

State Space Modeling

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Basic Concepts related to State Space

- **State Variables**

The smallest possible subset of system variables that can represent the entire state of the system at any given time

- **State Vector**

All the state variables $\{q_1, q_2, \dots, q_n\}$ can be looked on as components of state vector.

- **State Space**

A space whose coordinates consist of state variables is called a state space. Any state can be represented by a point in state space.

State Space Representation

Two parts:

- A set of **first order ODEs** that represents the **derivative** of each state variable x_i as an algebraic function of the set of state variables $\{x_i\}$ and the inputs $\{u_i\}$.

$$\begin{cases} \dot{q}_1 = f_1(x_1, x_2, x_3, \dots, x_n, u_1, u_2, u_3, \dots, u_m) \\ \dot{q}_2 = f_2(x_1, x_2, x_3, \dots, x_n, u_1, u_2, u_3, \dots, u_m) \\ \vdots \\ \dot{q}_n = f_n(x_1, x_2, x_3, \dots, x_n, u_1, u_2, u_3, \dots, u_m) \end{cases} \quad \dot{x} = Ax + Bu$$

- A set of equations that represents the output variables as algebraic functions of the set of state variables $\{x_i\}$ and the inputs $\{u_i\}$.

$$\begin{cases} y_1 = f_1(x_1, x_2, x_3, \dots, x_n, u_1, u_2, u_3, \dots, u_m) \\ y_2 = f_2(x_1, x_2, x_3, \dots, x_n, u_1, u_2, u_3, \dots, u_m) \\ \vdots \\ y_n = f_n(x_1, x_2, x_3, \dots, x_n, u_1, u_2, u_3, \dots, u_m) \end{cases} \quad y = Cx + Du$$

Differential Equations and State Space Representation

$$\ddot{y}(t) + a_1\dot{y}(t) + a_0y(t) = b_0u(t)$$

- A second-order differential equation requires two state variables:

$$x_1(t) = y(t) \quad x_2(t) = \dot{y}(t)$$

- We can reformulate the differential equation as a set of three equations:

$$\dot{x}_1(t) = x_2(t)$$

$$\dot{x}_2(t) = -a_0x_1(t) - a_1x_2(t) + b_0u(t)$$

$$y(t) = x_1(t)$$

- We can write these in matrix form as:

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -a_0 & -a_1 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ b_0 \end{bmatrix} u(t)$$

$$y(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$$

- This can be extended to an N^{th} -order differential equation of this type:
$$y^{(N)}(t) + \sum_{i=0}^{N-1} a_i y^{(i)}(t) = b_0 u(t)$$
- The state variables are defined as: $x_i(t) = y^{(i-1)}(t), \quad i = 1, 2, \dots, N$

Differential Equations and State Space Representation

- The resulting state equations are:

$$\begin{aligned}
 \dot{x}_1(t) &= x_2(t) \\
 \dot{x}_2(t) &= x_3(t) \\
 &\vdots \\
 \dot{x}_{N-1}(t) &= x_N(t) \\
 \dot{x}_N(t) &= -\sum_{i=0}^{N-1} a_i x_{i+1}(t) + b_0 u(t) \\
 y(t) &= x_1(t)
 \end{aligned}$$

$$\begin{aligned}
 \dot{\mathbf{X}} &= \mathbf{A}\mathbf{X} + \mathbf{B}u \\
 y &= \mathbf{C}\mathbf{X} + \mathbf{D}u
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{A} &= \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ -a_0 & -a_1 & -a_2 & \cdots & -a_{N-1} \end{bmatrix} & \mathbf{B} &= \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ b_0 \end{bmatrix} \\
 \mathbf{C} &= [1 \quad 0 \quad 0 \quad \cdots \quad 0] & \mathbf{D} &= 0
 \end{aligned}$$

Observable canonical form

Differential Equations and State Space Representation

- Next, consider a differential equation with a more complex forcing function:

$$\ddot{y}(t) + a_1\dot{y}(t) + a_0y(t) = b_1\dot{u}(t) + b_0u(t)$$

- The state model is:

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -a_0 & -a_1 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t) \quad y(t) = [b_0 \quad b_1] \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$$

$$\begin{aligned} \dot{X} &= AX + Bu \\ y &= CX + Du \end{aligned}$$

- We can verify this by expanding the matrix equation:

$$\dot{x}_1(t) = x_2(t)$$

$$\dot{x}_2(t) = -a_0x_1(t) - a_1x_2(t) + u(t)$$

$$y(t) = b_0x_1(t) + b_1x_2(t)$$

- To construct the original equation, differentiate the last equation:

$$\dot{y}(t) = b_0\dot{x}_1(t) + b_1\dot{x}_2(t)$$

$$= b_0x_2(t) + b_1[-a_0x_1(t) - a_1x_2(t) + u(t)]$$

$$= -a_1y(t) + (a_1b_0 - a_0b_1)x_1(t) + b_0x_2(t)$$

Differential Equations and State Space Representation

$$\begin{aligned}\dot{x}_1(t) &= x_2(t) \\ \dot{x}_2(t) &= -a_0x_1(t) - a_1x_2(t) + u(t) \\ y(t) &= b_0x_1(t) + b_1x_2(t)\end{aligned}$$

$$\begin{aligned}\dot{\mathbf{X}} &= \mathbf{A}\mathbf{X} + \mathbf{B}u \\ y &= \mathbf{C}\mathbf{X} + \mathbf{D}u\end{aligned}$$

- Differentiate the last equation again and substitute:

$$\begin{aligned}\ddot{y}(t) &= -a_1\dot{y}(t) + (a_1b_0 - a_0b_1)\dot{x}_1(t) + b_0\dot{x}_2(t) + b_1\dot{u}(t) \\ &= -a_1\dot{y}(t) + (a_1b_0 - a_0b_1)x_2(t) + b_0[-a_0x_1(t) - a_1x_2(t) + u(t)] + b_1\dot{u}(t) \\ &= -a_1\dot{y}(t) - b_0a_0x_1(t) - a_0b_1x_2(t) + b_0u(t) + b_1\dot{u}(t) \\ &= -a_1\dot{y}(t) - a_0(b_0x_1(t) + b_1x_2(t)) + b_0u(t) + b_1\dot{u}(t) \\ &= -a_1\dot{y}(t) - a_0y(t) + b_0u(t) + b_1\dot{u}(t)\end{aligned}$$

