

Frequency Response Techniques using Bode and Nyquist Plots

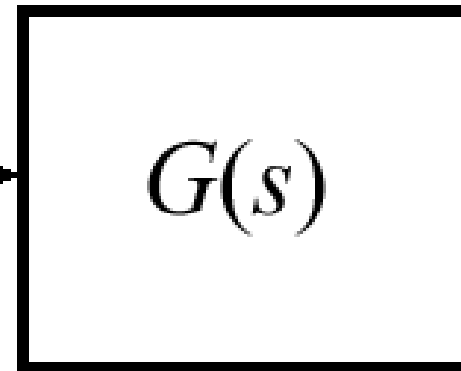
Concept of Frequency Response

Input $\sin(\omega t)$

System

Response

$$R(s) = \frac{\omega}{s^2 + \omega^2}$$



$C(s)$

Concept of Frequency Response

$$C(s) = \frac{\omega}{s^2 + \omega^2} G(s)$$

$$= \underbrace{\frac{K_1}{(s + j\omega)}}_{\text{Forced}} + \underbrace{\frac{K_2}{(s - j\omega)}}_{\text{Natural}} + \text{Fraction terms for } G(s)$$

Forced
Steady-state

Natural
Dies out

Concept of Frequency Response

$$K_1 = \frac{\omega}{(s - j\omega)} G(s) \Big|_{s=-j\omega} = \frac{1}{2} jG(-j\omega) = \frac{1}{2} M_G e^{-j(\phi_G - \pi/2)},$$

$$K_2 = \frac{\omega}{(s + j\omega)} G(s) \Big|_{s=j\omega} = -\frac{1}{2} jG(j\omega) = \frac{1}{2} M_G e^{j(\phi_G - \pi/2)}$$

where

$$M_G = |G(j\omega)|$$

$$\phi_G = \text{angle of } G(j\omega)$$

Concept of Frequency Response

$$C_{ss}(s) = \frac{K_1}{(s + j\omega)} + \frac{K_2}{(s - j\omega)}$$

$$= \frac{1}{2} M_G \left[\frac{e^{-j(\phi_G - \pi/2)}}{(s + j\omega)} + \frac{e^{j(\phi_G - \pi/2)}}{(s - j\omega)} \right]$$



$$c_{ss}(t) = \frac{1}{2} M_G \left[e^{-j(\omega t + \phi_G - \pi/2)} + e^{j(\omega t + \phi_G - \pi/2)} \right]$$

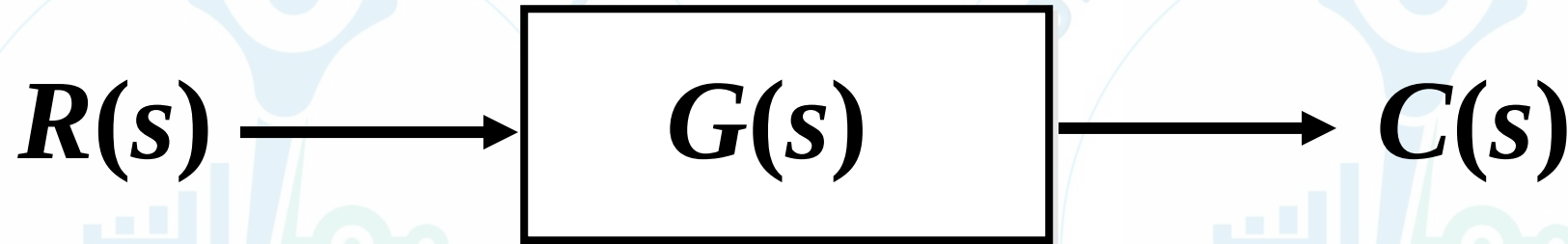
$$= M_G \sin(\omega t + \phi_G)$$

Concept of Frequency Response

Input

System

Response



$$R(t) = \sin(\omega t) \quad \rightarrow \quad c(t) = M_G \sin(\omega t + \phi_G)$$

where $M_G = |G(j\omega)|$ & $\phi_G = \text{angle of } G(j\omega)$

Concept of Frequency Response

System Parameters

- Magnitude $M_G = |G(j\omega)|$
- Phase angle $\phi_G = \text{angle of } G(j\omega)$

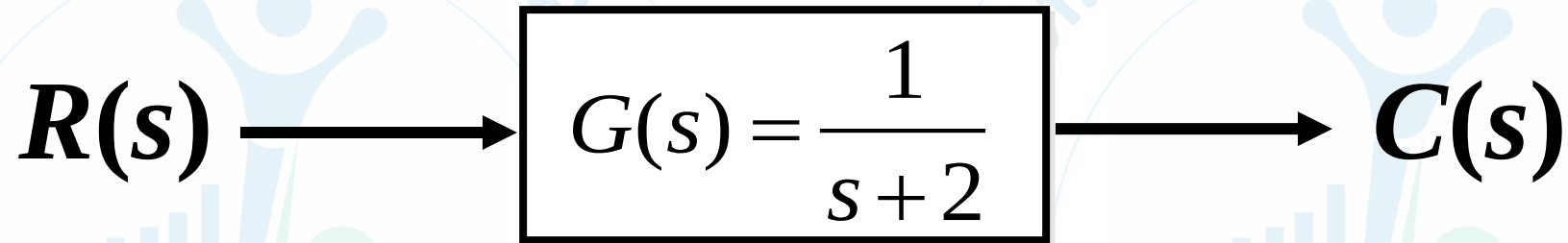
System Representation

Bode Diagram M_G & ϕ_G vs. ω

Nyquist Diagram M_G vs. ϕ_G for $0 < \omega < \infty$

Concept of Frequency Response

Example

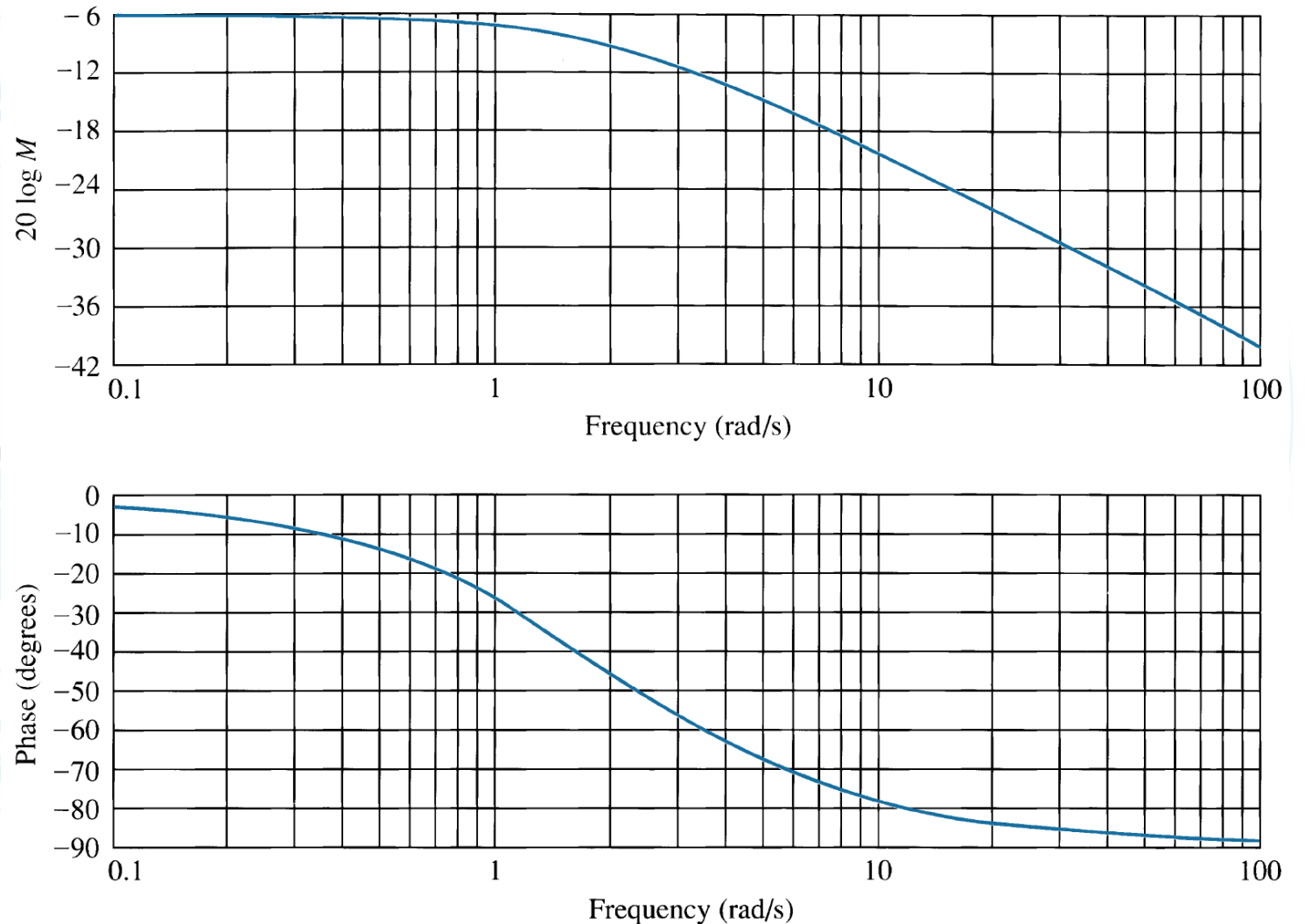


$$G(j\omega) = \frac{1}{j\omega + 2} = \frac{2 - j\omega}{\omega^2 + 4} = M_G e^{j\phi_G}$$


$$M_G = \frac{1}{\sqrt{\omega^2 + 4}}, \quad \text{and} \quad \phi_G = -\tan^{-1}\left(\frac{\omega}{2}\right)$$

