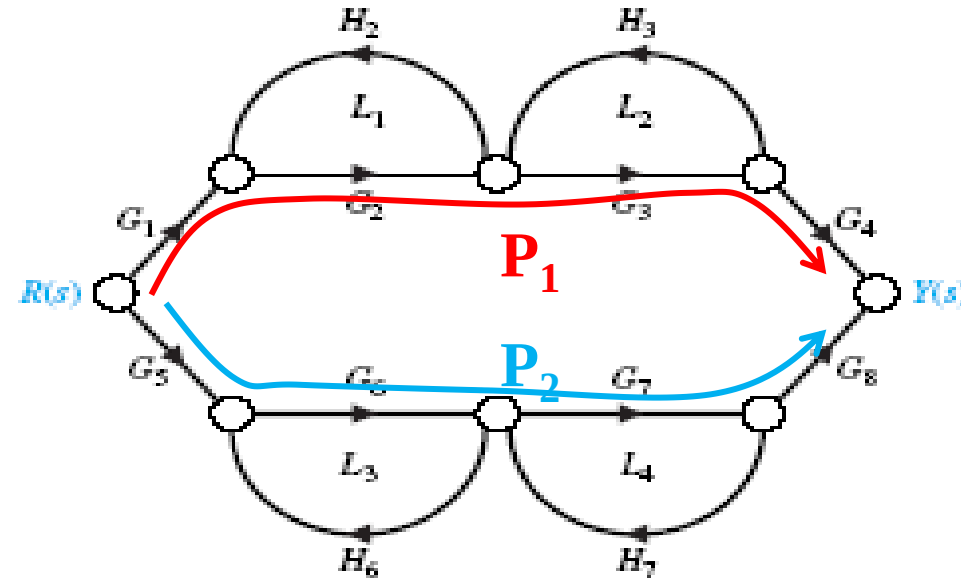
$$\Delta_2 = 1 - (\text{sum of all individual loop gains}) + \dots$$

$$\Delta_2 = 1$$

$$\frac{C}{R} = \frac{P_1 \Delta_1 + P_2 \Delta_2}{\Delta} = \frac{G_1 G_2 G_4 + G_1 G_3 G_4}{1 - G_1 G_4 H_1 + G_1 G_2 G_4 H_2 + G_1 G_3 G_4 H_2}$$

Signal-Flow Graph Models

Example.2



1. Calculate forward path gains for each forward path.
2. Calculate all loop gains.
3. Consider two non-touching loops.

$$L_1 L_3$$

$$L_1 L_4$$

$$L_2 L_4$$

$$L_2 L_3$$

Signal-Flow Graph Models

Example.2

4. Consider three non-touching loops.

None.

5. Calculate Δ from steps 2,3,4.

$$\Delta = 1 - (L_1 + L_2 + L_3 + L_4) + (L_1L_3 + L_1L_4 + L_2L_3 + L_2L_4)$$

$$\Delta = 1 - (G_2H_2 + H_3G_3 + G_6H_6 + G_7H_7) + \\ (G_2H_2G_6H_6 + G_2H_2G_7H_7 + H_3G_3G_6H_6 + H_3G_3G_7H_7)$$

Signal-Flow Graph Models

Example.2

Eliminate forward path-1

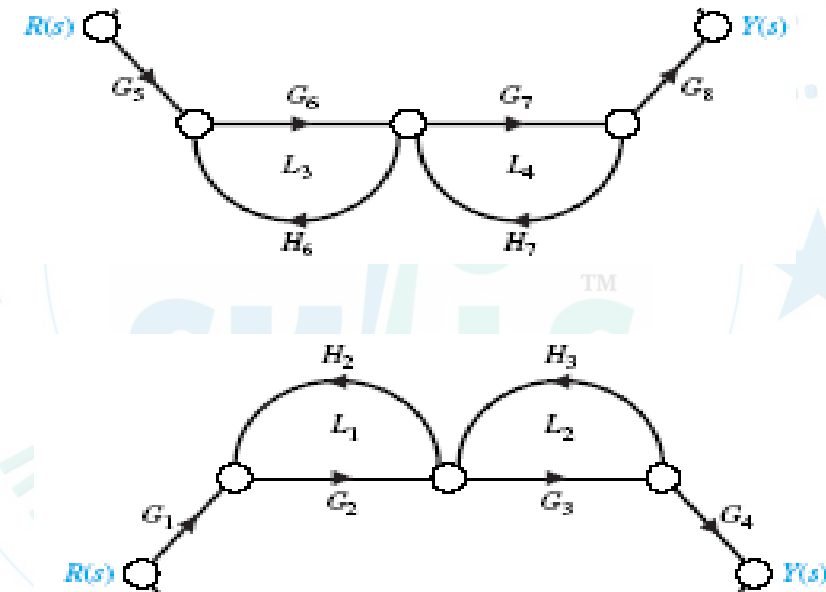
$$\Delta_1 = 1 - (L_3 + L_4)$$

$$\Delta_1 = 1 - (G_6 H_6 + G_7 H_7)$$

Eliminate forward path-2

$$\Delta_2 = 1 - (L_1 + L_2)$$

$$\Delta_2 = 1 - (G_2 H_2 + G_3 H_3)$$



Signal-Flow Graph Models

Example.2

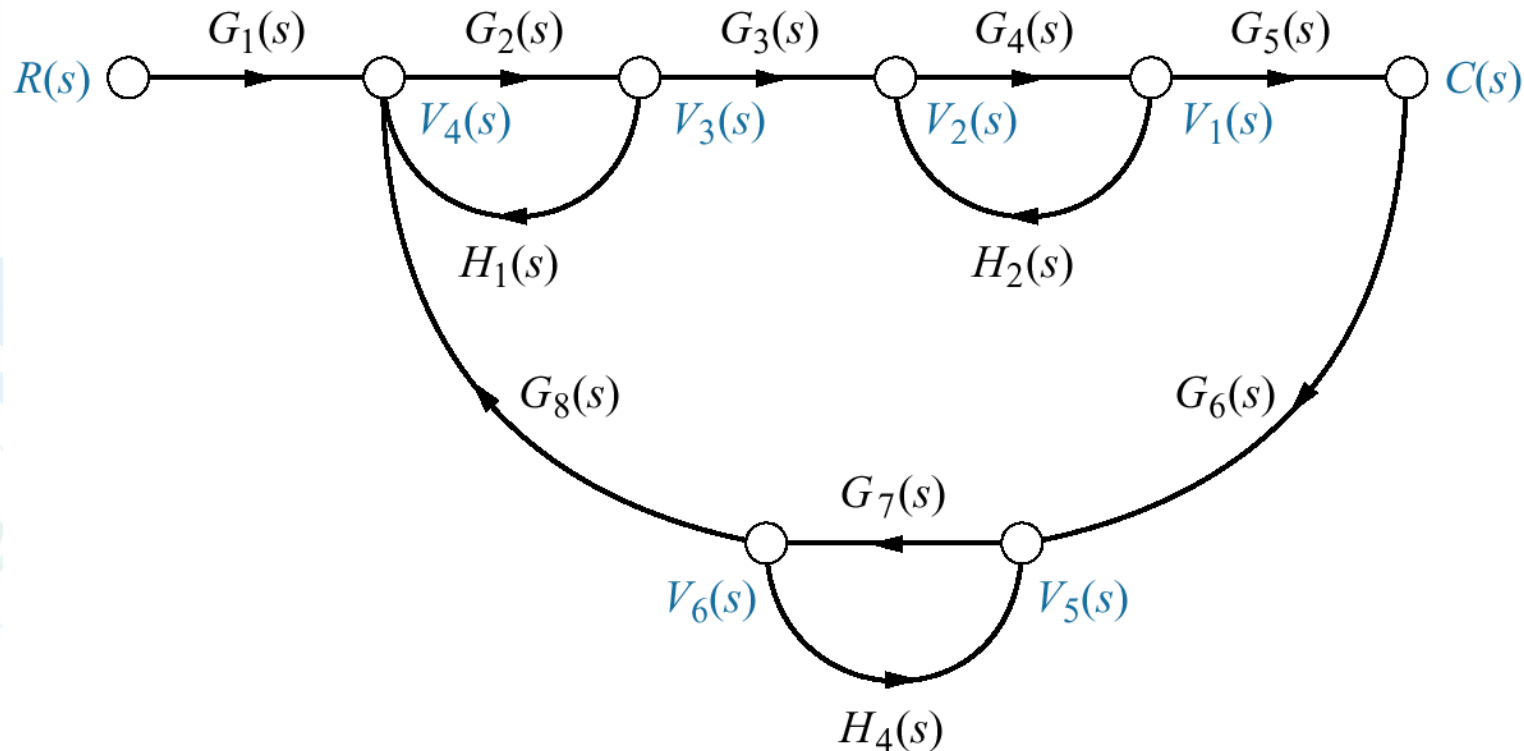
$$\frac{Y(s)}{R(s)} = \frac{P_1\Delta_1 + P_2\Delta_2}{\Delta}$$

$$\frac{Y(s)}{R(s)} = \frac{G_1G_2G_3G_4[1 - (G_6H_6 + G_7H_7)] + G_5G_6G_7G_8[1 - (G_2H_2 + G_3H_3)]}{1 - (G_2H_2 + H_3G_3 + G_6H_6 + G_7H_7) + (G_2H_2G_6H_6 + G_2H_2G_7H_7 + H_3G_3G_6H_6 + H_3G_3G_7H_7)}$$