- Hence, given a general LTI system:

$$\ddot{y}(t) + a_1\dot{y}(t) + a_0y(t) = b_0u(t) + b_1\dot{u}(t)$$

$$y^{(N)}(t) + \sum_{i=0}^{N-1} a_i y^{(i)}(t) = \sum_{i=0}^{N-1} b_i u^{(i)}(t)$$

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ -a_0 & -a_1 & -a_2 & \cdots & -a_{N-1} \end{bmatrix} \quad \mathbf{B} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}$$

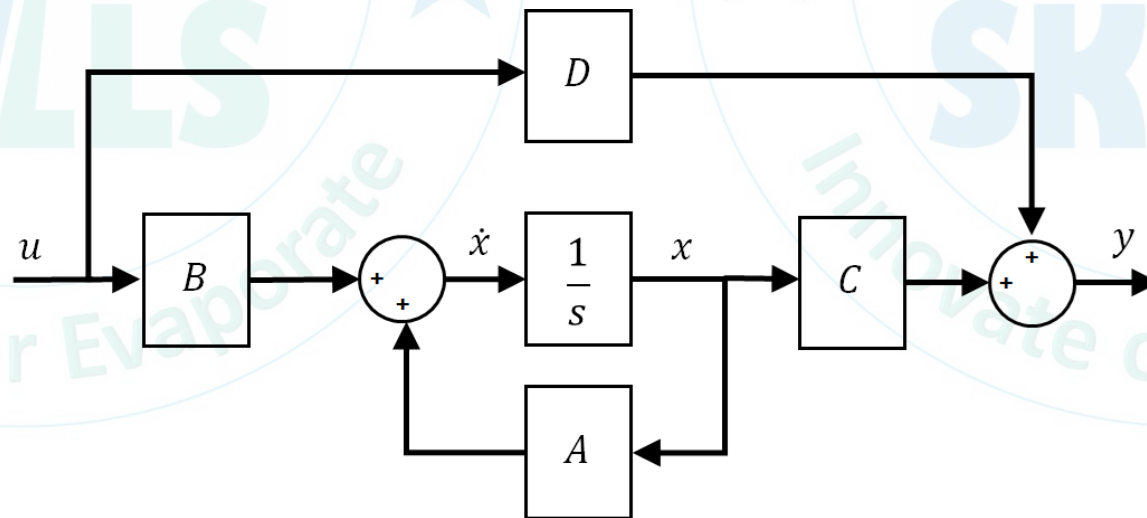
$$\mathbf{C} = [b_0 \quad b_1 \quad b_2 \quad \cdots \quad b_N] \quad \mathbf{D} = 0$$

Controllable canonical form

Differential Equations and State Space Representation

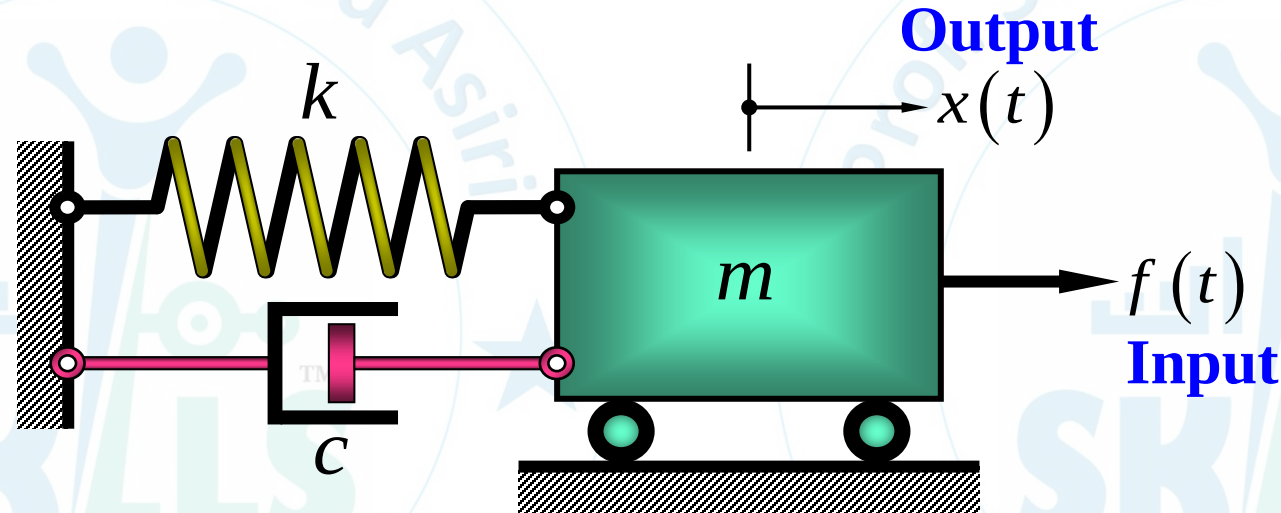
The state equations are:

$$\dot{X} = A X + B u$$
$$y = C X + D u$$



State Space Representation

Example:



$$m \ddot{x} + c \dot{x} + k x = F$$

F = Control force

State Space Representation

Example:

Let $x_1 = x, \quad x_2 = \dot{x}$

$$\underbrace{\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix}}_{\dot{\mathbf{X}}} = \underbrace{\begin{bmatrix} 0 & 1 \\ -k/m & -c/m \end{bmatrix}}_{\mathbf{A}} \underbrace{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}}_{\mathbf{X}} + \underbrace{\begin{bmatrix} 0 \\ 1/m \end{bmatrix}}_{\mathbf{B}} \underbrace{F}_{u}$$