Concept of Frequency Response

Example Bode Diagram



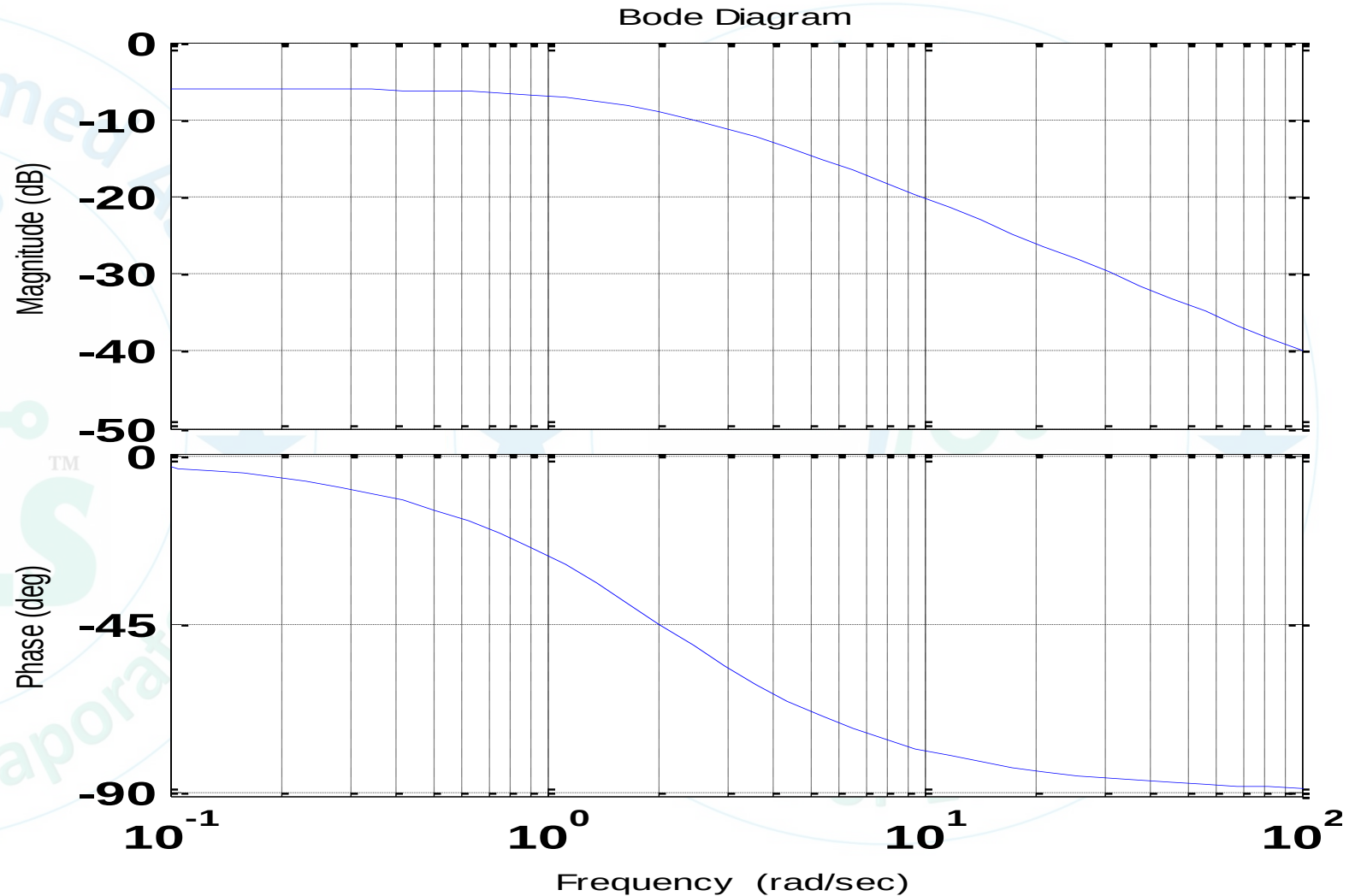
Frequency Response using Bode Plot

Example

```
>> n=1;  
>> d=[1 2];  
>> bode(n,d)  
>> grid
```

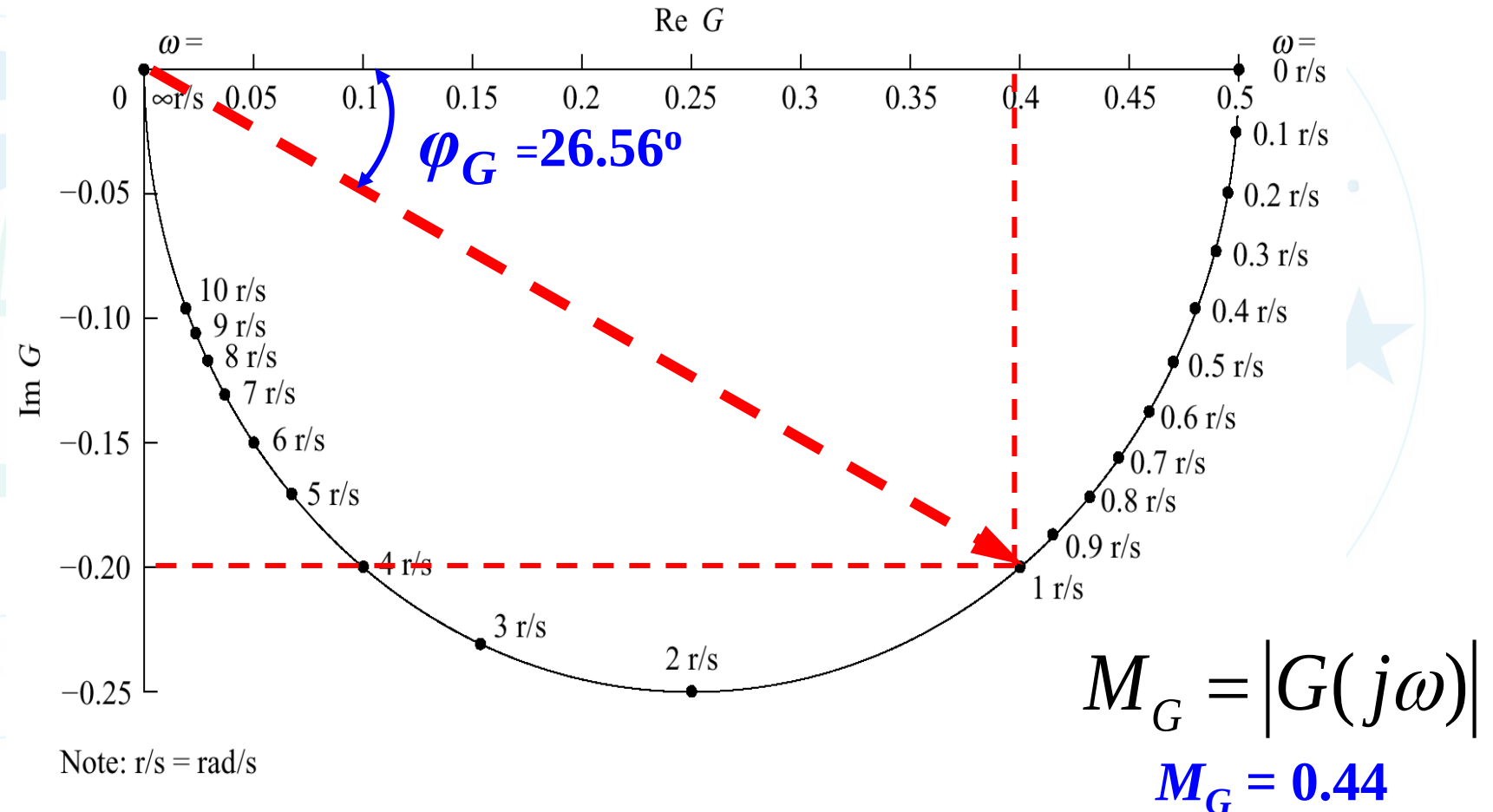


Hendrik Bode
1905 -1982



Frequency Response using Nyquist Plot

Example



Frequency Response using Nyquist Plot

Example

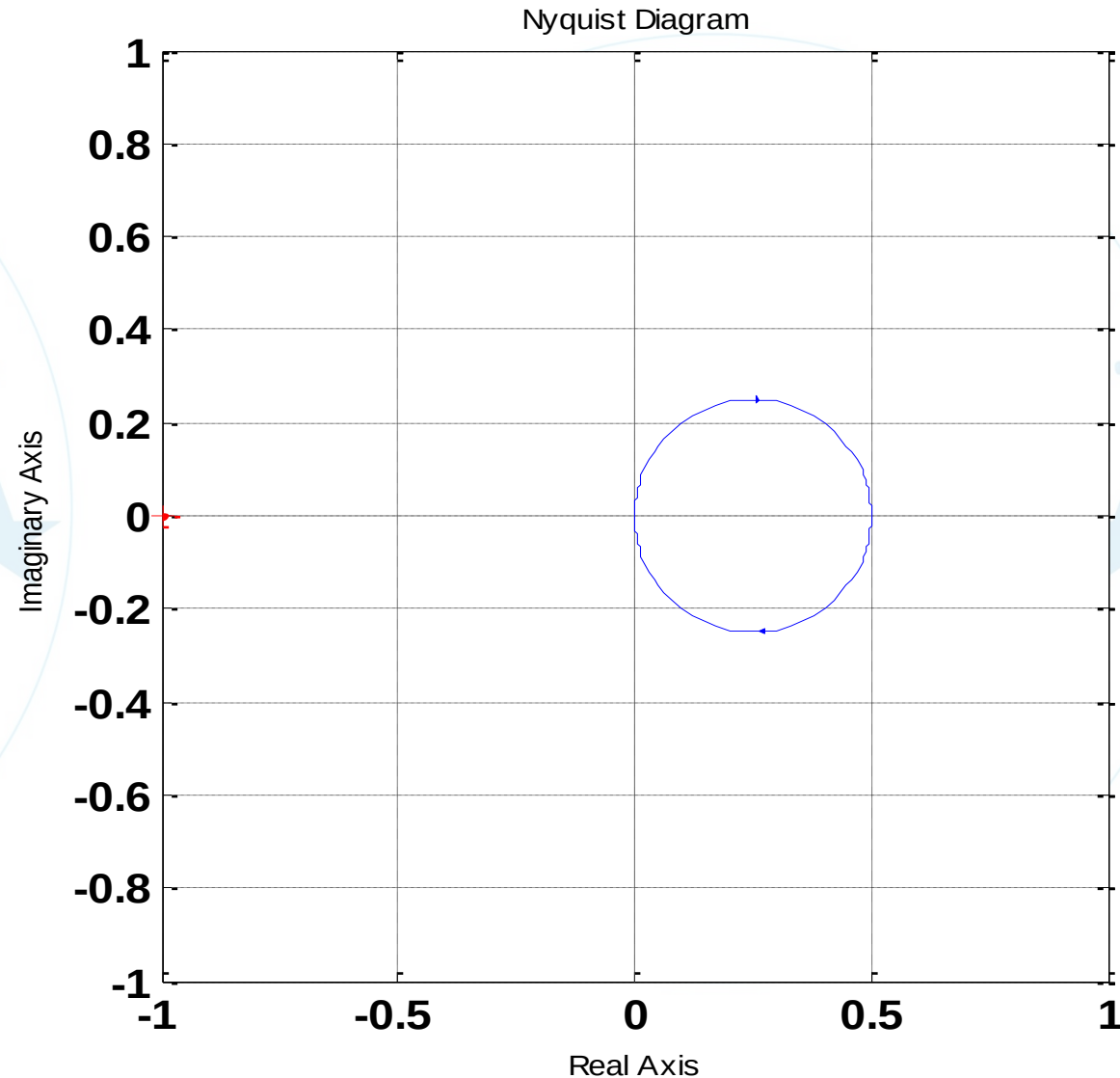
```
>> n=1;  
>> d=[1 2];  
>> nyquist(n,d)
```