Signal-Flow Graph Models

Example.3

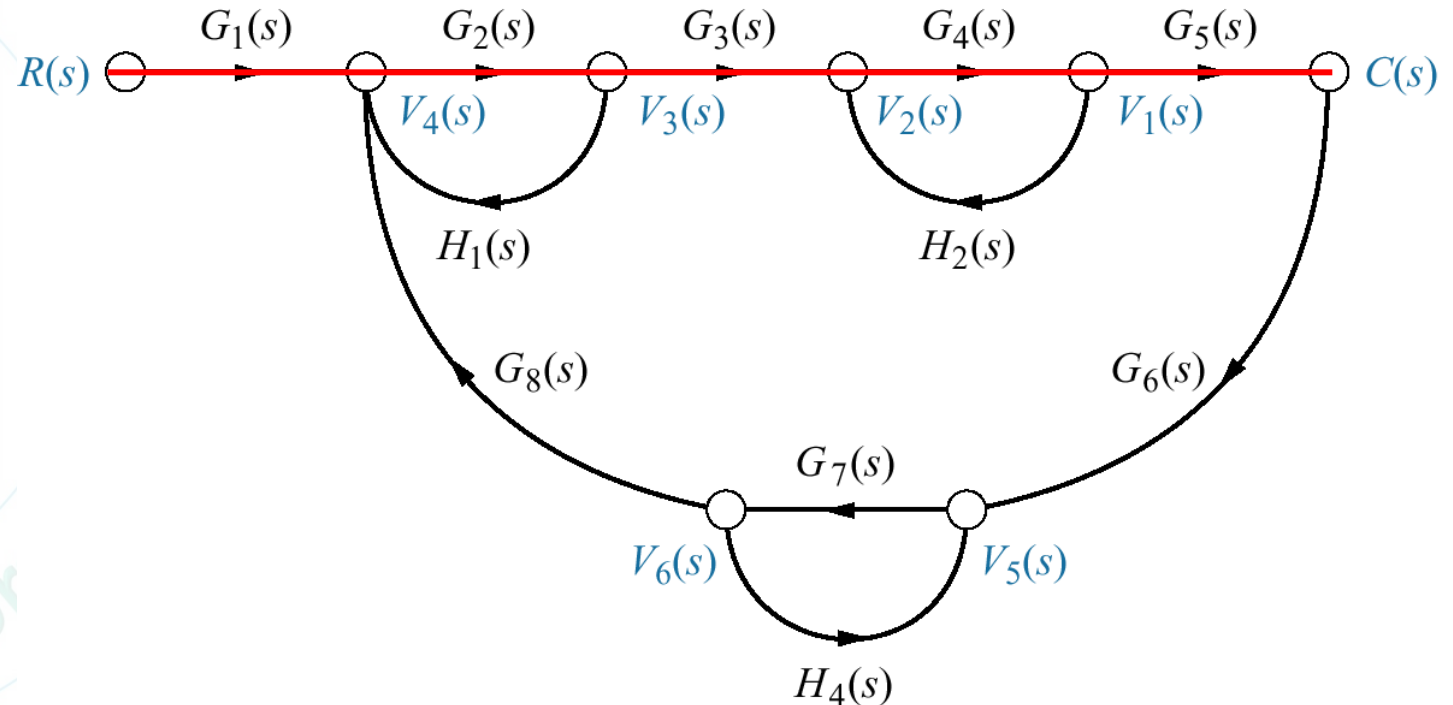
- Find the transfer function, $C(s)/R(s)$, for the signal-flow graph in figure below.



Signal-Flow Graph Models

Example.3

- There is only one forward Path.

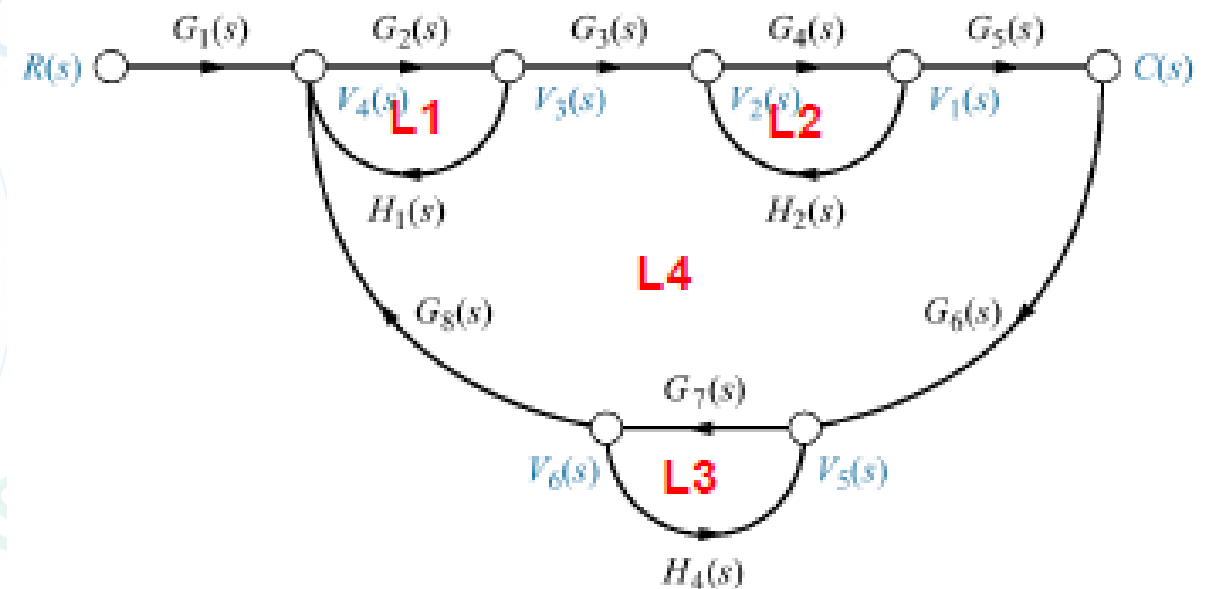


$$P_1 = G_1(s)G_2(s)G_3(s)G_4(s)G_5(s)$$

Signal-Flow Graph Models

Example.3

- There are four feedback loops.



$$L1. G_2(s)H_1(s)$$

$$L3. G_7(s)H_4(s)$$

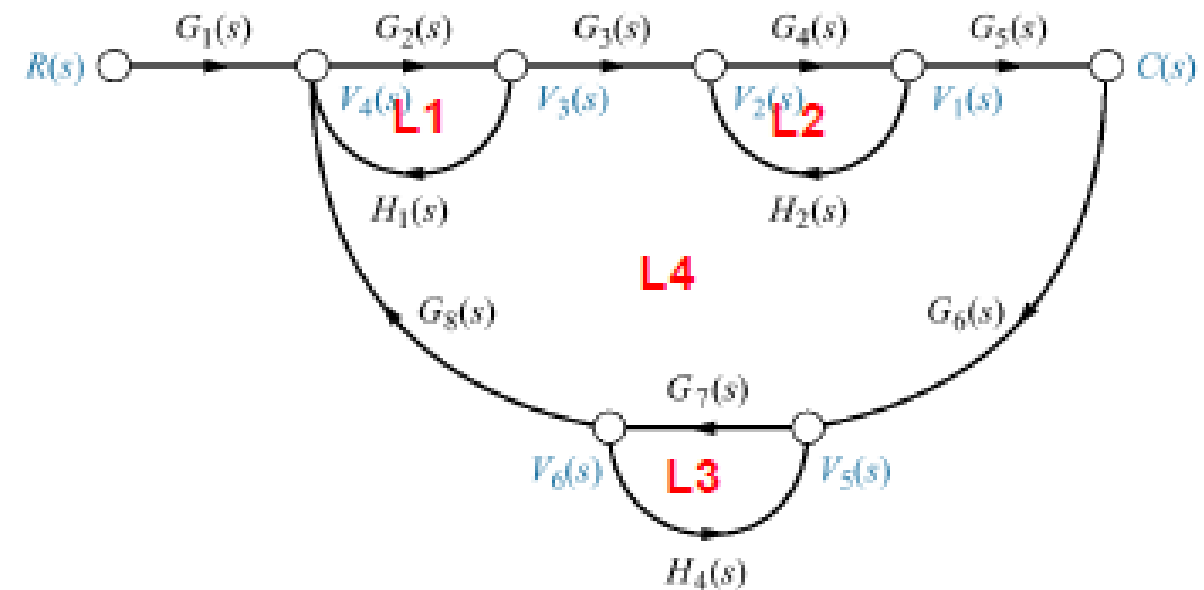
$$L2. G_4(s)H_2(s)$$

$$L4. G_2(s)G_3(s)G_4(s)G_5(s)G_6(s)G_7(s)G_8(s)$$

Signal-Flow Graph Models

Example.3

Non-touching loops taken two at a time.