State Space Representation

Example:

$$\dot{\mathbf{X}} = \mathbf{A}\mathbf{X} + \mathbf{B}u$$

where

$$\mathbf{X} = \begin{bmatrix} x \\ \dot{x} \end{bmatrix}, \quad \dot{\mathbf{X}} = \begin{bmatrix} \dot{x} \\ \ddot{x} \end{bmatrix}, \quad \mathbf{A} = \begin{bmatrix} 0 & 1 \\ -k/m & -c/m \end{bmatrix},$$

$$\mathbf{B} = \begin{bmatrix} 0 \\ 1/m \end{bmatrix}, \quad u = F$$

State Space Representation

$$\begin{aligned}\dot{\mathbf{X}} &= \mathbf{A}\mathbf{X} + \mathbf{B}u \\ \mathbf{y} &= \mathbf{C}\mathbf{X} + \mathbf{D}u\end{aligned}$$

where

$\mathbf{X}$ = state vector, $\mathbf{y}$ = Output vector, u = control vector

$\mathbf{A}$ = system matrix, $\mathbf{B}$ = Input matrix

$\mathbf{C}$ = Output matrix, $\mathbf{D}$ = Feedforward Matrix

State Space Representation

Output $y = x$



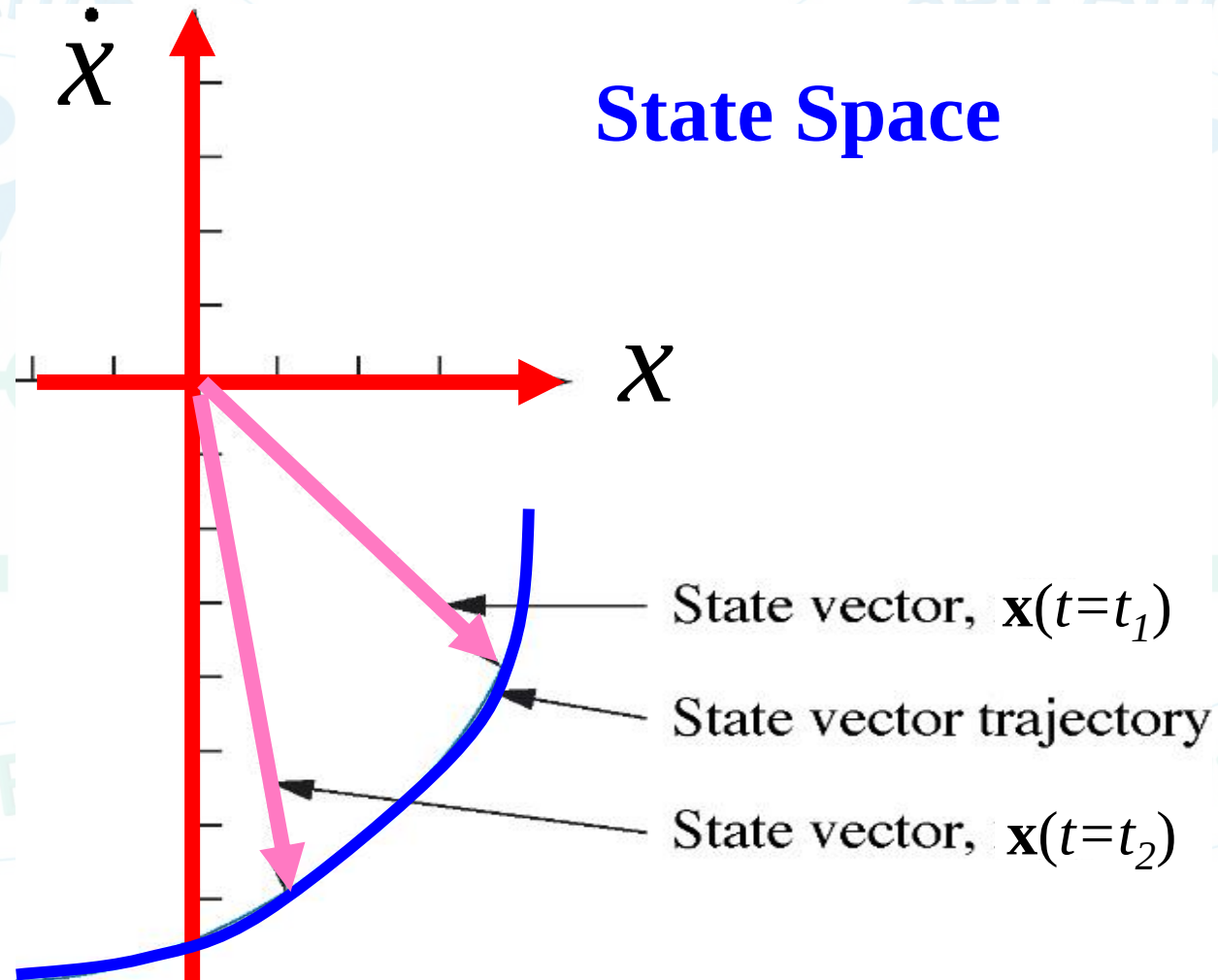
$$y = CX + Du$$

where

$$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, C = [10], D = 0, u = F$$

**Displacement
Sensor**

State Space Representation



State Space Representation

System Equation

$$L \frac{di}{dt} + Ri + \frac{1}{C} \int i dt = v$$

Output Equation

$$v_L = L \frac{di}{dt} = -Ri - \frac{1}{C} \int i dt + v$$

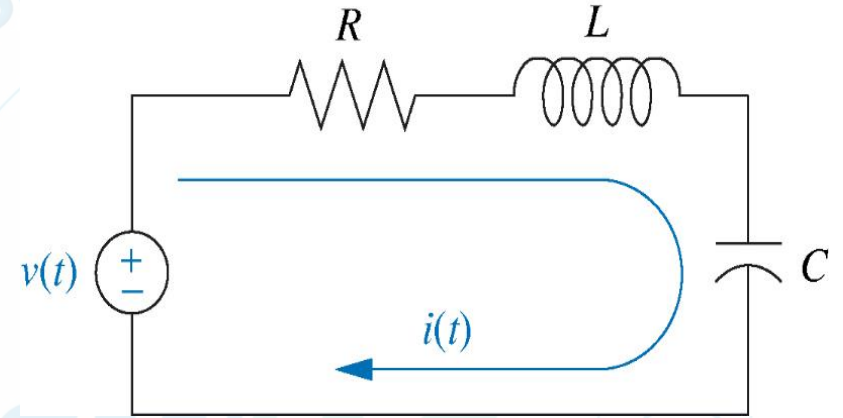
$$L \frac{di}{dt} + Ri + \frac{1}{C} \int i dt = v$$