Harry Nyquist

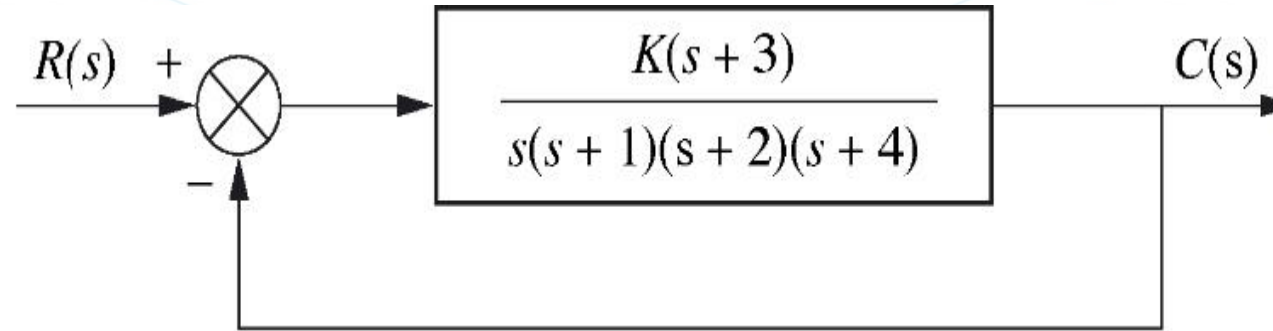
b. Feb. 7, 1889, Nilsby, Sweden

d. April 4, 1976, Harlingen, Texas, U.S.



Nyquist Stability

Example



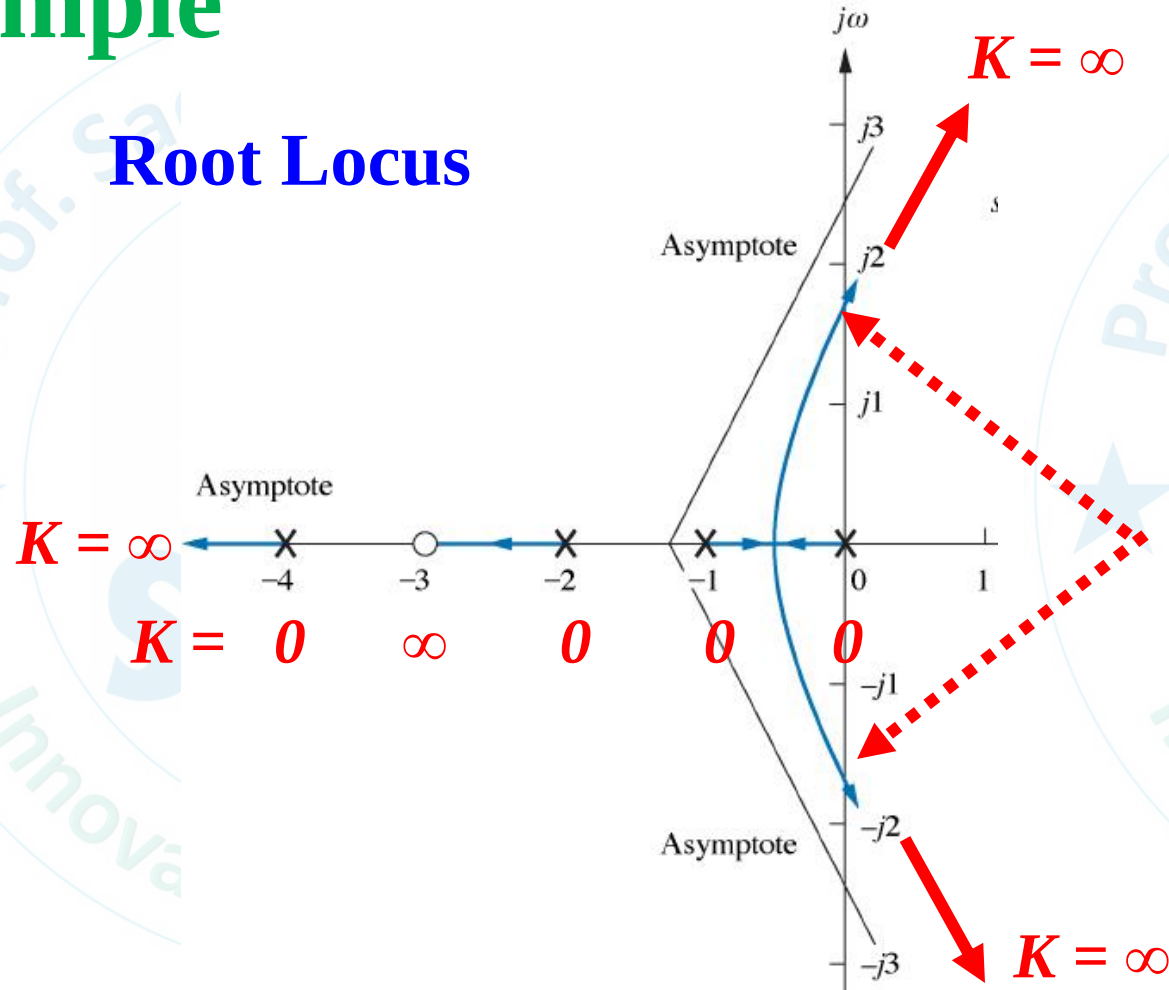
$$T = \frac{K(s+3)}{s^4 + 7s^3 + 14s^2 + (8+K)s + 3K}$$

s^4	1	14	$3K$
s^3	7	$8+K$	
s^2	$90-K$	$21K$	
s^1	$\frac{-K^2 - 65K + 720}{90-K}$		
s^0	$21K$		

Nyquist Stability

Example

Root Locus



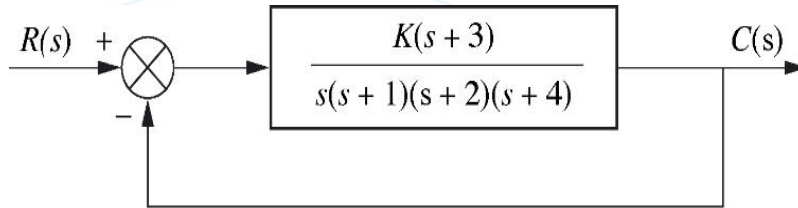
From Routh Table

$$-K^2 - 65K + 720 = 0$$

$$\rightarrow K = 9.65$$

Nyquist Stability

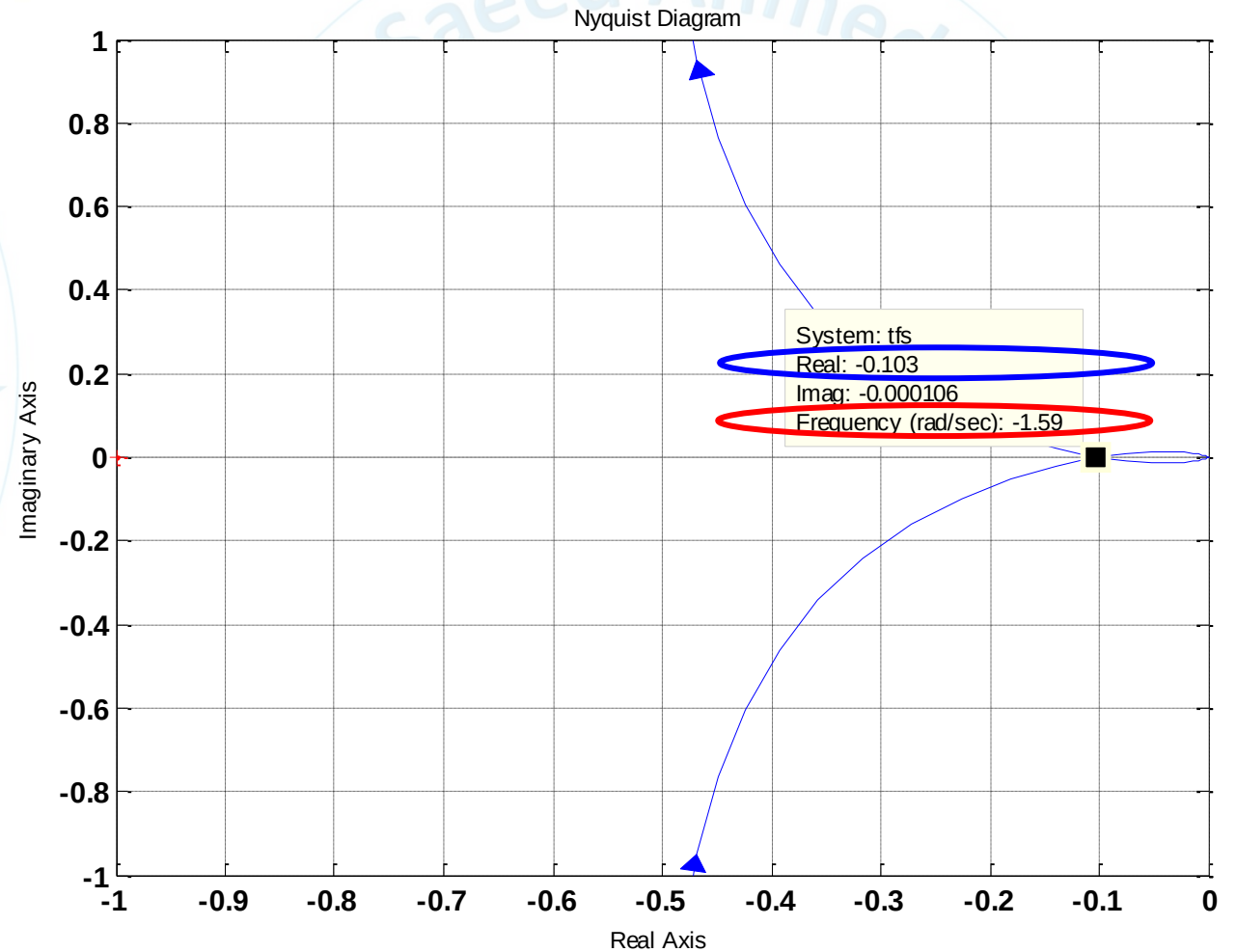
Example



```
>> n=[1 3];
>> tfs=zpk([-3],[0 -1 -2 -4],1);
>> nyquist(tfs)
```