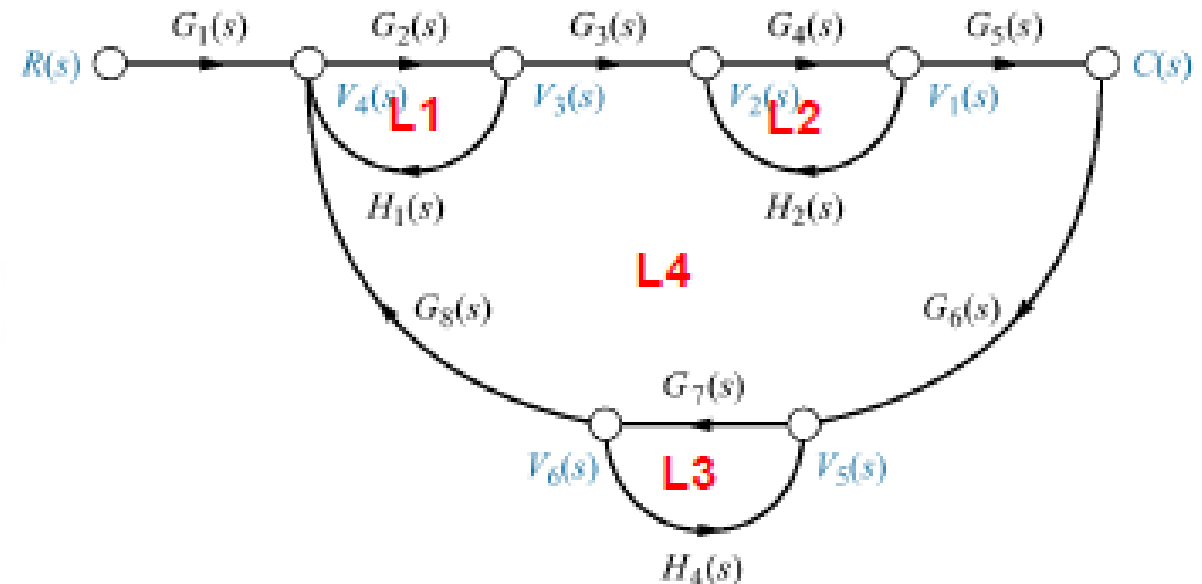
$$\text{L1 and L2: } G_2(s)H_1(s)G_4(s)H_2(s) \quad \text{L2 and L3: } G_4(s)H_2(s)G_7(s)H_4(s)$$

$$\text{L1 and L3: } G_2(s)H_1(s)G_7(s)H_4(s)$$

Signal-Flow Graph Models

Example.3

Non-touching loops taken three at a time.



L1, L2, L3: $G_2(s)H_1(s)G_4(s)H_2(s)G_7(s)H_4(s)$

Signal-Flow Graph Models

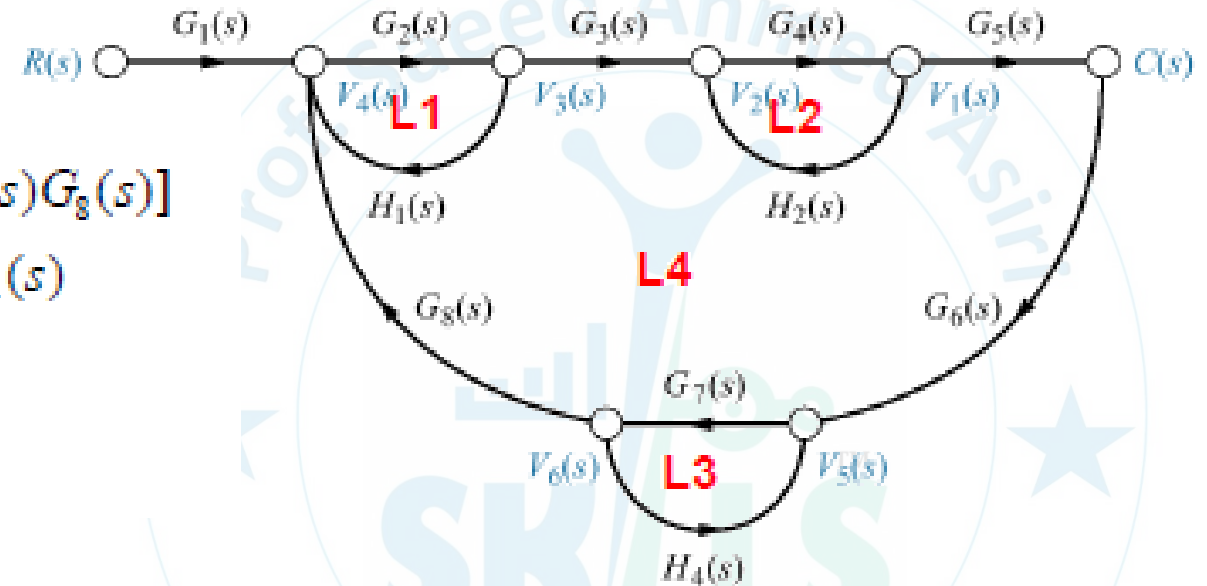
Example.3

$$\begin{aligned}\Delta = & 1 - [G_2(s)H_1(s) + G_4(s)H_2(s) \\ & + G_7(s)H_4(s) + G_2(s)G_3(s)G_4(s)G_5(s)G_6(s)G_7(s)G_8(s)] \\ & + [G_2(s)H_1(s)G_4(s)H_2(s) + G_2(s)H_1(s)G_7(s)H_4(s) \\ & + G_4(s)H_2(s)G_7(s)H_4(s)] \\ & - [G_2(s)H_1(s)G_4(s)H_2(s)G_7(s)H_4(s)]\end{aligned}$$

Eliminate forward path-1

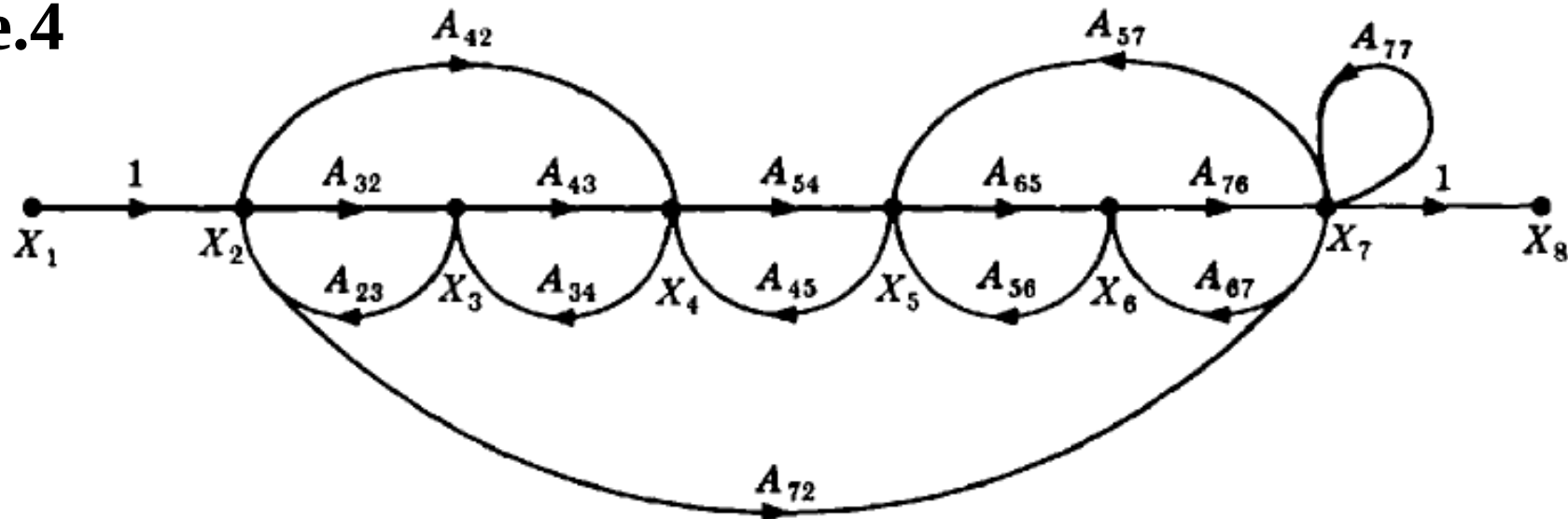
$$\Delta_1 = 1 - G_7(s)H_4(s)$$

$$\frac{C(s)}{R(s)} = \frac{P_1 \Delta_1}{\Delta} = \frac{G_1(s)G_2(s)G_3(s)G_4(s)G_5(s) - G_1(s)G_2(s)G_3(s)G_4(s)G_5(s)G_7(s)H_4(s)}{\Delta}$$