Let

$$\frac{dq}{dt} = i \Rightarrow L \frac{di}{dt} + Ri + \frac{1}{C} q = v$$

$$L \frac{di}{dt} = -\frac{1}{C} q - Ri + v$$

$$\frac{di}{dt} = -\frac{1}{LC} q - \frac{R}{L} i + \frac{1}{L} v$$



State Space Representation

$$\dot{\mathbf{X}} = \mathbf{A}\mathbf{X} + \mathbf{B}u$$

where

$$\mathbf{X} = \begin{bmatrix} q \\ i \end{bmatrix}, \quad \dot{\mathbf{X}} = \begin{bmatrix} \frac{dq}{dt} \\ \frac{di}{dt} \end{bmatrix}, \quad \mathbf{A} = \begin{bmatrix} 0 & 1 \\ -1/LC & -R/L \end{bmatrix},$$

$$\mathbf{B} = \begin{bmatrix} 0 \\ 1/L \end{bmatrix}, \quad u = v$$

$$\frac{dq}{dt} = i$$

$$\frac{di}{dt} = -\frac{1}{LC}q - \frac{R}{L}i + \frac{1}{L}v$$

State Space Representation

Output Equation

$$v_L = L \frac{di}{dt} = -Ri - \frac{1}{C} \int i dt + v$$

$$v_L = -\frac{1}{C} q - Ri + v$$

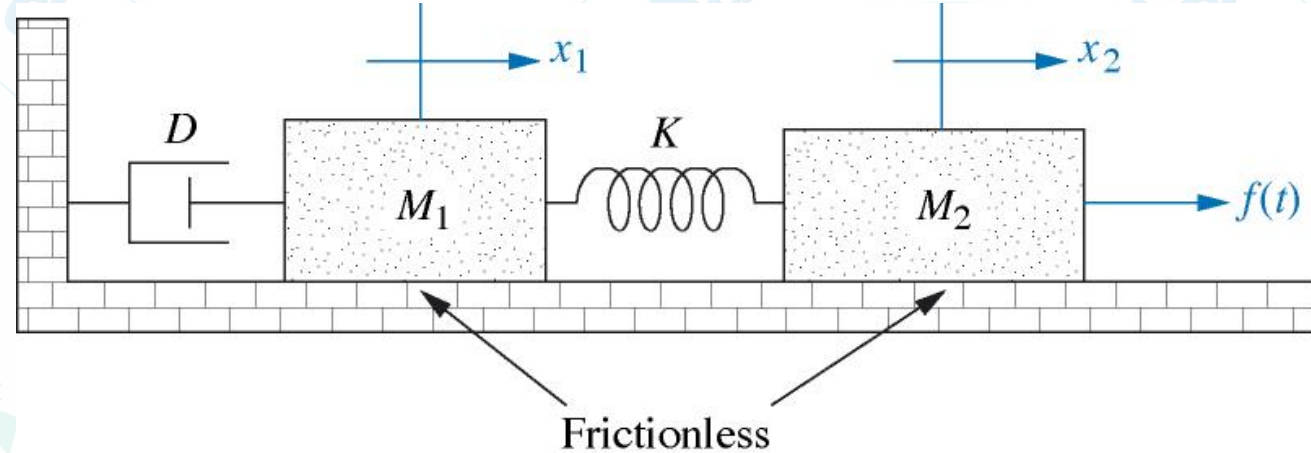
where

$$\mathbf{y} = \mathbf{C}\mathbf{X} + \mathbf{D}u$$

$$\mathbf{X} = \begin{bmatrix} q \\ i \end{bmatrix}, \mathbf{C} = [-1/C - R], \mathbf{D} = 1, u = v$$

State Space Representation

Example:



Equations of Motion

$$M_1 \ddot{x}_1 + D \dot{x}_1 + Kx_1 - Kx_2 = 0$$

$$-Kx_1 + M_2 \ddot{x}_2 + 0 \dot{x}_2 + Kx_2 = f$$

State Space Representation

Example:

Let

$$X = \begin{bmatrix} x_1 & \dot{x}_1 & x_2 & \dot{x}_2 \end{bmatrix}^T$$

$$\begin{bmatrix} \dot{x}_1 \\ \ddot{x}_1 \\ \dot{x}_2 \\ \ddot{x}_2 \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 1 & 0 & 0 \\ -K/M_1 & -D/M_1 & K/M_1 & 0 \\ 0 & 0 & 0 & 1 \\ K/M_2 & 0 & -K/M_2 & 0 \end{bmatrix}}_{\mathbf{A}} \begin{bmatrix} x_1 \\ \dot{x}_1 \\ x_2 \\ \dot{x}_2 \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ 0 \\ 0 \\ 1/M_2 \end{bmatrix}}_{\mathbf{B}} f$$

State Space Representation

Example:

$$y = x_1 = \underbrace{[1 \quad 0 \quad 0 \quad 0]}_{\text{Displacement Sensor}} \begin{bmatrix} x_1 \\ \dot{x}_1 \\ x_2 \\ \dot{x}_2 \end{bmatrix} + 0 \times f$$
$$= C X + D u$$

where $C = [1 \quad 0 \quad 0 \quad 0]$ and $D = 0$

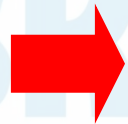
Transfer Function to State Space

System Equation

$$\frac{d^n y}{dt^n} + a_{n-1} \frac{d^{n-1} y}{dt^{n-1}} + \dots + a_1 \frac{dy}{dt} + a_0 y = b_0 u$$