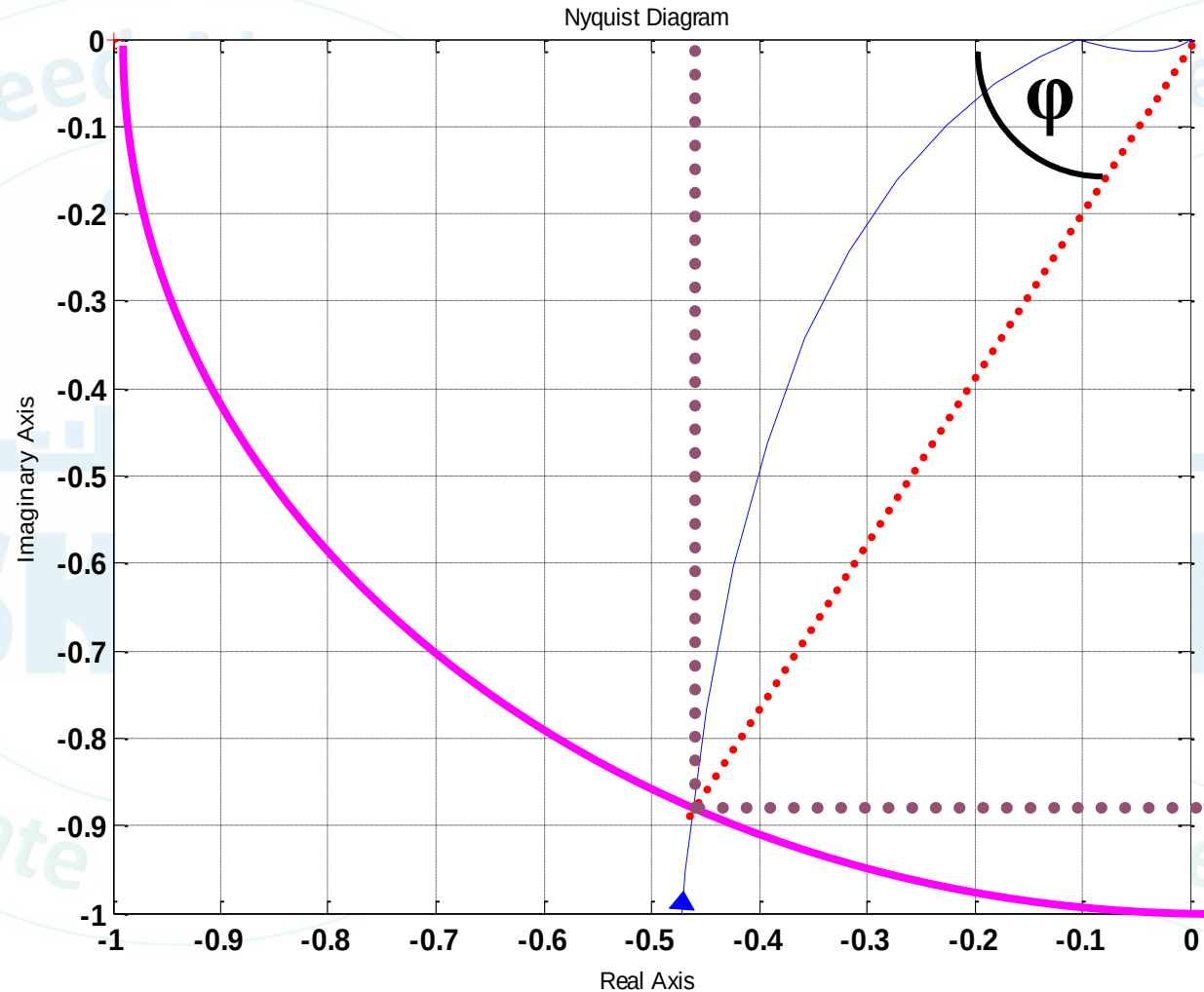
Gain Margin

$$GM = 1/0.103 = 9.7$$



Nyquist Stability

Example



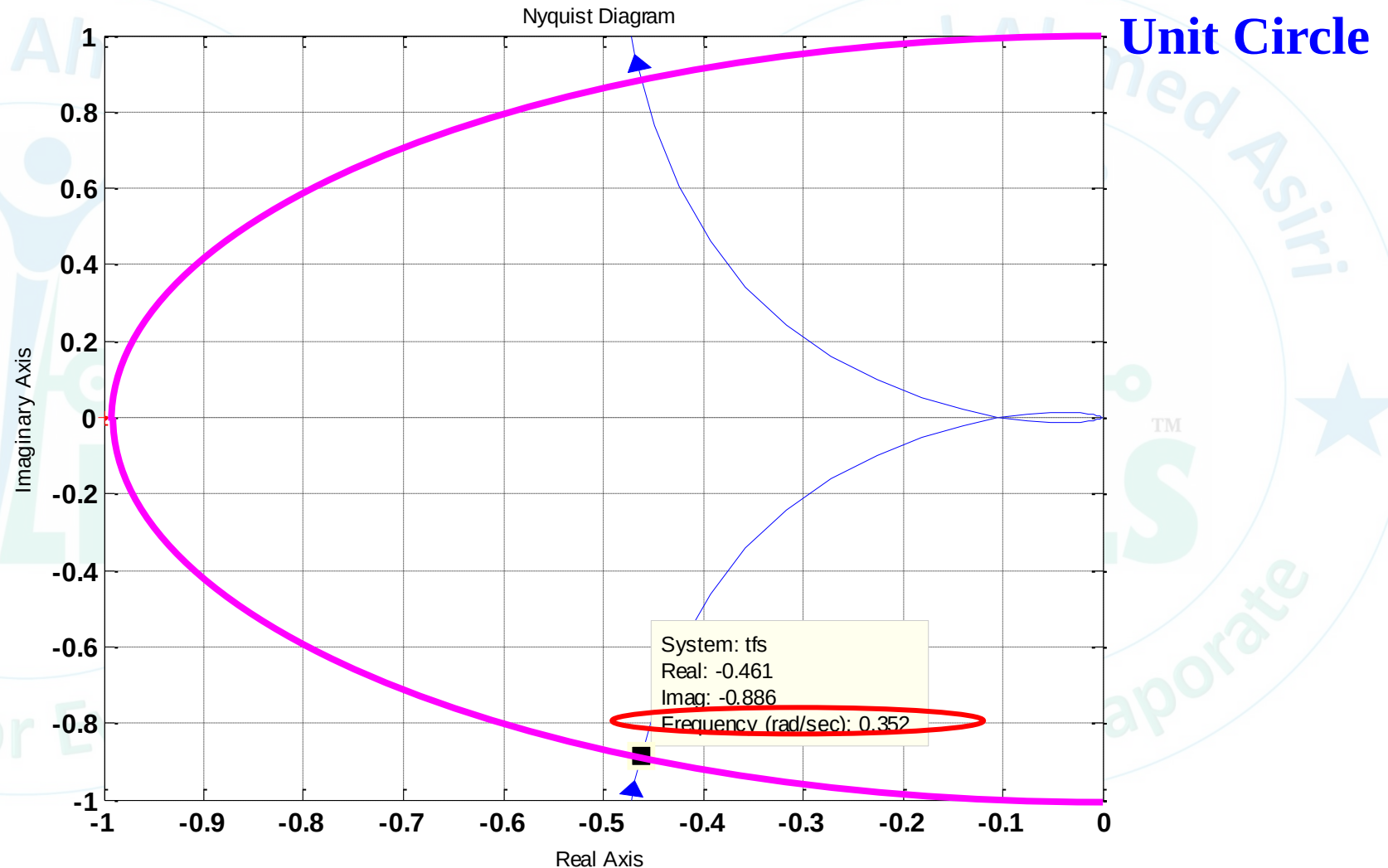
$$\phi = \tan^{-1}(0.88/0.47)$$
$$= 61.89^\circ$$