Signal-Flow Graph Models

Example.4



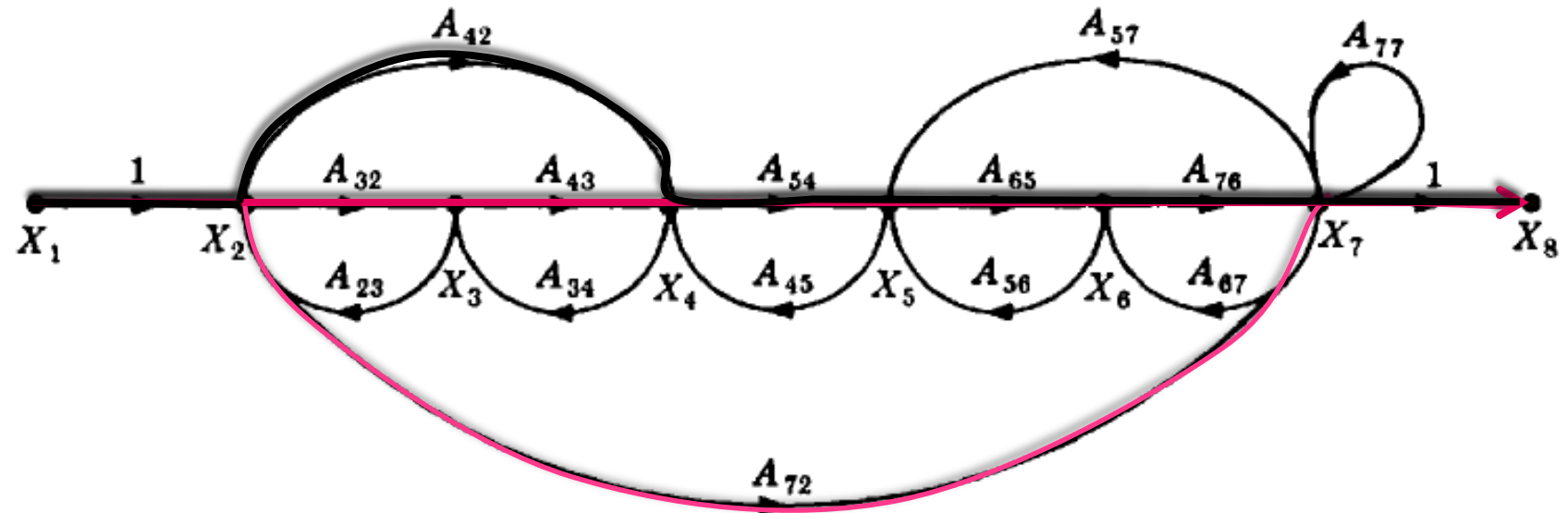
There are three forward paths, therefore $n=3$.

$$\frac{C(s)}{R(s)} = \frac{\sum_{i=1}^3 P_i \Delta_i}{\Delta} = \frac{P_1 \Delta_1 + P_2 \Delta_2 + P_3 \Delta_3}{\Delta}$$