Let

$$x_1 = y, x_2 = \frac{dy}{dt}, x_3 = \frac{d^2 y}{dt^2}, \dots, x_n = \frac{d^{n-1} y}{dt^{n-1}}$$



$$\dot{x}_1 = \frac{dy}{dt}, \dot{x}_2 = \frac{d^2 y}{dt^2}, \dot{x}_3 = \frac{d^3 y}{dt^3}, \dots, \dot{x}_n = \frac{d^n y}{dt^n}$$

or

$$\dot{x}_1 = x_2, \dot{x}_2 = x_3, \dots, \dot{x}_{n-1} = x_n,$$

$$\dot{x}_n = -a_0 x_1 - a_1 x_2 - \dots - a_{n-1} x_n + b_0 u$$

Transfer Function to State Space

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \vdots \\ \dot{x}_{n-1} \\ \dot{x}_n \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & \dots & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & \dots & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & 0 & \dots & 0 \\ -a_0 & -a_1 & -a_2 & -a_3 & -a_4 & -a_5 & \dots & -a_{n-1} \end{bmatrix}}_{\mathbf{A}} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_{n-1} \\ x_n \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ 0 \\ b_0 \end{bmatrix}}_{\mathbf{B}} u$$

$$y = \underbrace{[1 \quad 0 \quad 0 \quad \dots \quad 0]}_{\mathbf{C}} \begin{bmatrix} x_1 & x_2 & x_3 & \dots & x_{n-1} & x_n \end{bmatrix}^T$$

Transfer Function to State Space

Example:

$$\frac{C}{R} = \frac{24}{s^3 + 9s^2 + 26s + 24}$$



$$\ddot{c} + 9\dot{c} + 26c = 24r$$

Let

$$x_1 = c, \quad x_2 = \dot{c}, \quad x_3 = \ddot{c}$$



$$\dot{x}_1 =$$

$$x_2$$

$$\dot{x}_2 =$$

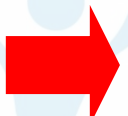
$$x_3$$

$$\dot{x}_3 = -24x_1 - 26x_2 - 9x_3 + 24r,$$

$$y = c = x_1$$

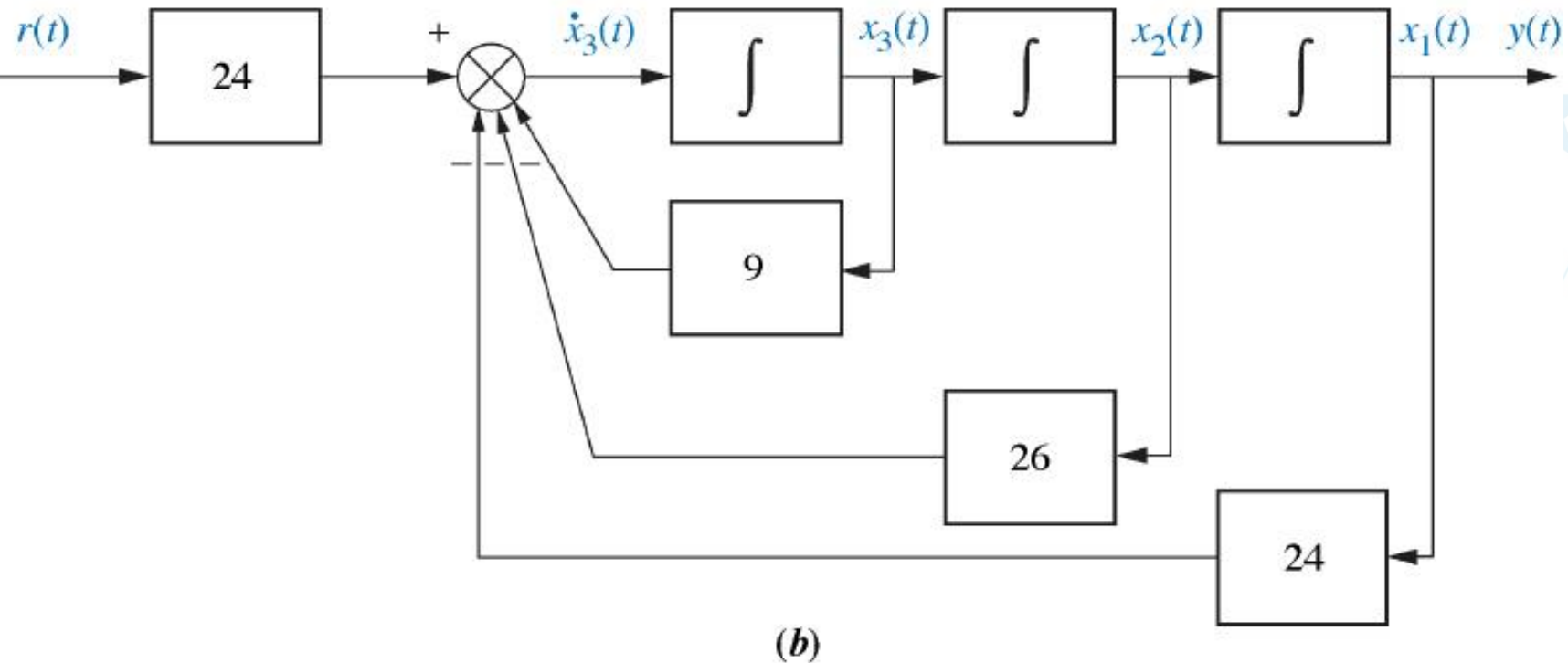
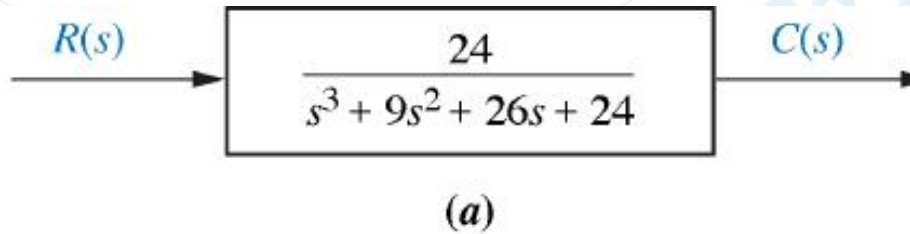
Transfer Function to State Space