Phase Margin

$$\phi = 61.89^\circ$$

Nyquist Stability

Example



Nyquist Stability

Example

```
>> [Gm,Pm,wg,wp]=margin(tfs)
```

Gm = 9.6456

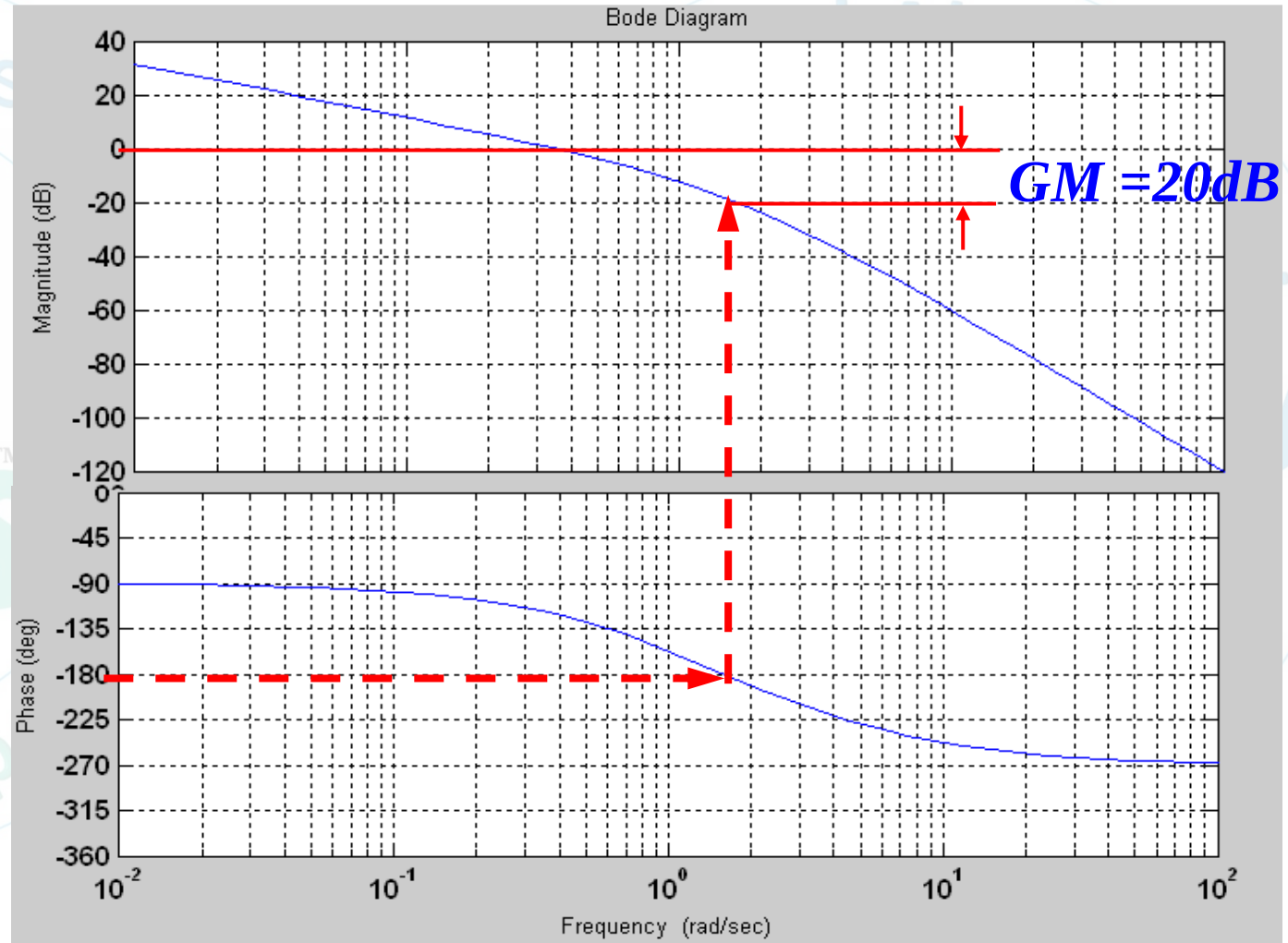
Pm = 62.4587

wg = 1.5877

wp = 0.3497

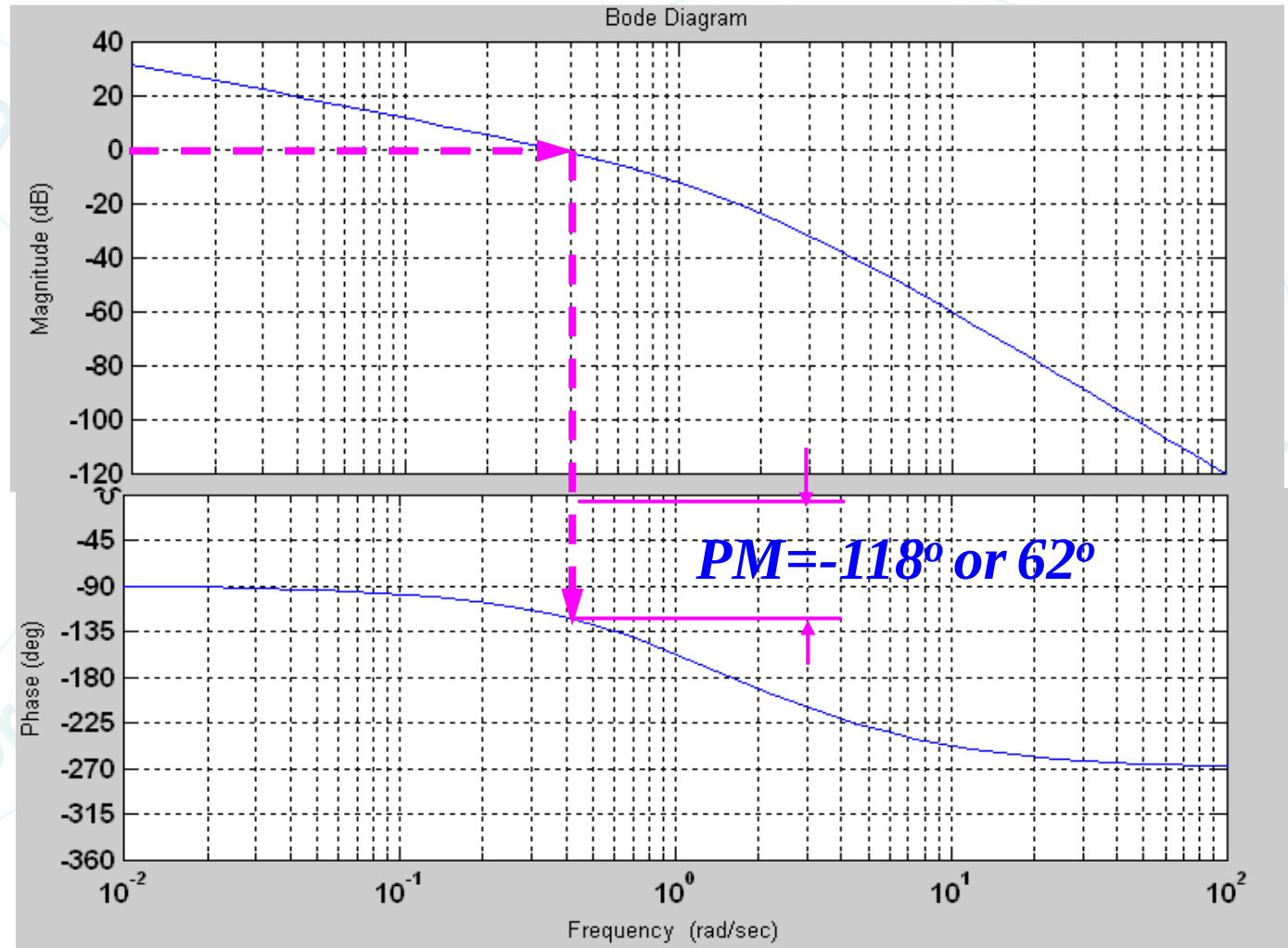
Nyquist Stability

Example



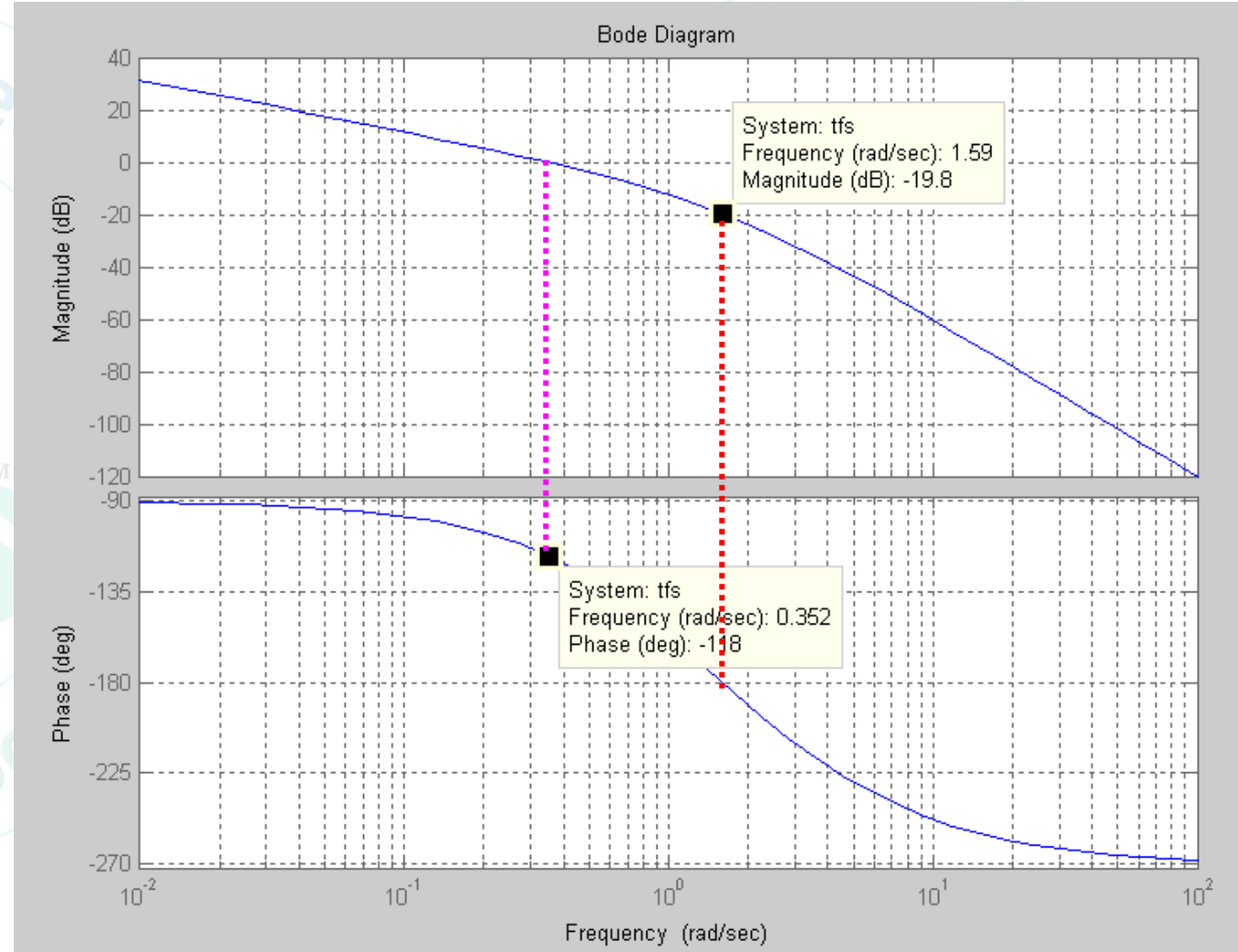
Nyquist Stability

Example

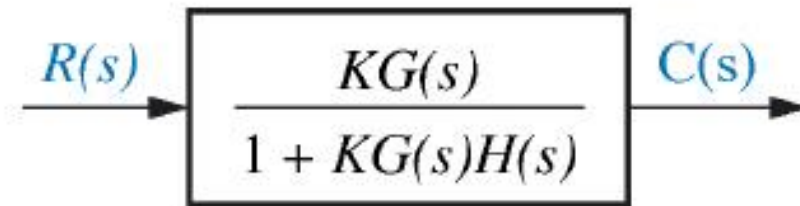
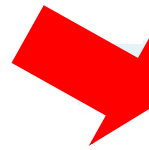