Signal-Flow Graph Models

Example.4

$$P_3 = A_{42}A_{54}A_{65}A_{76}$$

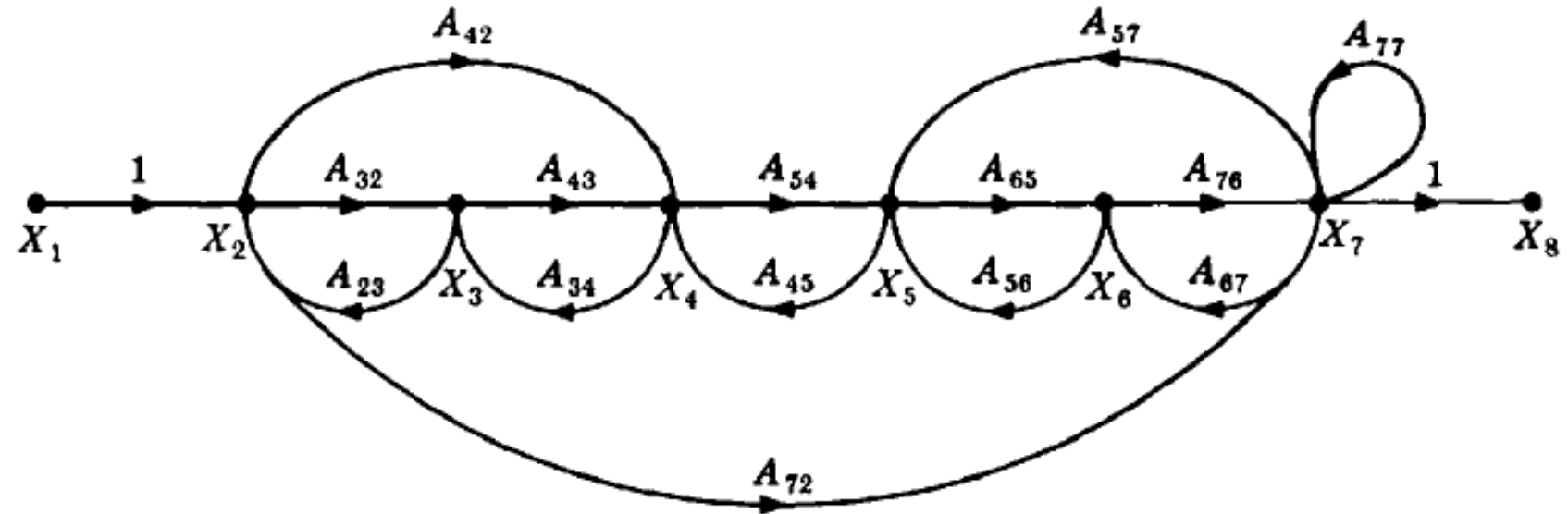


$$P_1 = A_{32}A_{43}A_{54}A_{65}A_{76}$$

$$P_2 = A_{72}$$

Signal-Flow Graph Models

Example.4



$$L_1 = A_{32}A_{23}$$

$$L_2 = A_{43}A_{34}$$

$$L_3 = A_{54}A_{45}$$

$$L_4 = A_{65}A_{56}$$

$$L_5 = A_{76}A_{67}$$

$$L_6 = A_{77}$$

$$L_7 = A_{42}A_{34}A_{23}$$

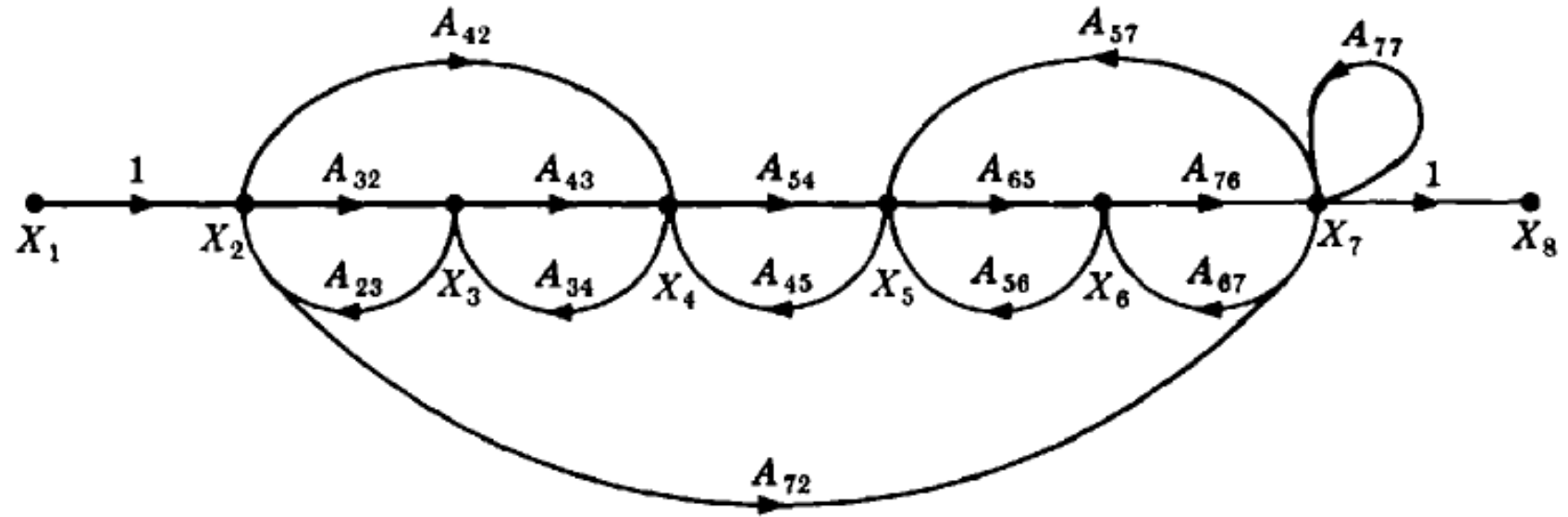
$$L_8 = A_{65}A_{76}A_{67}$$

$$L_9 = A_{72}A_{57}A_{45}A_{34}A_{23}$$

$$L_{10} = A_{72}A_{67}A_{56}A_{45}A_{34}A_{23}$$

Signal-Flow Graph Models

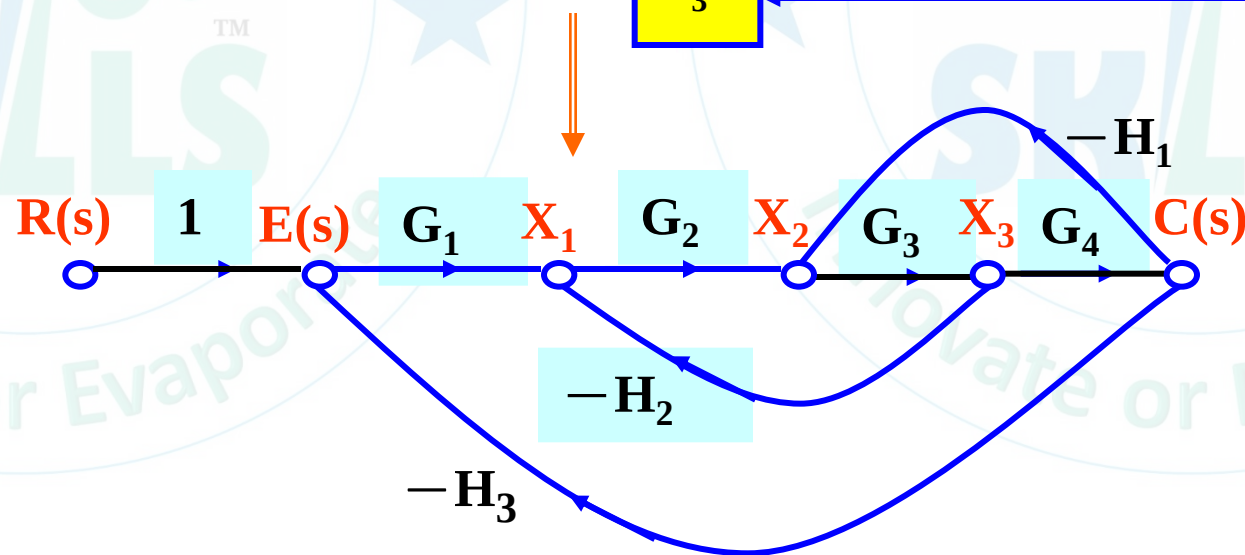
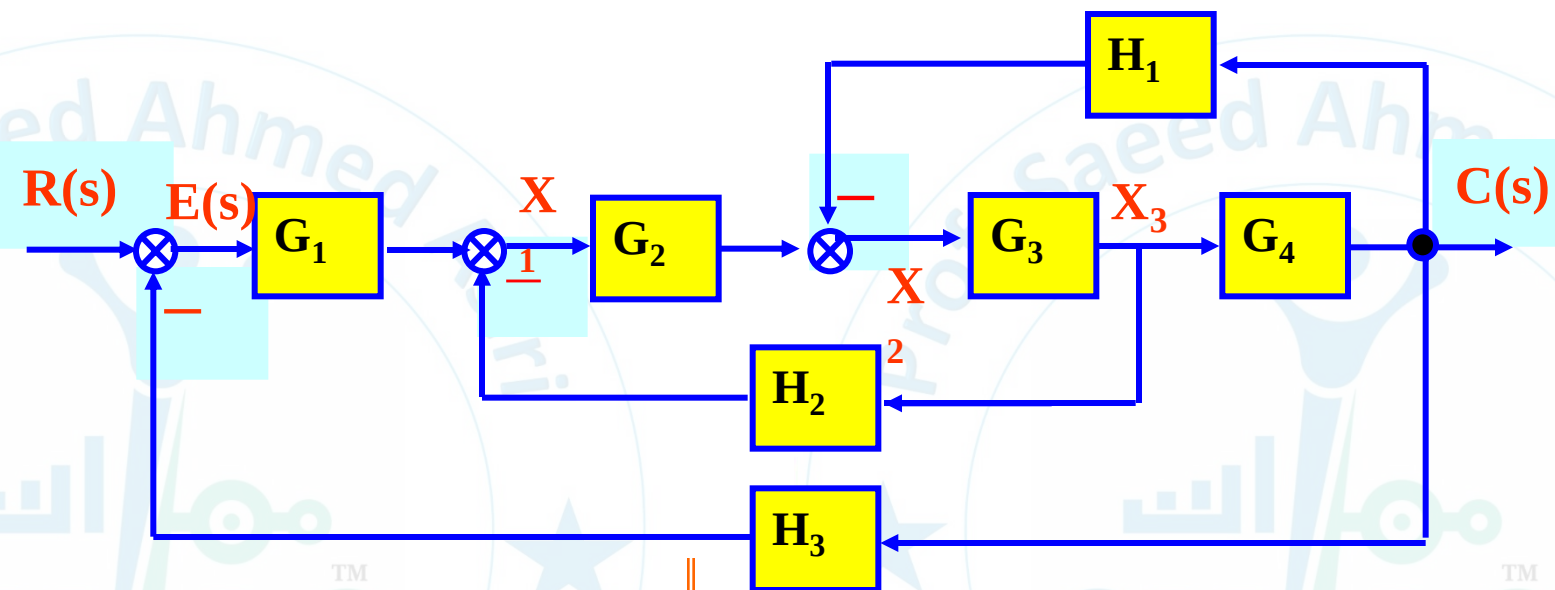
Example.4



$L_1 L_3$ $L_2 L_4$ $L_3 L_5$ $L_4 L_6$ $L_5 L_7$ $L_7 L_8$
 $L_1 L_4$ $L_2 L_5$ $L_3 L_6$ $L_4 L_7$
 $L_1 L_5$ $L_2 L_6$
 $L_1 L_6$ $L_2 L_8$
 $L_1 L_8$

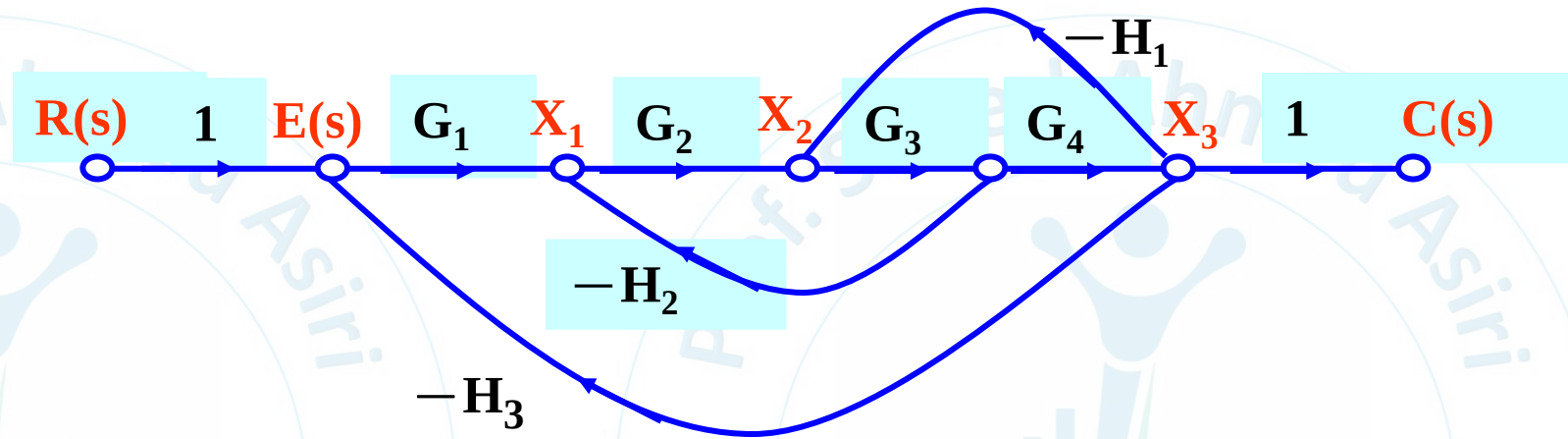
Block Diagram to Signal-Flow Graph

Example. 5:



Block Diagram to Signal-Flow Graph

Example. 5:



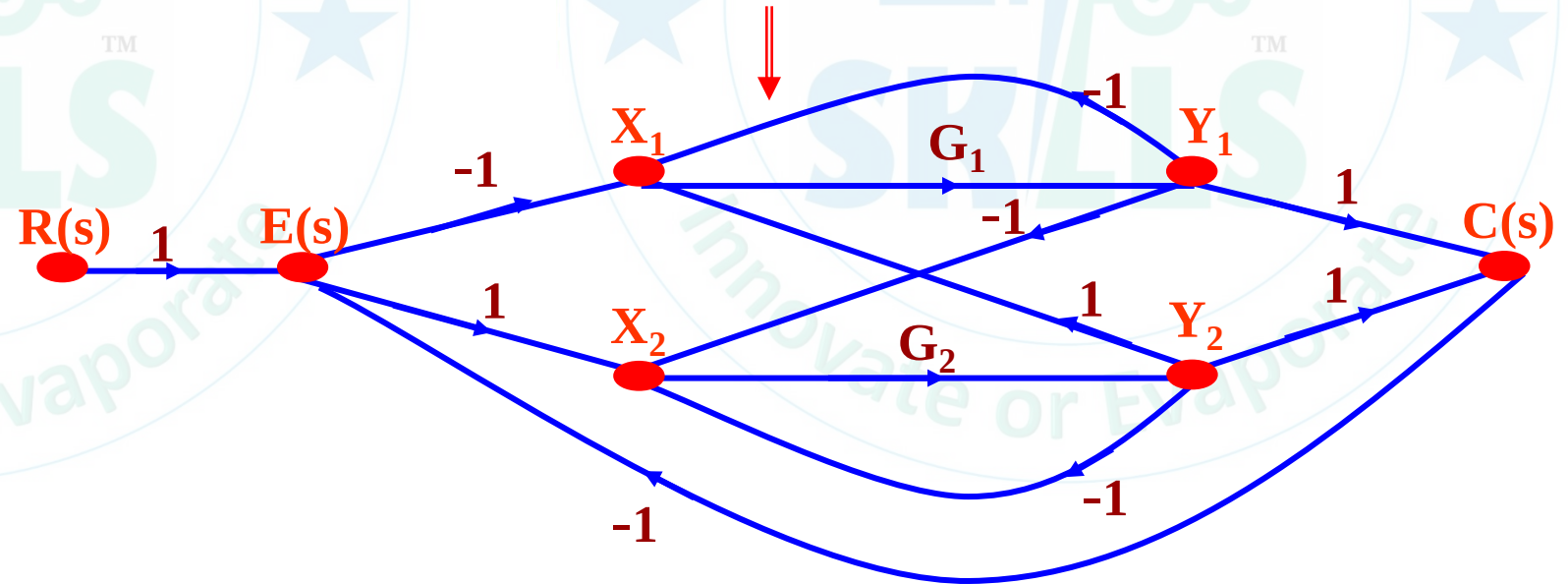
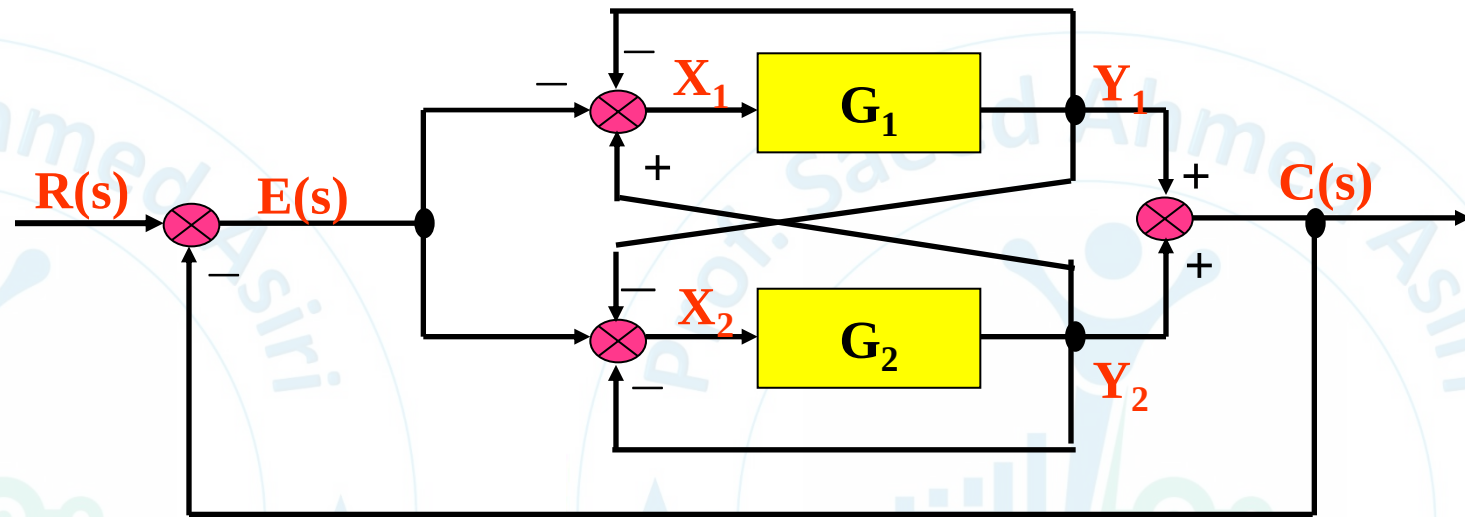
$$\Delta = 1 + (G_1 G_2 G_3 G_4 H_3 + G_2 G_3 H_2 + G_3 G_4 H_1)$$

$$P_1 = G_1 G_2 G_3 G_4; \quad \Delta_1 = 1$$

$$G = \frac{C(s)}{R(s)} = \frac{G_1 G_2 G_3 G_4}{1 + G_1 G_2 G_3 G_4 H_3 + G_2 G_3 H_2 + G_3 G_4 H_1}$$

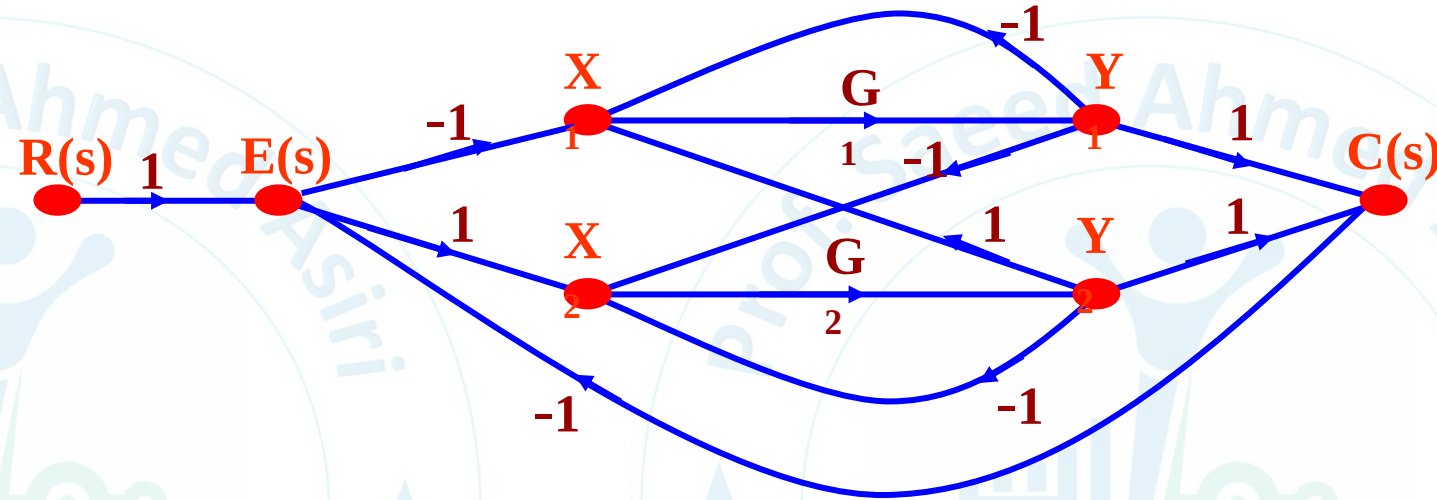
Block Diagram to Signal-Flow Graph

Example. 6:



Block Diagram to Signal-Flow Graph

Example. 6:



7 loops:

$$[G_1 \cdot (-1)]; \quad [G_2 \cdot (-1)]; \quad [G_1 \cdot (-1) \cdot G_2 \cdot 1]; \quad [(-1) \cdot G_1 \cdot 1 \cdot (-1)];$$

$$[(-1) \cdot G_1 \cdot (-1) \cdot G_2 \cdot 1 \cdot (-1)]; \quad [1 \cdot G_2 \cdot 1 \cdot (-1)]; \quad [1 \cdot G_2 \cdot 1 \cdot G_1 \cdot 1 \cdot (-1)].$$