Example:


$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -24 & -26 & -9 \end{bmatrix}}_{\mathbf{A}} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ 0 \\ 24 \end{bmatrix}}_{\mathbf{B}} u$$
$$y = \underbrace{[1 \quad 0 \quad 0]}_{\mathbf{C}} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

Transfer Function to State Space

Example:



Transfer Function to State Space

Example:

MATLAB

```
>> n=24;  
>> d=[1 9 26 24];  
>>  
[A,B,C,D]=tf2ss(n,d)
```

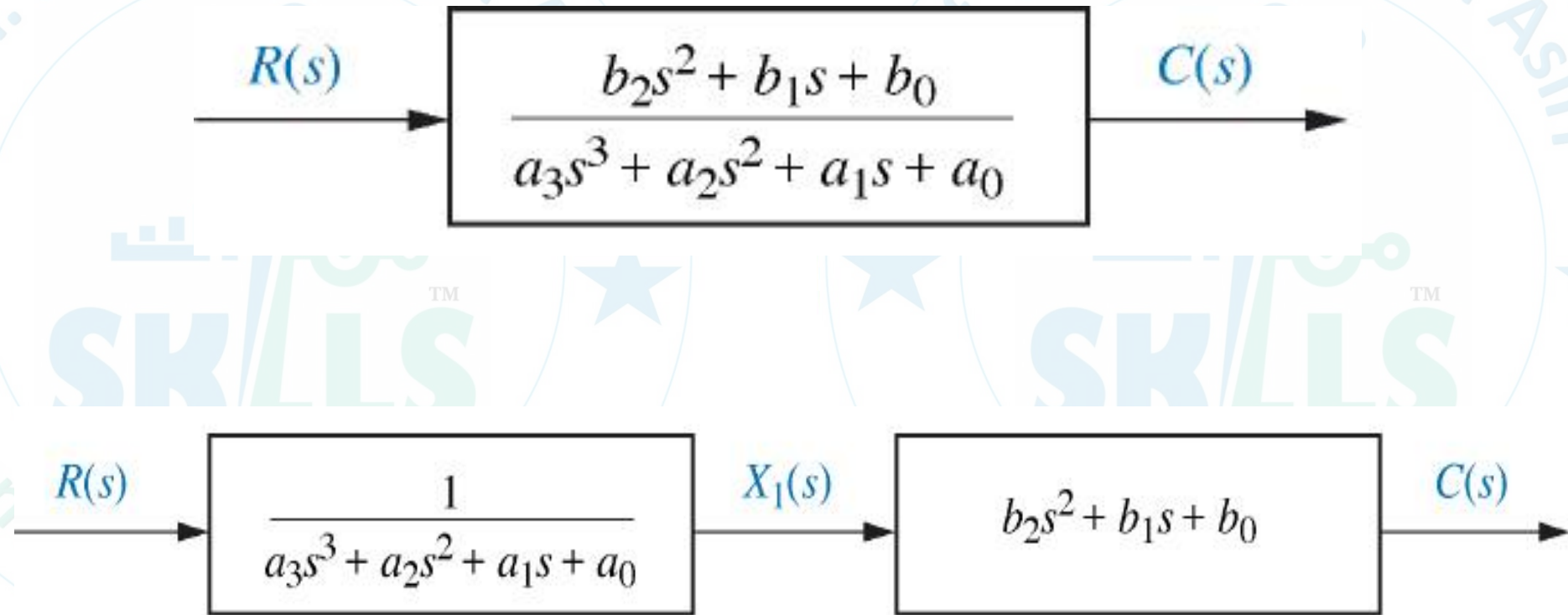