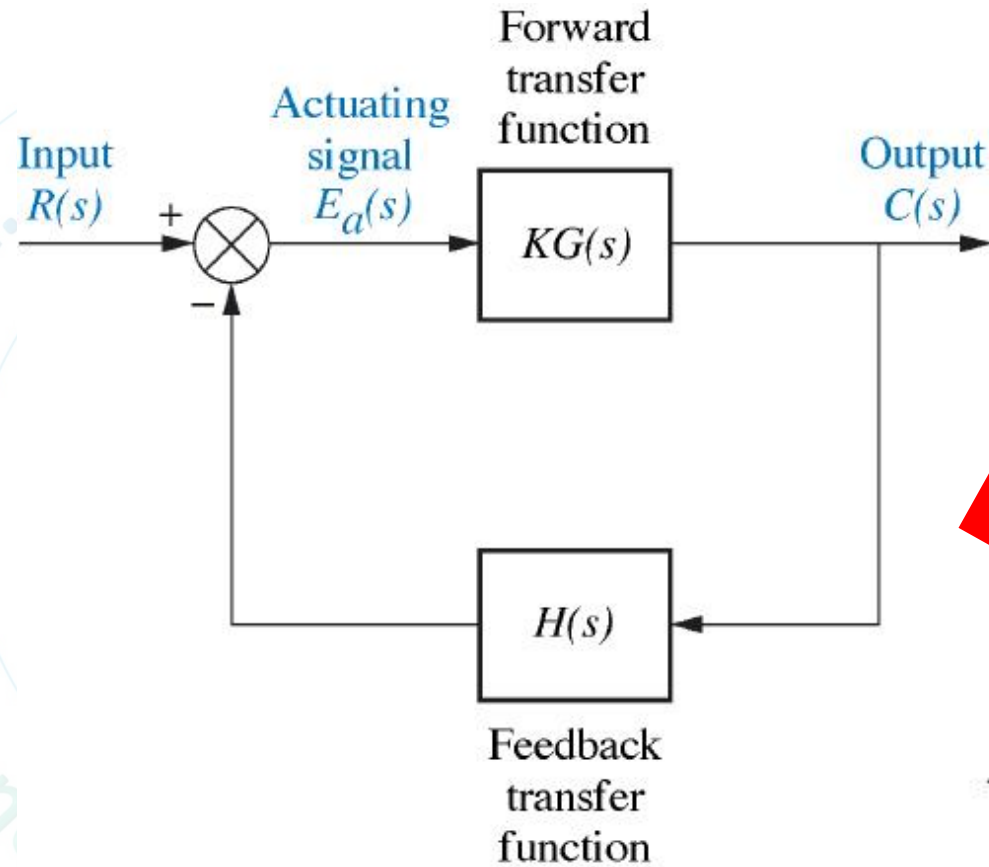


Nyquist Stability

Example



Nyquist Stability



Nyquist Stability

When $1 + KG(s)H(s) = 0$

System output $\rightarrow \infty$



a. Magnitude Property

$$KG(s)H(s) = -1,$$

or

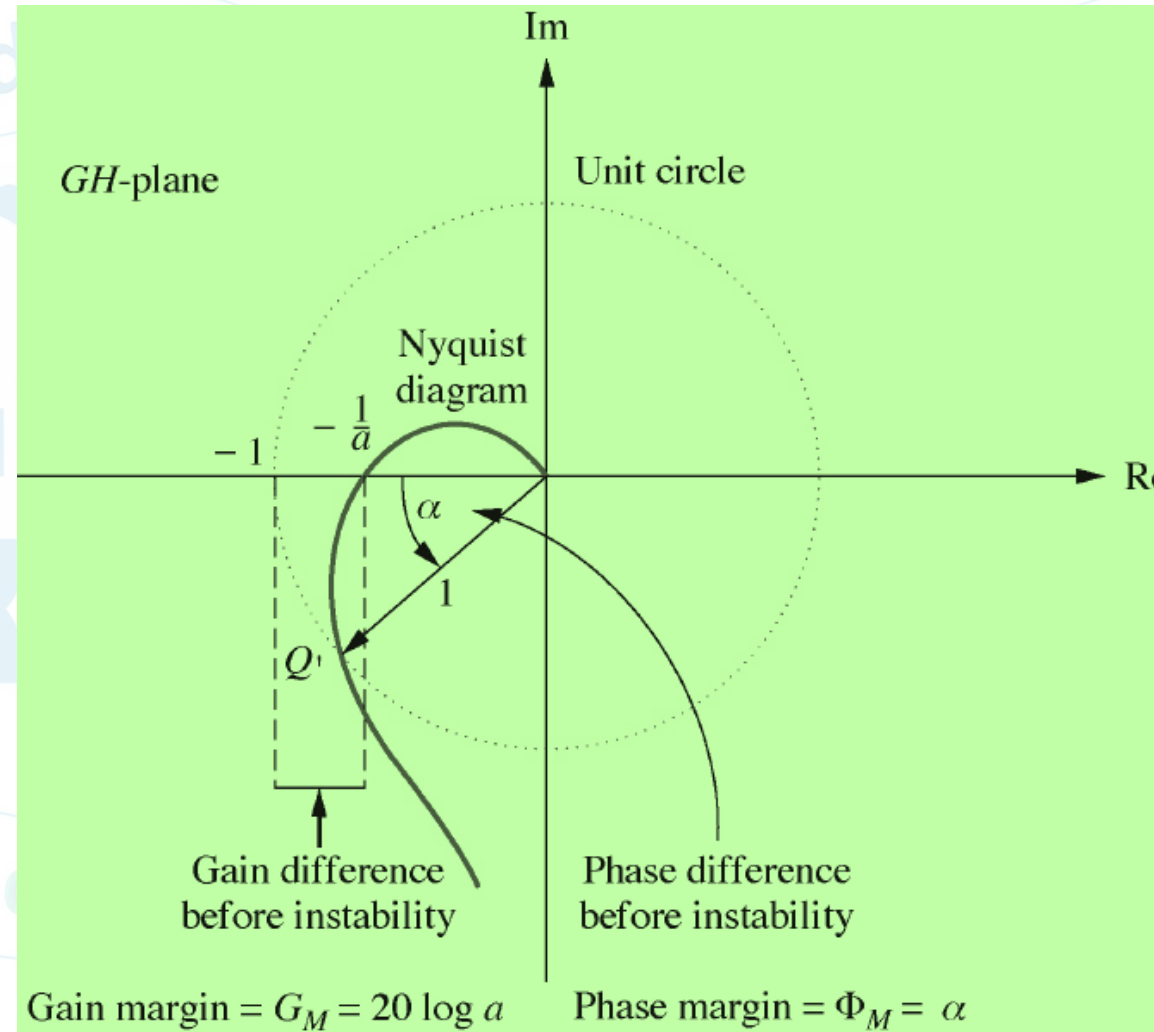
$$K = \frac{1}{|G(s)H(s)|}$$

Gain Margin

b. Phase angle Property

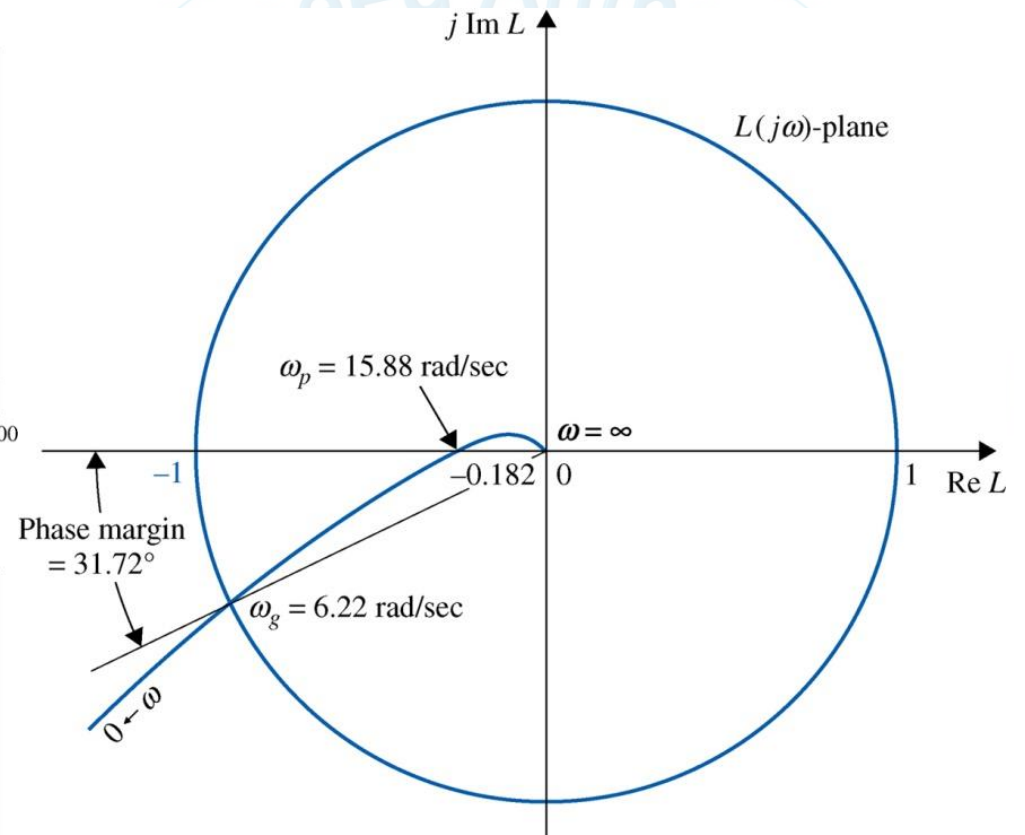
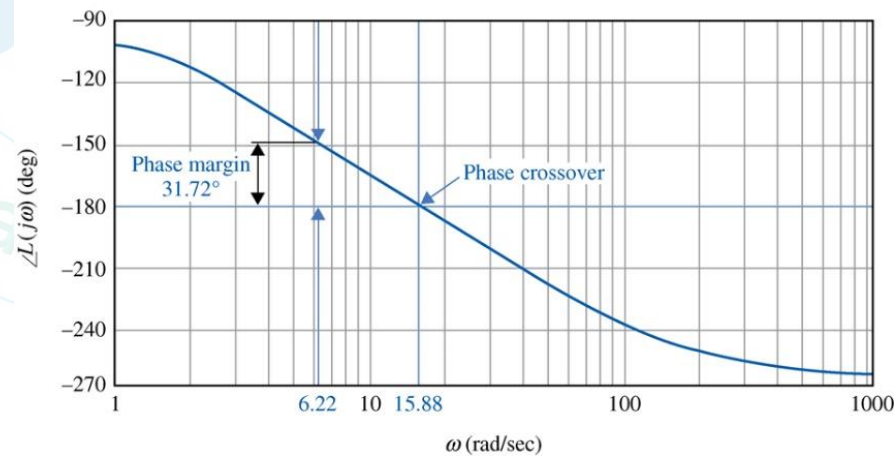
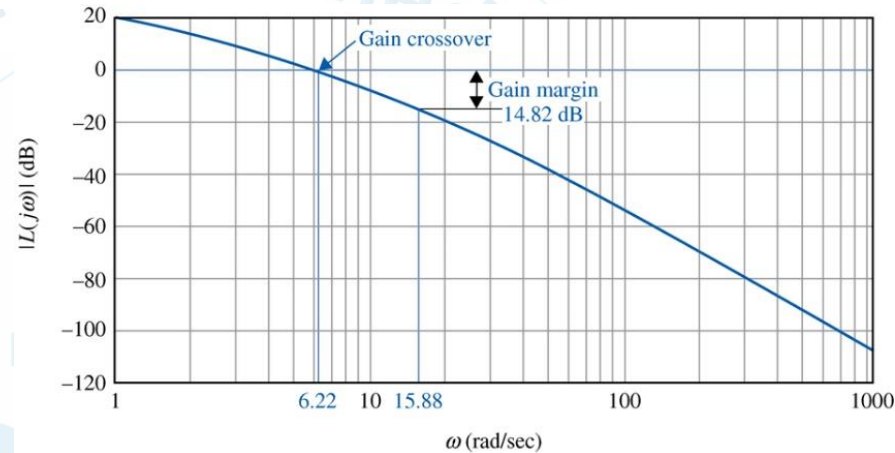
$$\angle [G(s)H(s)] = \pi$$