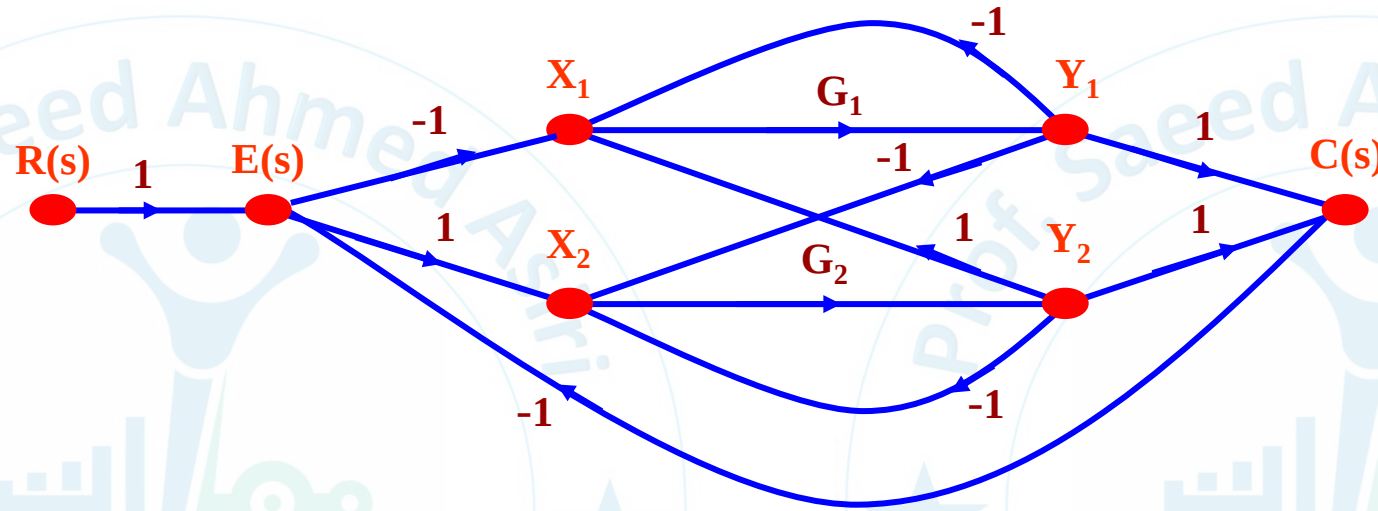
3 '2 non-touching loops':

$$[G_1 \cdot (-1)] \cdot [G_2 \cdot (-1)]; \quad [(-1) \cdot G_1 \cdot 1 \cdot (-1)] \cdot [G_2 \cdot (-1)];$$

$$[1 \cdot G_2 \cdot 1 \cdot (-1)] \cdot [G_1 \cdot (-1)].$$

Block Diagram to Signal-Flow Graph

Example. 6:



Then:

$$\Delta = 1 + 2G_2 + 4G_1G_2$$

4 forward paths:

$$p_1 = (-1) \cdot G_1 \cdot 1$$

$$\Delta_1 = 1 + G_2$$

$$p_2 = (-1) \cdot G_1 \cdot (-1) \cdot G_2 \cdot 1$$

$$\Delta_2 = 1$$

$$p_3 = 1 \cdot G_2 \cdot 1$$

$$\Delta_3 = 1 + G_1$$

$$p_4 = 1 \cdot G_2 \cdot 1 \cdot G_1 \cdot 1$$

$$\Delta_4 = 1$$

Block Diagram to Signal-Flow Graph

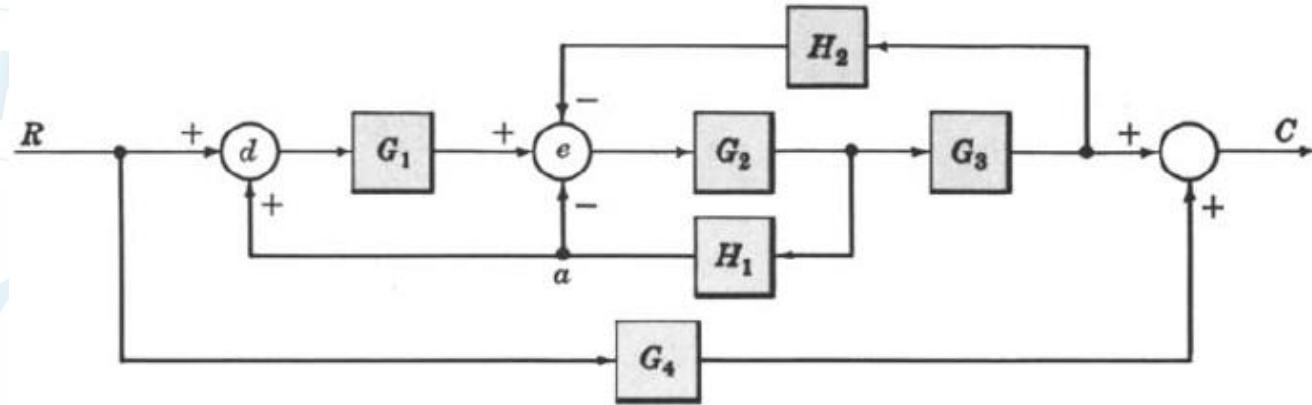
Example. 6:

We have

$$\frac{C(s)}{R(s)} = \frac{\sum p_i \Delta_i}{\Delta} = \frac{G_2 - G_1 + 2G_1G_2}{1 + 2G_2 + 4G_1G_2}$$

Block Diagram to Signal-Flow Graph

Example. 7:



- The signal flow graph of the above block diagram is shown below.

- There are two forward paths. The path gains are

$$P_1 = G_1 G_2 G_3 \text{ and } P_2 = G_4$$

- The three feedback loop gains are

$$P_{11} = -G_2 H_1, P_{21} = G_1 G_2 H_1, P_{31} = -G_2 G_3 H_2.$$

- No loops are non-touching, hence

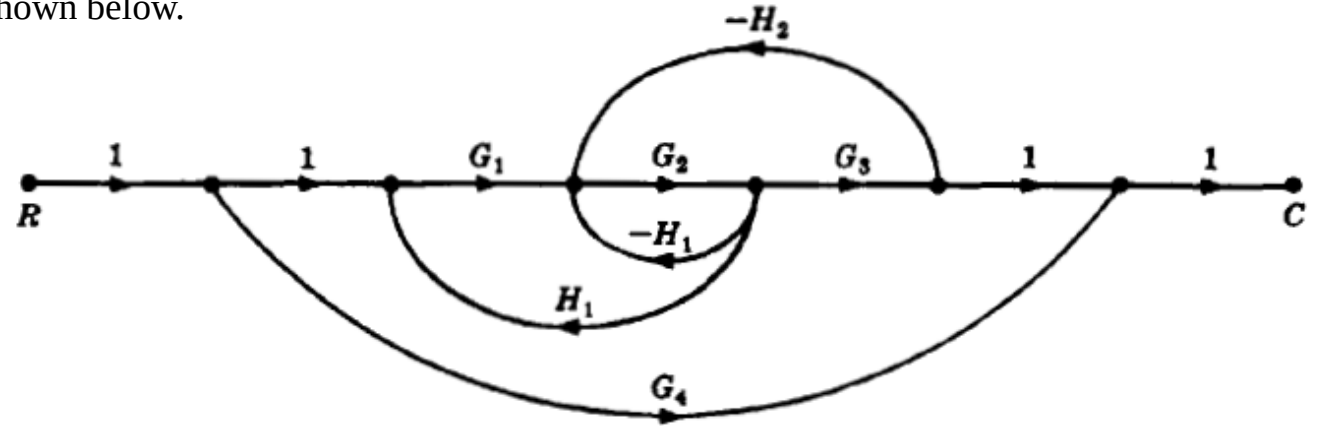
$$\Delta = 1 - (P_{11} + P_{21} + P_{31})$$