A =
-9 -26 -24
1 0 0
0 1 0

B =
1
0
0

C = 0 0 24

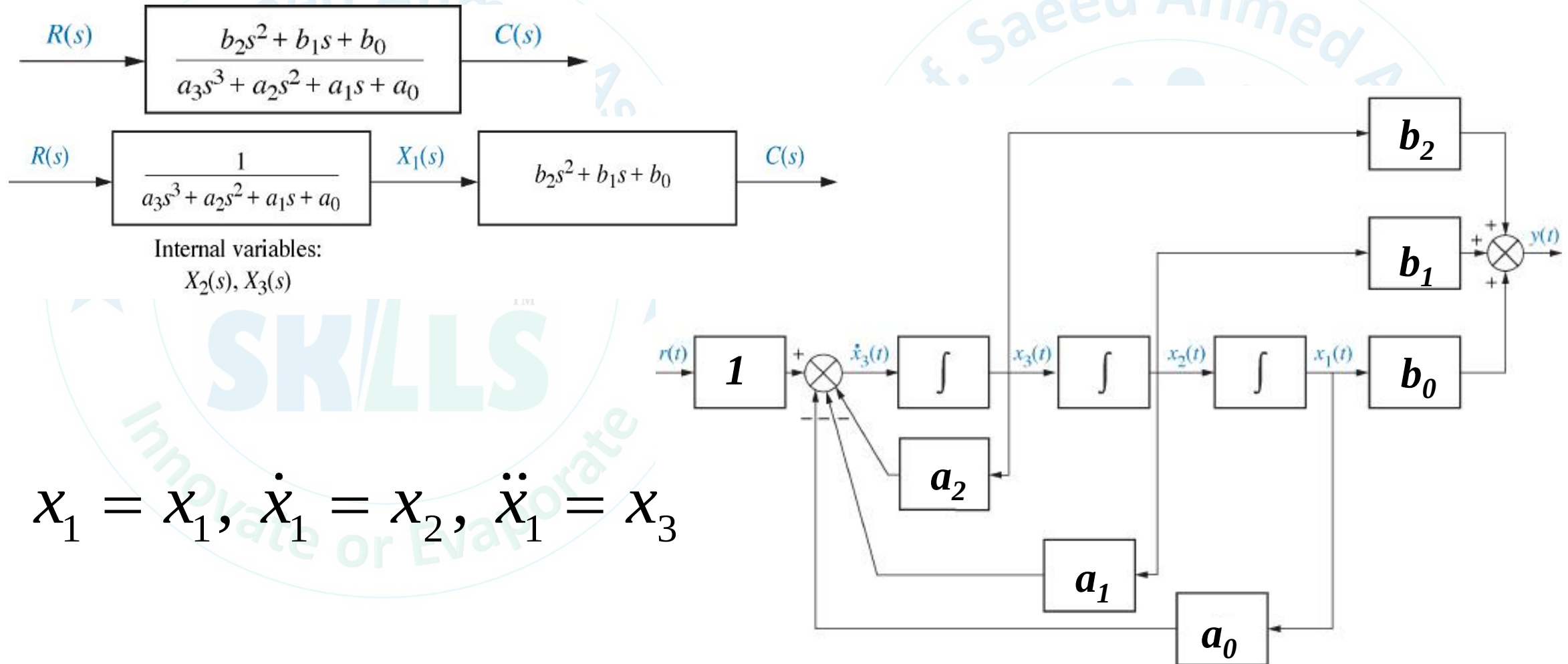
D = 0

T. Fn. with numerator Poly. to S-S

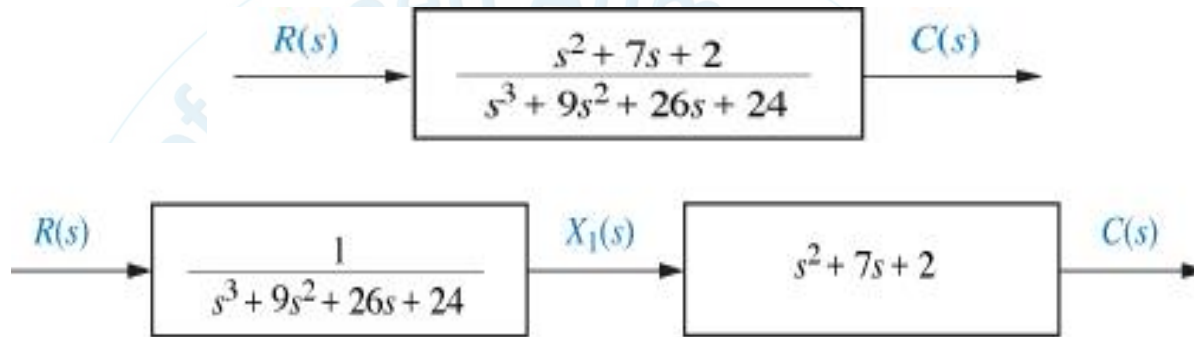


Internal variables:
 $X_2(s), X_3(s)$

T. Fn. with numerator Poly. to S-S



T. Fn. with numerator Poly. to S-S

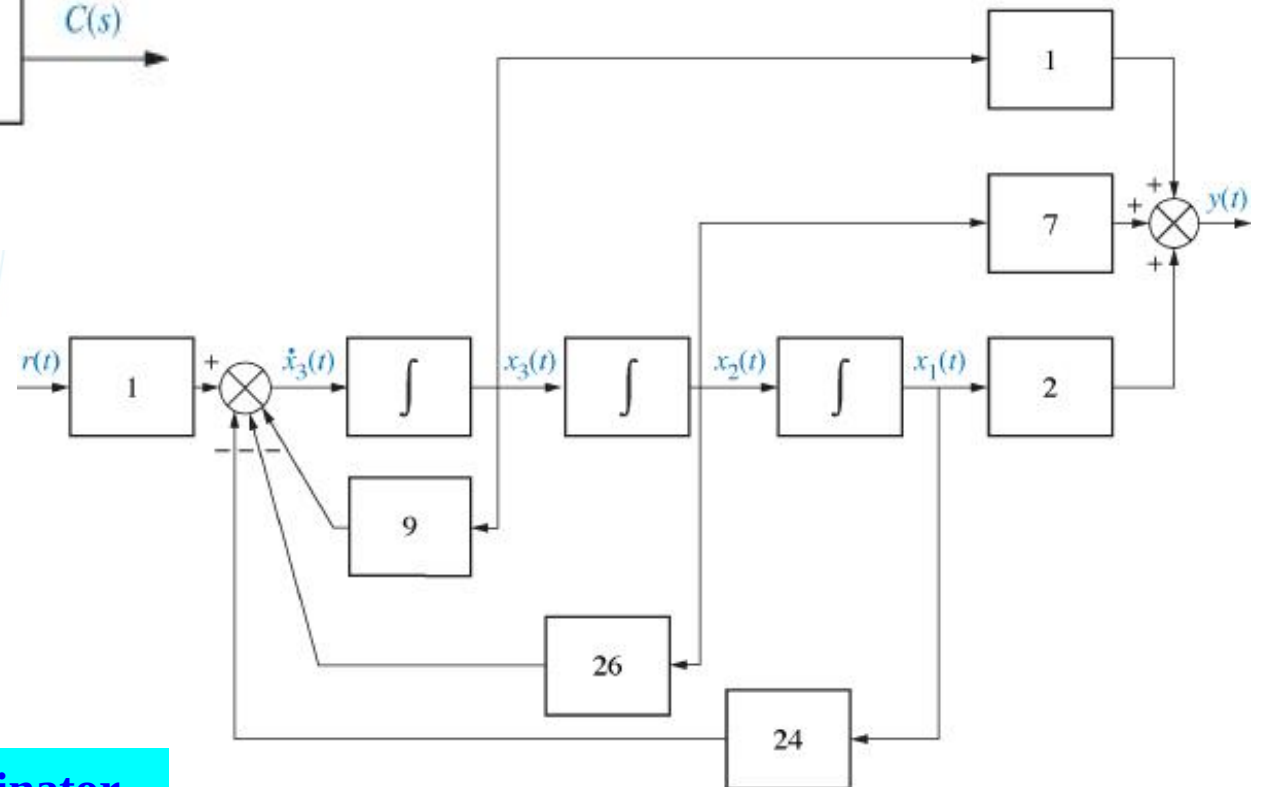


Internal variables:
 $X_2(s), X_3(s)$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -24 & -26 & -9 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u$$