Nyquist Stability



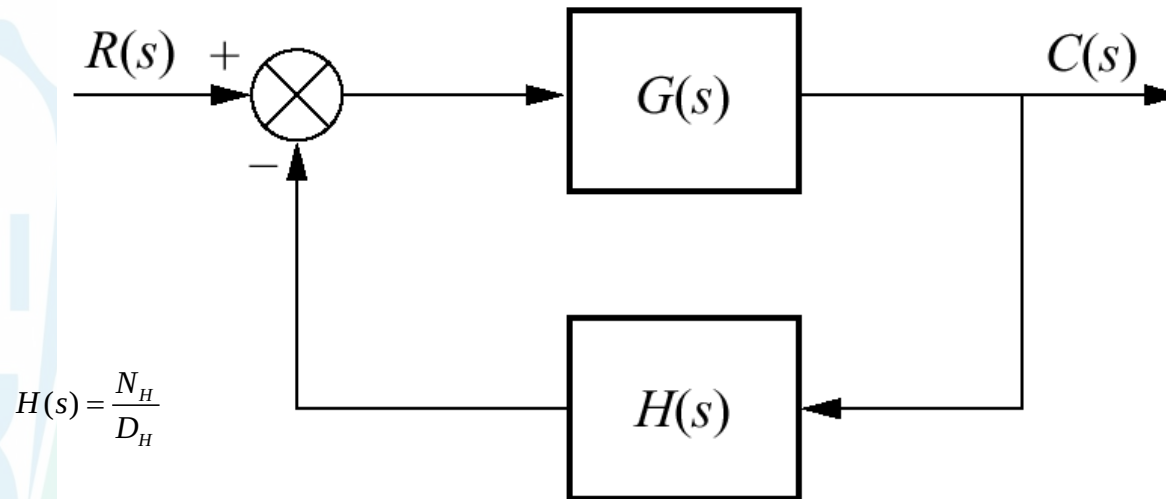
Nyquist Stability

$$L(s) = \frac{2500}{s(s+5)(s+50)}$$

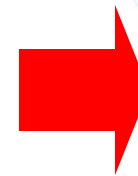


Nyquist Stability

Defines number of closed-loop poles in the right half-plane

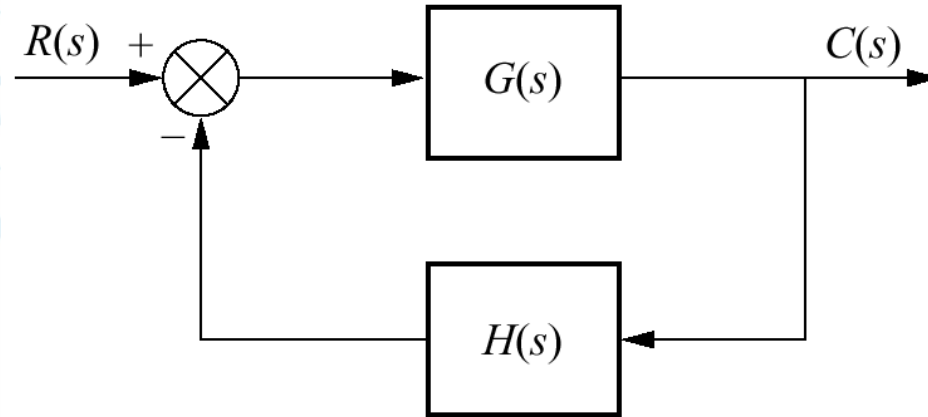


$$G(s) = \frac{N_G}{D_G} \text{ \& }$$

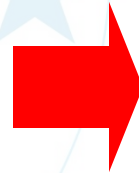


$$G(s)H(s) = \frac{N_G}{D_G} \frac{N_H}{D_H}$$

Nyquist Stability



$$G(s)H(s) = \frac{N_G}{D_G} \frac{N_H}{D_H}$$



$$1 + G(s)H(s) = \frac{N_G N_H + D_G D_H}{D_G D_H}$$



$$T(s) = \frac{G(s)}{1 + G(s)H(s)} = \frac{N_G D_H}{D_G D_H + N_G N_H}$$

Nyquist Stability

Relationships between:

Poles of $1+G(s)H(s)$ & Poles of $G(s)H(s)$

$$G(s)H(s) = \frac{N_G}{D_G} \frac{N_H}{D_H}$$

$$1 + G(s)H(s) = \frac{N_G N_H + D_G D_H}{D_G D_H}$$

Zeros of $1+G(s)H(s)$ & Poles of $T(s)$

$$T(s) = \frac{G(s)}{1 + G(s)H(s)} = \frac{N_G D_H}{D_G D_H + N_G N_H}$$

Nyquist Diagram and Analysis

Nyquist Criterion

- Let us first introduce the most **important equation** when dealing with the Nyquist criterion:

$$Z = N + P$$

- Where:
 - Z** is the number of zeros of the characteristic equation (and therefore the number of poles of the closed-loop transfer function) that are in the right-half of the s plane.
 - N** is the number of encirclements of the $(-1, 0)$ point.
 - P** is the number of poles of the open-loop characteristic equation that are in the right-half of the s plane.

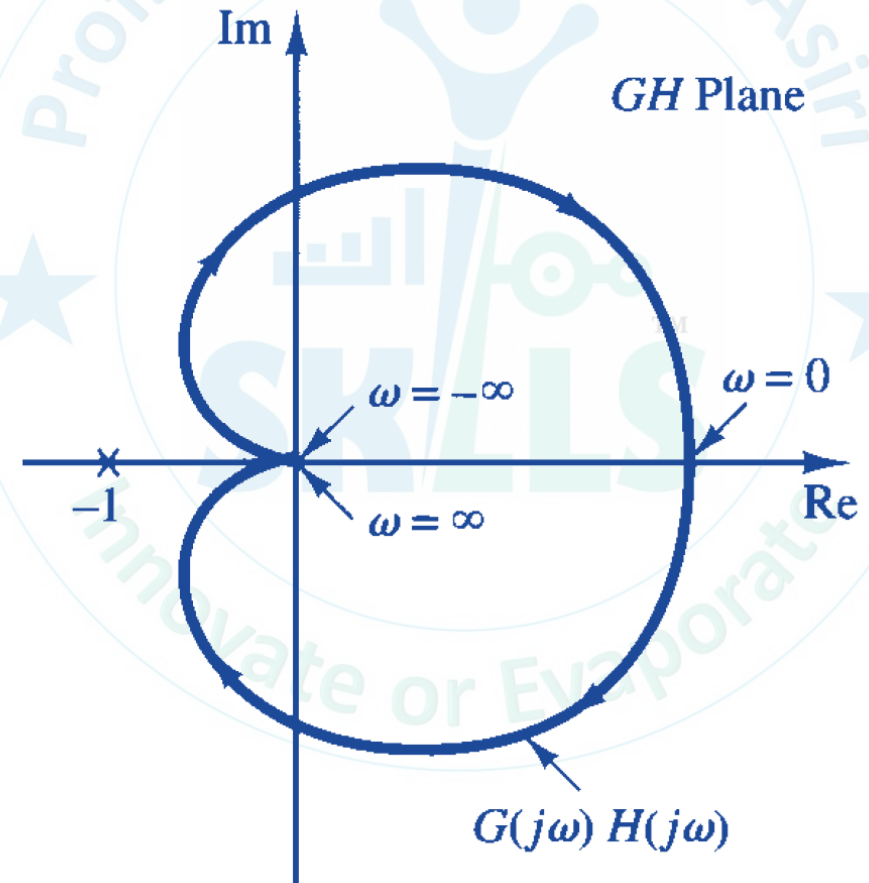
Nyquist Diagram and Analysis

Example

Investigate the stability of $GH(s) = \frac{K}{(1+T_1s)(1+T_2s)}$ using Nyquist Stability Criteria

Since $P = 0$, so we need $N=Z=0$ for stability.

As shown in the figure, there is no encirclement for -1. Hence, the closed loop system is stable for any positive value of K, T_1, T_2

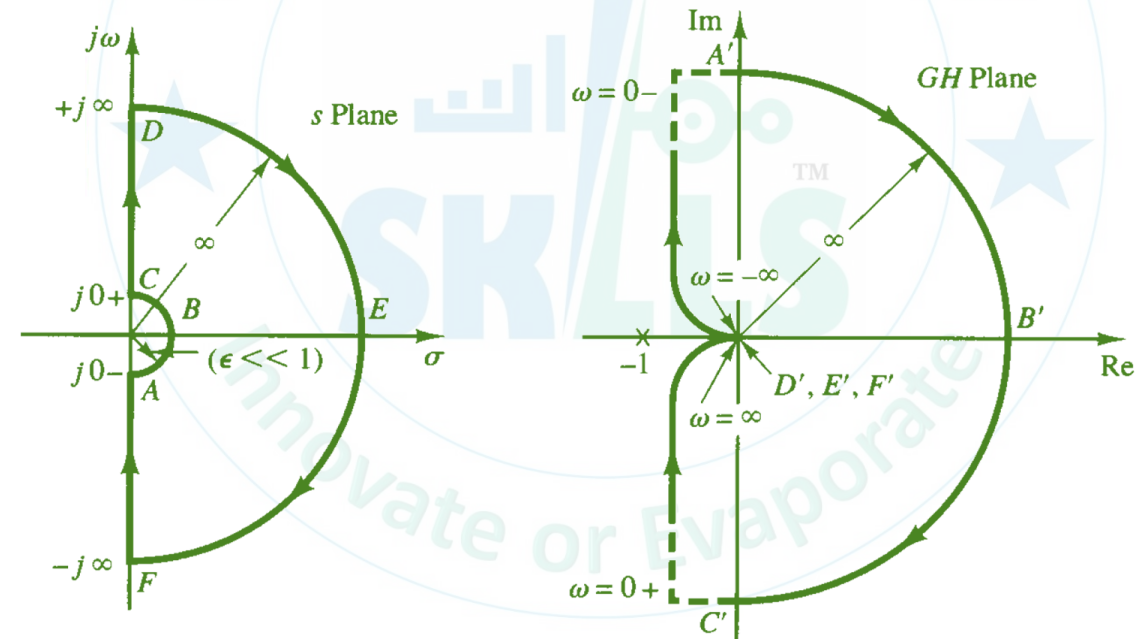


Nyquist Diagram and Analysis

Example

Investigate the stability of $G(s) = K/[s(1 + Ts)]$ using Nyquist Stability Criteria

Since $P = 0$, so we need $N=Z=0$ for stability.
As shown in the figure, there is no encirclement for -1. Hence, the closed loop system is stable.