- Because the loops touch the nodes of P1, hence

$$\Delta_1 = 1$$

- Since no loops touch the nodes of P2, therefore

$$\Delta_2 = \Delta.$$

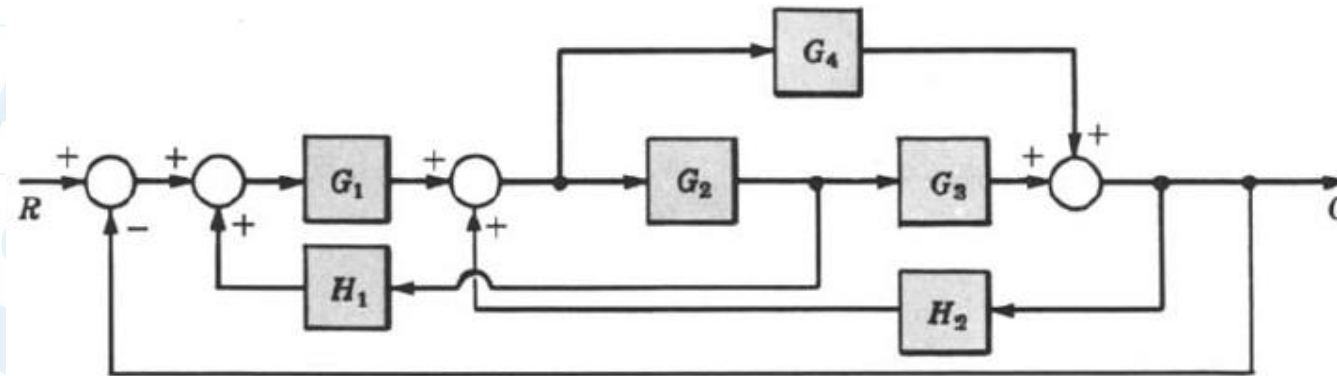


- Hence the transfer function $T = C/R$ is

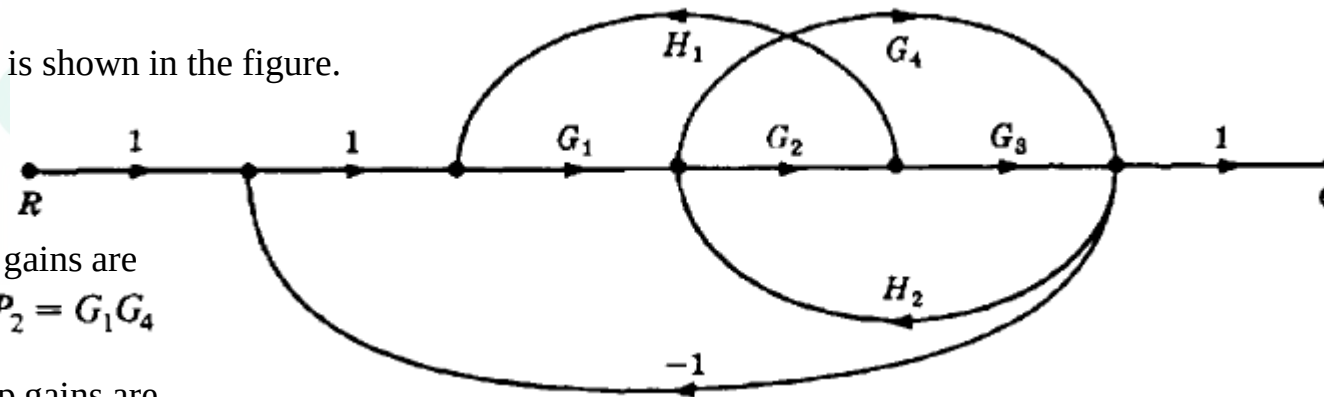
$$T = \frac{P_1 \Delta_1 + P_2 \Delta_2}{\Delta} = \frac{G_1 G_2 G_3 + G_4 + G_2 G_4 H_1 - G_1 G_2 G_4 H_1 + G_2 G_3 G_4 H_2}{1 + G_2 H_1 - G_1 G_2 H_1 + G_2 G_3 H_2}$$

Block Diagram to Signal-Flow Graph

Example. 7:



- The signal flow graph is shown in the figure.



- The two forward path gains are $P_1 = G_1G_2G_3$ and $P_2 = G_1G_4$
- The five feedback loop gains are $P_{11} = G_1G_2H_1$, $P_{21} = G_2G_3H_2$, $P_{31} = -G_1G_2G_3$, $P_{41} = G_4H_2$, and $P_{51} = -G_1G_4$.
- All feedback loops touches the two forward paths, hence $\Delta_1 = \Delta_2 = 1$

- There are no non-touching loops, hence

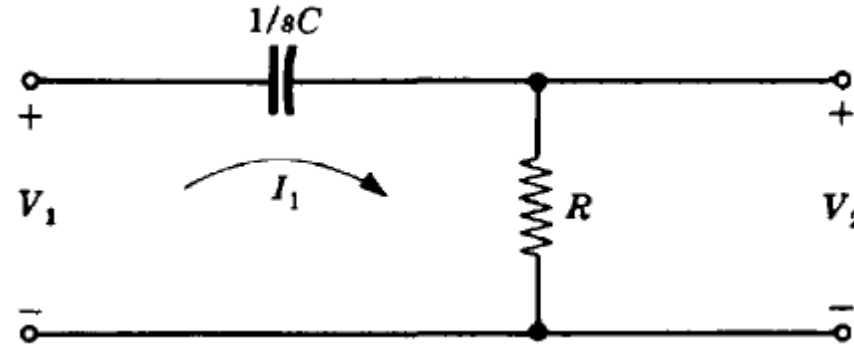
$$\Delta = 1 - (P_{11} + P_{21} + P_{31} + P_{41} + P_{51})$$

$$= 1 + G_1G_2G_3 - G_1G_2H_1 - G_2G_3H_2 - G_4H_2 + G_1G_4$$

$$\frac{C}{R} = \frac{P_1\Delta_1 + P_2\Delta_2}{\Delta} = \frac{G_1G_2G_3 + G_1G_4}{1 + G_1G_2G_3 - G_1G_2H_1 - G_2G_3H_2 - G_4H_2 + G_1G_4}$$

Electrical System to Signal-Flow Graph

Example. 8:

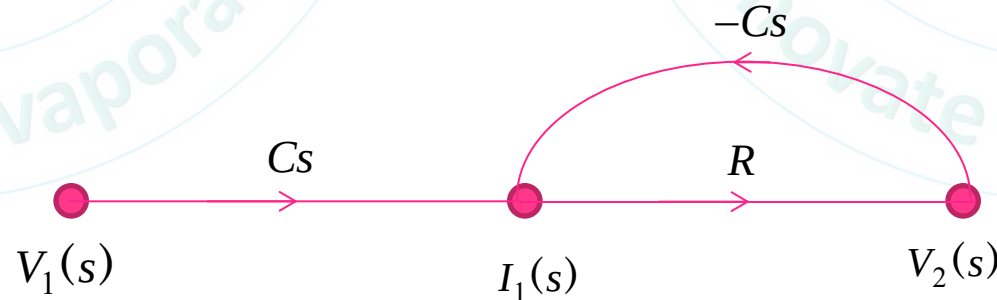


$$V_1(s) = \frac{1}{Cs} I_1(s) + I_1(s)R$$

$$V_2(s) = I_1(s)R$$

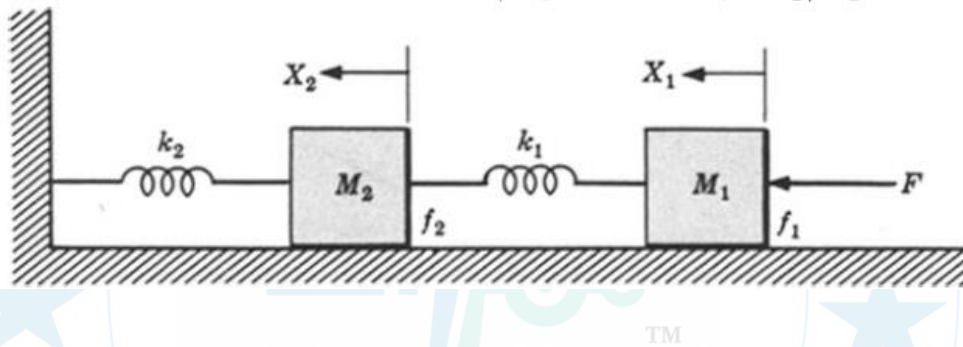


$$CsV_1(s) - CsV_2(s) = I_1(s)$$



Electrical System to Signal-Flow Graph

Example. 9:



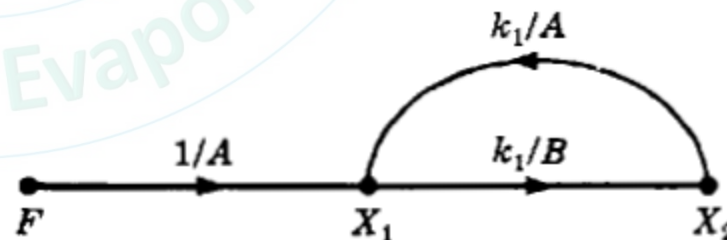
$$\begin{aligned} \text{(i)} \quad & F + k_1 X_2 = (M_1 s^2 + f_1 s + k_1) X_1 \\ \text{(ii)} \quad & k_1 X_1 = (M_2 s^2 + f_2 s + k_1 + k_2) X_2 \end{aligned}$$

$$F = M_1 s^2 X_1 + k_1 (X_1 - X_2)$$

$$0 = M_2 s^2 X_2 + k_1 (X_2 - X_1) + k_2 X_2$$

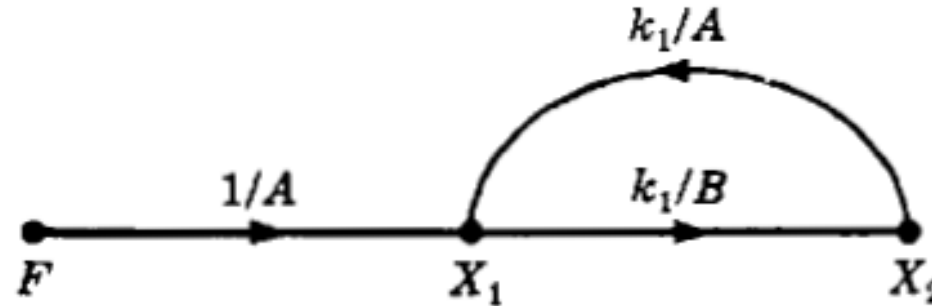
$$\text{(iii)} \quad \left(\frac{1}{A} \right) F + \left(\frac{k_1}{A} \right) X_2 = X_1$$

$$\text{(iv)} \quad \left(\frac{k_1}{B} \right) X_1 = X_2$$



Electrical System to Signal-Flow Graph

Example. 9:



The forward path gain is $P_1 = k_1/AB$. The feedback loop gain is $P_{11} = k_1^2/AB$. then $\Delta = 1 - P_{11} = (AB - k_1^2)/AB$ and $\Delta_1 = 1$. Finally,

$$\frac{X_2}{F} = \frac{P_1 \Delta_1}{\Delta} = \frac{k_1}{AB - k_1^2} = \frac{k_1}{(M_1 s^2 + f_1 s + k_1)(M_2 s^2 + f_2 s + k_1 + k_2) - k_1^2}$$