A

B



Same as Transfer function with polynomial denominator

T. Fn. with numerator Poly. to S-S

But $C(s) = (b_2 s^2 + b_1 s + b_0) X_1(s) = (s^2 + 7s + 2) X_1(s)$

→ $c = \ddot{x}_1 + 7\dot{x}_1 + 2x_1$

Let $x_1 = x_1, \dot{x}_1 = x_2, \ddot{x}_1 = x_3$

→ $y = c = x_3 + 7x_2 + 2x_1 = b_2 x_3 + b_1 x_2 + b_0 x_1$

or $y = [b_0 \ b_1 \ b_2][x_1 \ x_2 \ x_3]^T = \underbrace{[2 \ 7 \ 1]}_C [x_1 \ x_2 \ x_3]^T$

T. Fn. with numerator Poly. to S-S

```
>> n=[1 7 2];  
>> d=[1 9 26 24];  
>> [A,B,C,D]=tf2ss(n,d)
```

A =

-9	-26	-24
1	0	0
0	1	0

B =

1
0
0

C =

1	7	2
---	---	---

D =

0

T. Fn. with numerator Poly. to S-S

SUMMARY

- Denominator of Tr. Fn. (**Poles**) affects the **A** matrix
- Numerator of Tr. Fn. (**Zeros**) affects the **C** matrix
- Numerator **gain** affects **B** matrix

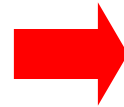
State-Space to Transfer Function

State-Space Model

$$\begin{aligned}\dot{\mathbf{X}} &= \mathbf{A}\mathbf{X} + \mathbf{B}u, \\ \mathbf{y} &= \mathbf{C}\mathbf{X} + \mathbf{D}u\end{aligned}$$

Laplace Transform

$$\begin{aligned}s\mathbf{X} &= \mathbf{A}\mathbf{X} + \mathbf{B}u, \\ \mathbf{y} &= \mathbf{C}\mathbf{X} + \mathbf{D}u\end{aligned}$$



$$\mathbf{X} = (s\mathbf{I} - \mathbf{A})^{-1} \mathbf{B}u$$

or

$$\mathbf{Y}(s) = [\mathbf{C}(s\mathbf{I} - \mathbf{A})^{-1} \mathbf{B} + \mathbf{D}] \mathbf{U}(s)$$

State-Space to Transfer Function

Example

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -2 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 10 \\ 0 \\ 0 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

State-Space to Transfer Function

Example

$$(s\mathbf{I} - \mathbf{A}) = \begin{bmatrix} s & 0 & 0 \\ 0 & s & 0 \\ 0 & 0 & s \end{bmatrix} - \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -2 & -3 \end{bmatrix} = \begin{bmatrix} s & -1 & 0 \\ 0 & s & -1 \\ 1 & 2 & s+3 \end{bmatrix}$$

$$(s\mathbf{I} - \mathbf{A})^{-1} = \frac{\text{adj}(s\mathbf{I} - \mathbf{A})}{\det(s\mathbf{I} - \mathbf{A})} = \frac{\begin{bmatrix} (s^2 + 3s + 2) & (s + 3) & 1 \\ -1 & s(s + 3) & s \\ -s & -(2s + 1) & s^2 \end{bmatrix}}{s^3 + 3s^2 + 2s + 1}$$

$$\mathbf{C} = [1 \ 0 \ 0], \quad \mathbf{D} = 0$$

State-Space to Transfer Function

Example

$$(s\mathbf{I} - \mathbf{A})^{-1} = \frac{\text{adj}(s\mathbf{I} - \mathbf{A})}{\det(s\mathbf{I} - \mathbf{A})} = \frac{\begin{bmatrix} (s^2 + 3s + 2) & (s + 3) & 1 \\ -1 & s(s + 3) & s \\ -s & -(2s + 1) & s^2 \end{bmatrix}}{s^3 + 3s^2 + 2s + 1}$$

$$\mathbf{B} = [10 \ 0 \ 0]^T, \mathbf{C} = [1 \ 0 \ 0], \mathbf{D} = 0$$

$$\mathbf{Y}(s) = \left[\mathbf{C}(s\mathbf{I} - \mathbf{A})^{-1} \mathbf{B} + \mathbf{D} \right] u = \frac{10(s^2 + 3s + 2)}{(s^3 + 3s^2 + 2s + 1)} u$$

Transfer Function

State-Space to Transfer Function

Example

Method 1

```
>> syms s
>> A=[0 1 0;0 0 1;-1 -2, -3];
>> B=[10 0 0]';
>> C=[1 0 0];
>> AS=s*eye(3)-A;
>> ASI=inv(AS);
>> G=C*ASI*B
```

$$G = 10*(s^2+3*s+2)/(s^3+3*s^2+2*s+1)$$

State-Space to Transfer Function

Example

Method 2

```
>> A=[0 1 0;0 0 1;-1 -2, -3];
```

```
>> B=[10 0 0]';
```

```
>> C=[1 0 0];
```

```
>> D=0;
```

```
>> [n,d]=ss2tf(A,B,C,D)
```

```
n =      0    10.0000    30.0000    20.0000
```

```
d =     1.0000     3.0000     2.0000     1.0000
```

```
>> G=tf(n,d)
```

Transfer function: $\frac{10 s^2 + 30 s + 20}{s^3 + 3 s^2 + 2 s + 1}$

END