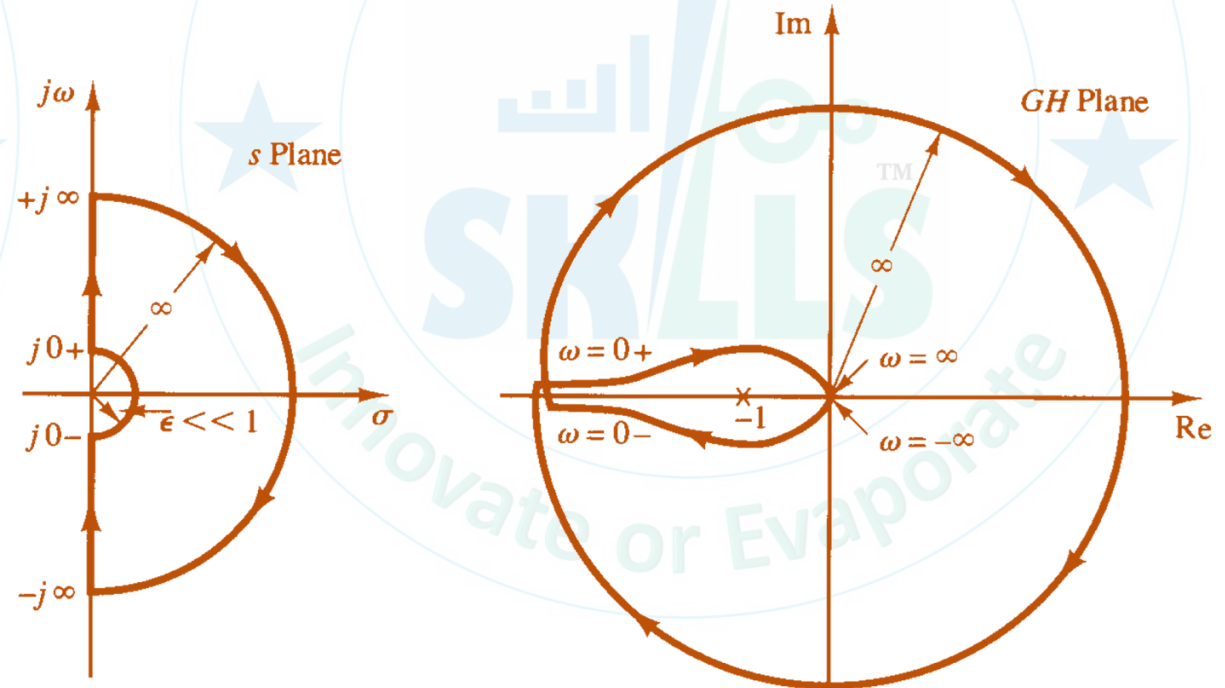
Nyquist Diagram and Analysis

Example

Investigate the stability of $G(s) = K / [s^2 (1 + T s)]$ using Nyquist Stability Criteria

Since $P = 0$ for positive T , so we need $N=Z=0$ for stability.

But as shown in the figure, there is two clockwise encirclements for -1 so $N = 2$. Hence, the closed loop system is unstable.

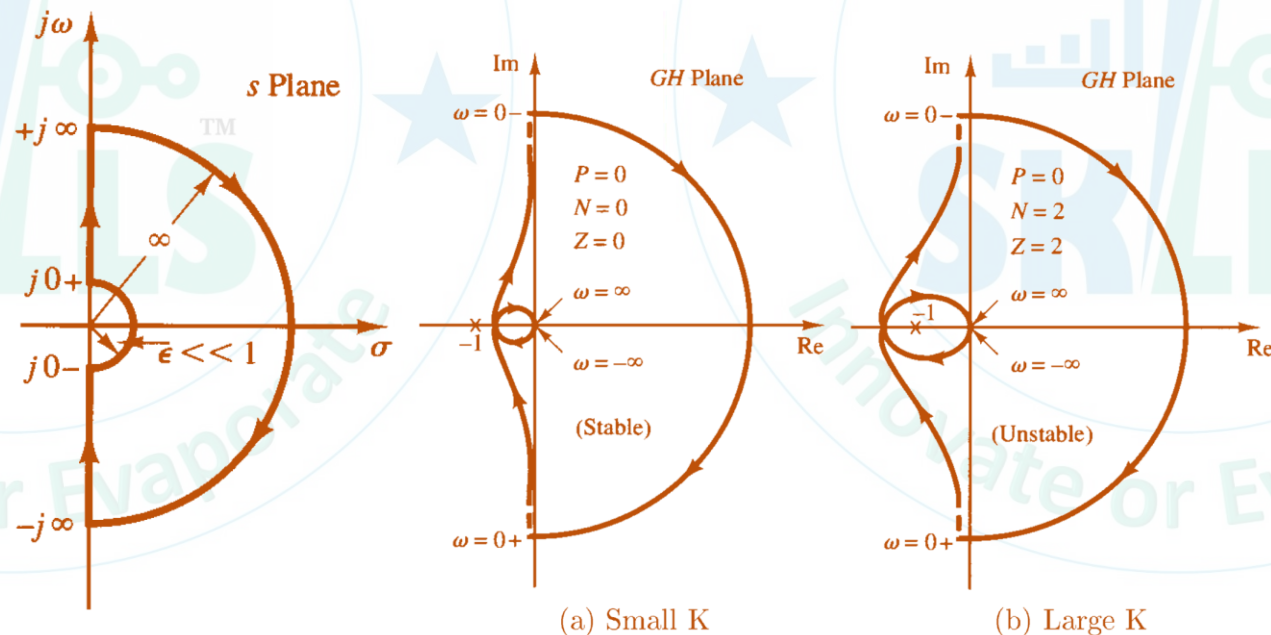


Nyquist Diagram and Analysis

Example

Investigate the stability of $GH(s) = \frac{K}{s(1+T_1s)(1+T_2s)}$ using Nyquist Stability Criteria

Since $P = 0$ for positive T_1 and T_2 , so we need $N=Z=0$ for stability.



Nyquist Diagram and Analysis

Example

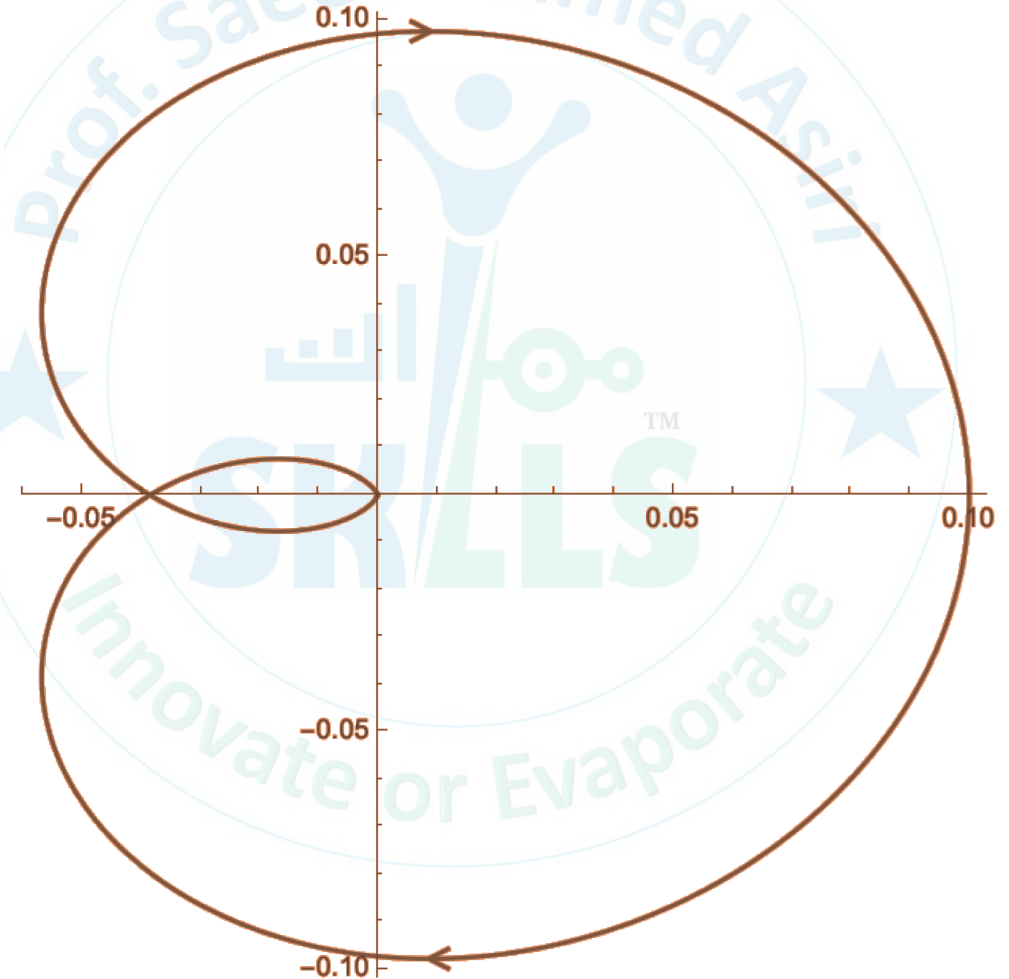
Consider the following system:

$$G(s) = \frac{1}{(s+2)(s^2+2s+5)}$$

Suppose this is in unity feedback with a constant gain controller K . In other words, we have a negative feedback loop where the forward gain is $KG(s)$ and the loop gain is also $KG(s)$.

Investigate the stability of the system using

- a) Routh Array
- b) Nyquist Plot



END